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Résumé

Le contexte actuel et récurrent des taux bas incite les assureurs-vie à mieux prendre en compte les risques auxquels ils sont durablement exposés. En effet les contraintes liées au secteur de l'assurance vie sont nombreuses. Elles sont notamment réglementaires via par exemple le besoin en capital de Solvabilité II ou le taux de minimum de participation aux bénéfices ; ainsi que contractuelles (ex : le taux minimum garanti par l'assureur à ses assurés).

La bonne gestion de l'actif pour un assureur est un enjeu majeur. Il est notamment important, du fait de la complexité et de la durée des contrats d'assurance, de correctement modéliser les interactions actif et passif afin de représenter au mieux la réalité dans les modèles de projection.

Le contexte économique actuel contraint les assureurs à prendre en compte les taux faibles voire négatifs que la zone euro connait aujourd'hui, notamment car certains contrats d'assurance garantissent un taux minimum garanti positif qui pénalise énormément les assureurs lorsque les taux deviennent négatifs. Les assureurs recherchent ainsi le meilleur compromis pour obtenir un rendement élevé tout en maitrisant les risques auxquels ils sont sensibles.
La première étude de ce mémoire permettra, à l'aide d'un cas simplifié,[1] de mesurer les impacts de différentes allocations de portefeuilles d'actifs sur le capital économique réglementaire et la rentabilité d'un assureur-vie pour un produit d'épargne Euro. Cette étude révélera notamment que les indicateurs habituellement suivis tels que le profit ne permettront pas à eux seuls d'expliquer les résultats d'une telle étude dans un contexte de taux négatifs.

Dans un second temps, l'étude permettra de montrer l'intérêt de la gestion des portefeuilles d'actifs au sein des modèles de projection, notamment par l'implémentation d'une nouvelle stratégie de réallocation d'actifs plus alignée avec les pratiques observées sur le marché. Cette nouvelle stratégie sera comparée à la stratégie initialement implémentée permettant ainsi de mesurer les impacts de ce type de changements dans un modèle actif-passif. Les résultats obtenus seront étudiés et le paramétrage de cette stratégie fera l'objet d'une optimisation prenant en considération les indicateurs réglementaires usuels tel que le SCR. Une attention particulière sera prêtée aux provisions et notamment à la PRE (Provision pour Risque d'Exigibilité) pour expliquer les résultats obtenus. L'étude soulignera l'importance d'un paramétrage recalibré régulièrement et propre aux spécificités de chaque assureur.

[1] On considérera dans cette étude le cas simplifié d'un assureur vie avec un portefeuille fictif.

Abstract

The current and recurring context of low rates drives life insurers to take better account of the risks to which they are durably exposed. Indeed, there are many constraints inherent to life insurance sector. They are mainly regulatory with for example the minimum rate of profit sharing or the Solvency II capital requirement but also contractual (e.g. the minimum rate guaranteed by the insurer to its policyholders).

A good knowledge of asset management and a better modelling of the interactions between assets and liabilities aligned with reality are seen as powerful levers for life insurers given the complexity and the extended durations of the business.

The current economic context forces insurers to deeply consider low or even negative rates when making strategic decisions, especially since some insurance products guarantee a positive minimum guaranteed rate enormously penalising insurers when rates become negative. Insurers are therefore looking for the best compromise to obtain a high return while controlling the risks to which they are exposed.
The first study of this report will measure, through a simplified case,[2] the impact of different asset portfolio allocations on the economic capital and profitability of a life insurer for a Euro fund product. In particular, this study will reveal the usually monitored indicators, such as the profit and the SCR, are not sufficient to explain the results. Especially since the study is conducted in a context of negative rates.

The second part of the report will show the interest of asset portfolio management within projection models, particularly through the implementation of a new asset allocation strategy, more aligned with the practices observed on the market. This new strategy will be compared to the strategy initially implemented and gives the opportunity to measure impacts such changes can have on the expected results. The results will be challenged and the configuration optimized considering the usual indicators such as the SCR. The insurer's provisions such as the liquidity risk reserve will also be studied. This study outlines the importance of a regular and insurer-specific calibration in the management of more complex assets modelling.

[2] In this study, a simplified case of a life insurer with a fictional portfolio will be considered.

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Introduction

The current and recurrent context of low rates encourages life insurers to take better account of the risks to which they are durably exposed. Indeed, there are many constraints linked to life insurance sector.

Life insurance has always been the favourite investment for French people. Due to the available guarantees offered by Euro funds products, this type of investment still attracts in 2020 and therefore drives the insurer to seek for better financial returns on the market while always mastering his exposure to risk.

This situation leads insurers to focus on their asset and liability management, indeed asset portfolio management in projection models is a powerful tool for life insurers.

The purpose of this report is to discuss the impacts asset and liability modelling in a low interest rates environment has on insurers' solvency and profitability.

It must be mentioned that the ALM model used for this study is only used for studies purposes. Therefore, due to a lack of information on its existing configuration, it required time to be apprehended for this study. This time should be considered when analysing the results of the following studies.

This report is divided into five parts. The two first parts carry out an introduction to life insurance as well as a presentation of the current economic context and the regulatory framework. The third part gives the reader a detailed insight of the ALM model used for this study (inputs, provision calculation, validation tests performed). Then the fourth part presents the study conducted to find the optimal portfolio allocation in a low interest rate environment. Finally, the last part is devoted to the presentation of the modelling of the new implemented asset allocation strategy. For the studies discussed in chapters 4 and 5, the impacts on the economic capital and profitability will be detailed.

Chapter 1

Regulation context - Solvency II

Directive 2009/138/EC of the European Parliament, known as Solvency II, is an extension of the Basel III reform for the insurance sector and deeply modifies the existing regime. It became effective on January 1, 2016.
Unlike the previous Directive, Solvency I, Solvency II encourages insurance organizations to better understand and assess their risks, by adapting regulatory requirements to the risks that companies incur in their business. To do so, the new standards require insurers to take a forward-looking approach to their business.
Solvency II consists of 3 pillars. The first one focuses on quantitative requirements, the second on governance and risk management requirement and finally the third on the principle of communication and transparency required for insurers concerning their diffusion of public information.

1.1 Solvency II's three pillars

1.1.1 Pillar 1: Quantitative requirements on Own Funds and provisions

Pillar 1 deals with the capital requirements that apply to insurance companies and the definition of technical standards for calculating the Own Funds and related technical provisions. Theses quantitative requirements are reflected through the calculation of the MCR (Minimal Capital Requirement) and the SCR (Solvency Capital Requirement).

The SCR is the first level of capital enabling an insurance or reinsurance company to absorb significant losses. It is equivalent to the amount an insurer must have in order to limit the risk of ruin to 0.5% within 1 year. It allows policyholders to have reasonable confidence in the insurer's ability to meet its commitments.
Its calculation can be made following four techniques:

- The standard formula with parameters given by the EIOPA, the regulator will provide an accurate methodology to calculate the SCR which will be detailed in section 1.3.
- The standard formula with the company's own parameters (USP). The application of Undertaking Specific Parameters offers an alternative to companies not having the possibility to build an internal model and who consider that the standard formula overestimates their SCR. Companies with sufficient historical data on their past experience can apply USPs so that their capital requirements are calculated using their own assumptions and not those of the market.
- The use of an internal model, under certain conditions, the legislator will leave some freedom to the company to determine its regulatory capital.
- The use of a partial model where only a part of the risks is estimated with an internal model.

The MCR represents the second level of capital, which is the amount of eligible capital below which the insurance or reinsurance company is exposed to an unacceptable risk in continuing its activity. Therefore, when the available capital is below the MCR, the insurance company is forbidden to practice its activity.

The level of the minimum capital requirement should be adjusted to the real risks to which insurers are exposed. Therefore, riskier are the assets or liabilities held by the organizations, higher will be the corresponding minimum capital requirement.

1.1.2 Pillar 2: Risk's governance and qualitative requirements

Defined by Article 45 of the Solvency II Directive, Pillar 2 brings together the qualitative requirements (governance rules and risk management) and the Own Risk and Solvency Assessment (ORSA).
The ORSA provides a global view of the company, its profitability, solvency and medium-term risk management objectives, unlike SCR which is calculated on a one-year horizon. This last point is important and makes it possible to compensate for a limitation of the standard formula by considering short-term risks and including the claims experience for a number of business lines. The specific features of each company are thus better represented.
The ORSA carries a strategic aspect placing it as a real steering tool for the management. The ORSA exercise is an opportunity for every company to instil the risk management culture and ensure the transparency of strategic ambitions and the associated risks.
In order to restructure these objectives, the regulatory texts explain a certain number of ORSA requirements. There are three requirements given as follows:

– Assessment of the Overall Solvency Needs

– Assessment of continuous compliance with the regulatory requirements concerning the coverage of the SCR and the MCR and the requirements concerning the calculation of technical provisions

– Estimation of the gap between the company's risk profile and the SCR calculation assumptions

The **Overall Solvency Needs** measure the insurer's future capital requirements, in order to ensure that the company is solvable and that all the resources required to implement the Business Plan are consistent with its risk profile and risk appetite. It must consider changes in the risk profile and the quality and quantity of capital. It is necessary to monitor the situation in the event of a crisis by performing stress tests. This assessment also allows for an evaluation of the risks that are not taken into consideration in the SCR calculation, but to which the company may be sensitive.

Continuous compliance with capital requirements and technical provisions enables companies to ensure ongoing compliance with capital requirements such as the SCR and MCR and the requirements on technical provisions. Therefore, companies must adopt a forward-looking view and be able to test the resilience of their economic balance sheet in case of unfavourable changes arising from risk factors (level of rates, spreads, real estate, etc.). These tests, known as stress tests, allow for an analysis of the resilience of the balance sheet and can be instantaneous or long-term forecasting. When unfavourable stresses are applied, the behaviour of the company indicators should be assessed.

The **deviation of the risk profile** consists in measuring the gap between the insurer's risk profile and the assumptions underlying the SCR calculation. It also includes mapping the risks to which the company is sensitive. Pillar 2 takes notice of a wider range of risks than those covered in Pillar 1.

Companies must be able to assess whether their risk profile is well represented in the assumptions made for the SCR calculation by the regulator, in particular through the "fit with the standard formula" study.

1.1.3 Pillar 3: Communication and transparency

Pilar 3 focuses on the financial communication provided by the insurers, through prudential reporting to supervisory authorities, shareholders or general public.
The publication includes annual and quarterly quantitative statements (QRT29 - Quantitative Reporting Templates) as well as annual narrative reports such as the SFCR (Solvency and Financial Conditions Report), the RSR (Regulator Supervisory Report), the ORSA report and the actuarial report.
Some of these reports are public (example the SFCR and QRT) and others are only communicated to local regulators (RSR or actuarial report for example).

1.2 Prudential balance sheet

The rules for the valuation of assets and liabilities within the framework of the Solvency II Directive are detailed in Article 64 of the Directive. The regulation requires an economic view of the balance sheet, including the unrealized gains and losses and the valuation of the assets using market prices.
The diagram below shows the composition of the economic balance sheet of an insurance company:

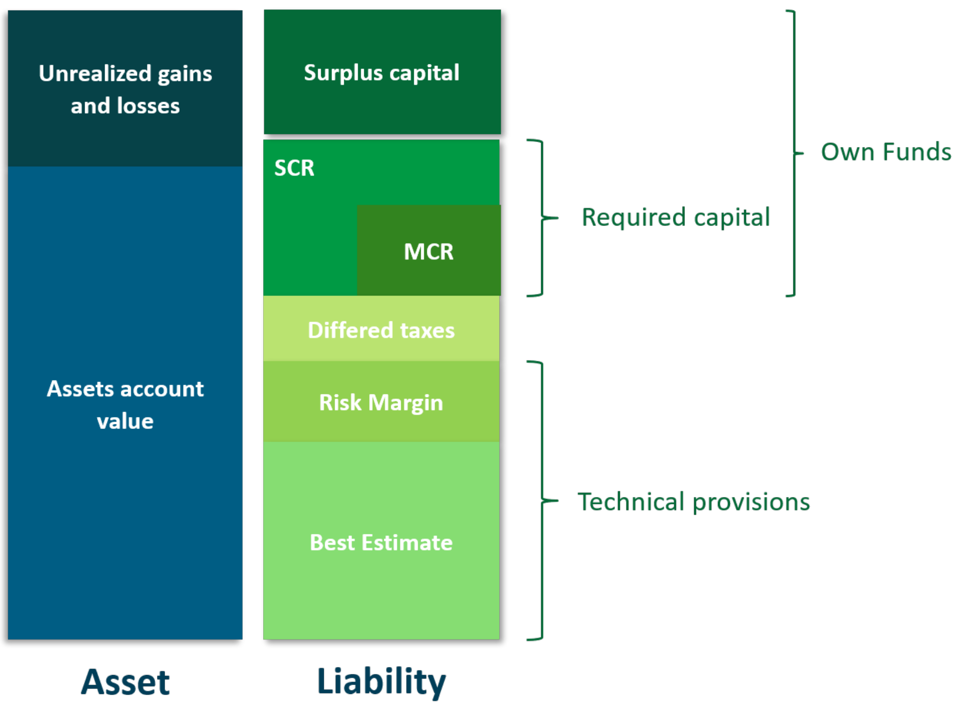


Figure 1.1: Simplified economic SII balance sheet

- Technical provisions are calculated as the sum of a "Best Estimate of Liabilities" (BEL) and a Risk Margin (RM). The Best Estimate is the fairest assessment of the economic value of the business. Its detailed calculation is given in the following subsection.
- Net Asset Value (NAV) is defined as the difference between the market value of the insurer's assets and liabilities without the risk margin of technical provisions and subordinated debts.
- Own Funds are calculated as the difference between assets and liabilities, calculated at fair value.

$$\text{Own Funds} = \text{MV} - (\text{BEL} + \text{RM} + \text{Deferred Taxes})$$

where MV represents the market value of the assets.

Economic Own Funds can also be seen as the sum of the Solvency Capital Requirement and the surplus capital, also known as free Own Funds.

1.2.1 Best Estimate of Liabilities (BEL)

In life insurance, the economic balance sheet must be balanced, and it is necessary to make an asset/liability adjustment represented by the Best Estimate of Liabilities. The aim is to measure the insurer's liabilities at the valuation date of the assets expected on average.
The average BEL is calculated for all assets by plotting a number of asset trajectories (i.e. 1000 or more) according to theoretical scenarios for which the BEL will be given by the average of the 1000 [1] BEL calculated.
The BEL is the present value of expected future cash flows, discounted using a risk-free yield curve, it is thus the average of the discounted sum of the insurer's future engagements. The liabilities are obtained by calculating the difference between the projected outflows and inflows according to the assumptions calibrated in the model.

$$BEL = \sum_{i=1}^{1000} \sum_{t \geq 0} \frac{CF_t^i}{(1+r_t)^t} \text{ where i the i}^{th} \text{ simulation}$$

The fair value measurement of the insurer's commitments is based on the following principles:

- A projection of all cash flows related to the portfolio's products from the valuation date to the date of extinction of all risks related to the products. These projections consider the behaviour of policyholders (surrenders, arbitrages, mortality experience, etc.) and management actions (investment strategy, profit-sharing distribution policy, etc.).
- Discounting of the relevant risk-free yield curve.

The cash-flows used in the calculation of the best estimate are the flows associated with existing products in the portfolio over the projection period. New business is therefore not considered in the calculation of the BEL.

1.2.2 Risk Margin and Cost of Capital

The risk margin is used to cover the risk related to the uncertainties associated with the best estimate valuation of provisions. It represents the cost of capitalizing the capital covering the insurer's commitments.
It can be calculated with the following formula using the "Cost of Capital" method [2] :

$$RM = CoC \sum_{t \geq 0} \frac{SCR_t}{(1+r_t)^{t+1}}$$

Where:

- SCR_t: SCR at date t, valuated with projections
- CoC: the Cost of Capital
- r_t: the risk-free rate at date t

These assumptions assume that the portfolio has been disposed of, and that the assets have been cleared and reinvested in risk-free securities.
The Cost of Capital (CoC) rate is set at 6% to reflect the cost of holding an eligible amount of capital for an insurance or reinsurance company. It is identical for all companies.
This rate was chosen to represent the additional rate a BBB-rated company would have to bear to raise eligible equity capital compared to the risk-free rate.

[1] In the following studies, the number of asset trajectories simulated is 1000.
[2] Article 34 - Commission Delegated Regulation

1.2.3 European Embedded Value

The purpose of the European Embedded Value (EEV), also known as the Market Consistency Value (MCEV), is to provide a fair representation of the value of a life insurance company through shareholders' point of view.
The EEV is considered to be the sum of the revalued net assets and the Value of Inforce (VIF) of the company's portfolio.

$$\text{EEV} = \text{VIF} + \text{NAV}$$

Also, from a theoretical point of view, the available Own Funds resulting from the SII prudential balance sheet correspond very closely to the company's MCEV. The result of this reasoning is due to an equivalence between the two approaches, which sometimes makes it possible to estimate the intrinsic value of the company via a solvency ratio calculation.

Focus on the Value of In Force

The VIF is obtained by calculating the Present Value of Future Profit (PVFP) after taking into account the cost of capital, the cost of unhedgeable risks and the cost of options and guarantees.

$$\text{VIF} = \text{PVFP -TVOG-CoC}$$

where:

- PVFP, is the present value of cash-flows produced by the portfolio for the shareholders. Financial income is equal to the risk-free market rate at the date of calculation. The technical assumptions (redemption rate, amount of expenses, etc.) are established according to the best possible assessment and take into account the contractual terms of the products, regulations and the company's management policy. These future results are discounted at the valuation date using a risk-free market rate curve that is consistent with the financial income projected in the assets.

- TVOG, Time of Value of Options and Guarantees, is an indicator to estimate guarantees and options given to life-insurance products. These commitments directly depend on the characteristics of the products. Therefore, its valuation can be complex if there are multiple options and guarantees.
  TVOG reflects the value of the uncertainty of the duties taken by the life-insurance company towards its policyholders. The main guarantee in terms of value for euro fund products is the minimum guaranteed rate.
  Stochastic modelling is used to estimate the TVOG. Indeed, it is the difference between the stochastic BEL and the deterministic BEL.

- CoC, Cost of Capital is detailed in the previous section

1.3 SCR calculation: standard formula

The standard formula approach is based on an economic valuation of assets and liabilities and on assumptions and calculation methods fixed in Solvency II Directive. Insurance companies are required to apply marginal shocks leading to a ruin situation within 1-year with 0,5% probability. The architecture retained by the standard formula for the calculation of the SCR is as follows:

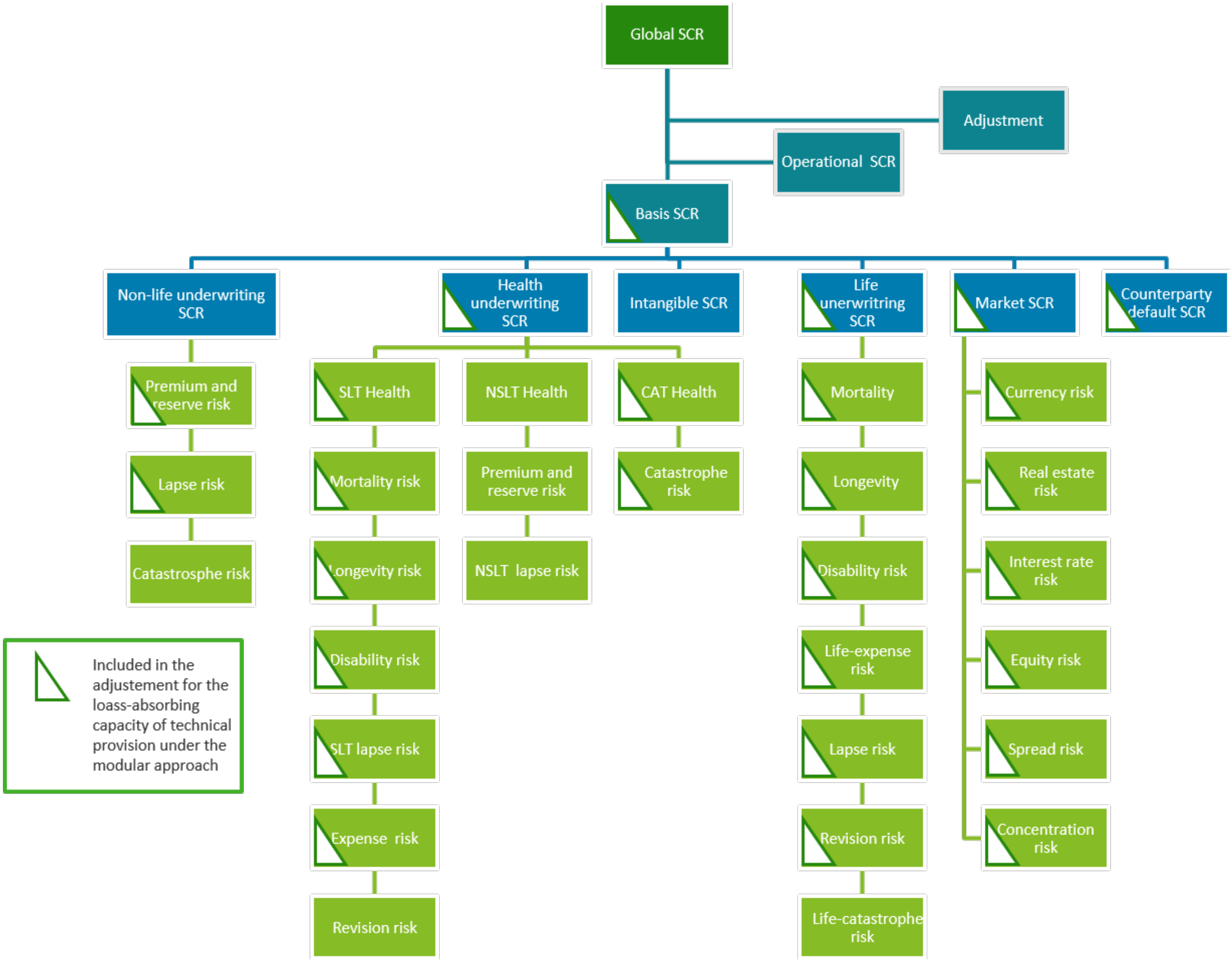


Figure 1.2: Risk modules defined by the EIOPA

The overall SCR for the entity is obtained using the following formula:

$$\text{SCR} = \text{BSCR} - \text{Adj} + SCR_{Op}$$

Where:

– The BSCR[3] corresponds to the economic capital before integrating the operational risk and shall be equal to the following:

$$BSCR = \sqrt{\sum_{i,j} Corr_{ij} \text{ x } SCR_i \text{ x } SCR_j} + SCR_{intangible}$$

where SCR_i denotes the risk module i and where "i,j" means that the sum of the different terms should cover all possible combinations of i and j related risk modules. The intangible risk SCR is the required capital for risk related to intangible assets.

– The Adjustment module enables an adjustment of the SCR considering the loss-absorption capacity of technical provisions and deferred taxes. This process is detailed in section 1.3.1.

– SCR_{Op} is the capital requirement for operational risk.

[3] Basic Solvency Capital Requirement

For the calculation of the gross SCRs, cash flows relating to the participation in future profits remain the same as in the central scenario. In the event of stress on the yield curve, these same cash flows must be discounted using the unshocked yield curve.

There are two approaches to calculate the SCR for modules and sub-modules risks. The first is to use closed formulas, while the second is based on the valuation of the variation of the net asset value (Net Asset Value or shareholders' Own economic Funds) of the company between the unshocked and shocked scenarios.

We chose the use of NAV (Net Asset Value) variation approach. The amount of capital required is given by the difference between the NAV of the central scenario and the NAV of the shocked scenario:

$$SCR_{simplerisk} = \Delta NAV = NAV_{beforeshock} - NAV_{aftershock}$$

Once the required capital for each of the risks has been estimated and detailed, the standard formula requires to aggregate these using a correlation matrix. The idea of this aggregation is to consider that certain risks are correlated. Thus, to consider the simple sum of all these risks would be a strong overestimation of the risk inherent to the firm. The correlation matrix applied for the standard formula is provided by EIOPA.

	Market SCR	Default SCR	Life SCR	Health SCR	Non-Life SCR
Market SCR	1	0.25	0.25	0.25	0.25
Default SCR	0.25	1	0.25	0.25	0.5
Life SCR	0.25	0.25	1	0.25	0
Health SCR	0.25	0.25	0.25	1	0
Non-Life SCR	0.25	0.5	0	0	1

Table 1.1: Correlation matrix defined by the EIOPA

This decomposition of the SCR is based on three strong assumptions:

- The correlation coefficients used to aggregate risks in order to obtain an overall risk value are considered constant whatever the economic scenario. These correlation coefficients were calibrated on extreme economic scenarios.
- The SCR covers all quantifiable risks for the next 12 months. This is obviously an approximation as some risks are not taken into account in this figure 1.1.
- The SCR must be representative of the capital required to deal with a risk that theoretically occurs once every 200 years (99.5% confidence level). Therefore, the calculation of the capital requirement for each sub module is done at this same level of risk.

1.3.1 Absorption of the capital requirement by technical provisions and taxes

The notion of "adjustments" presented previously is essential: it reflects the loss-absorption capacity of technical provisions on the one hand (via the leverage of profit-sharing) and deferred taxes on the other hand. Indeed, in the event of a loss, it's a way for the insurer to pass on part of the losses to the insured and the state. Section 9 of Solvency II Directive and more specifically Articles 206 and 207 detail the calculations.
The following graph illustrates this general mechanism in the case of a fall of the equity market.

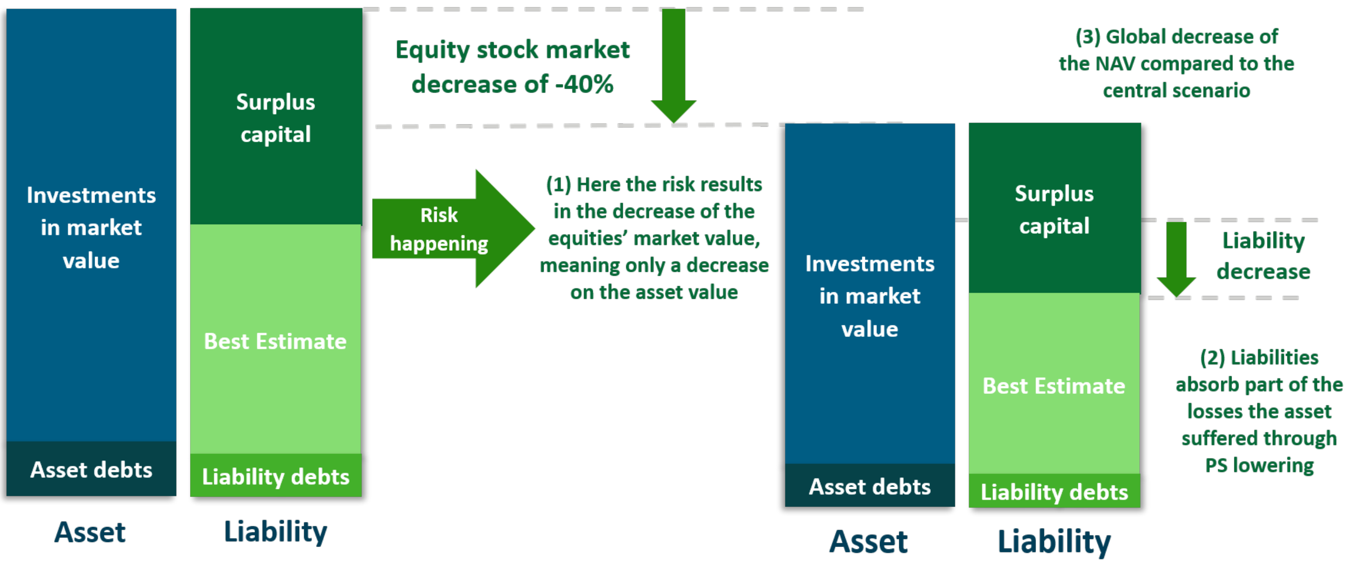


Figure 1.3: Absorption through profit-sharing (PS) example

Indeed, the policyholder bears part of the shock by having a reduced profit-sharing. This mechanism is only correct for the Euro fund. The adjustment is composed of two elements:

$$\text{Adjustment} = \text{Adjustment}_{PS} + \text{Adjustment}_{DT}$$

Adjustment_{PS} = adjustment through the loss-absorption by profit-sharing
Adjustment_{DT} = adjustment through the loss absorption of deferred taxes

a) **Absorption through profit-sharing**

The capacity of the insurer to share his loss with the policyholders comes from his ability to manage the profit-sharing distributed each year. If, in theory, this leverage allows the insurer to share a loss with the insured, this capacity to share losses is limited by the presence of technical rates, guaranteed minimum rate or even the regulatory profit-sharing mechanism.

Under the standard formula, the SCR calculation includes the mechanism of loss-absorption by the profit-sharing. The SCR is calculated gross and net of the loss absorption effect for the risks identified by a green triangle in figure 1.2.
Under the standard formula, where the impact on the insurer's Own Funds of the different risk factors are measured independently (see the modular structure of the presented above), it is necessary to ensure that the absorption by the provisions capital requirement resulting from the aggregation of the modular calculations does not exceed a predefined limit.

b) **Absorption through differed taxes**

The deferred tax adjustment is used to represent the absorption capacity of the company's losses through taxes.

The tax absorption adjustment follows the same logic. When a company realizes a loss, it can give rise to a tax credit. Then, it can be considered that this loss will only impact the insurer's Own Funds to the extent of this loss amount net of taxes. In this case, a capital requirement can be partially absorbed by the tax by applying to the SCR the tax rate to which the company is subject. However, in order to be able to justify such a capacity of tax absorption, the insurer must show that the tax credit generated by the loss can be attributed to future taxes. In other words, it should be justified that the loss can be used to generate a tax saving for the insurer.

1.4 Market risk

The Solvency II Directive defines market risk as the risk of loss, or of adverse change in the insurer's financial situation, resulting directly or indirectly from fluctuations observed in market values (or prices) of financial instruments in the asset portfolio.
Market risk encompasses all risks inherent to the life insurer's financial assets. In euro funds products, the insurer collects premiums and guarantees capital to policyholders. In the event of a loss, the policyholders' capital will be returned to them in its whole as well as the interest generated by this capital, it is therefore necessary for the insurer to turn to financial markets to ensure his capacity to fulfil his commitments.
This risk can be divided into several risk sub-modules. These modules will be aggregated by a correlation matrix. Market risk is composed of the following sub-categories risks (as defined in the Delegated Acts[7]):

- Interest rate risk
- Equity risk
- Property risk
- Spread risk
- Concentration risk
- Currency risk

Each module is associated with a shock or extreme scenario that is applied to both the assets and liabilities of the portfolio. The value of the module will be the difference between the shocked value and the unshocked value of the NAV as previously explained.
Two different correlation matrices are used for the aggregation of the different elementary risks.
An aggregation for each interest rate shock is performed using the correlation matrix corresponding to the shock. The market SCR [13] then corresponds to the maximum of the market SCR between the values associated with the interest rate fall and the interest rate rise.

	Interest Rate	Equity	Property	Spread	Change	Concentration
Interest Rate	1	0/0.5	0/0.5	0/0.5	0.25	0
Equity	0/0.5	1	0.75	0.75	0.25	0
Property	0/0.5	0.75	1	0.5	0.25	0
Spread	0/0.5	0.75	0.5	1	0.25	0
Change	0.25	0.25	0.25	0.25	1	0
Concentration	0	0	0	0	0	1

Table 1.2: Correlation matrix for the market SCR

1.4.1 Interest rate risk

According to Solvency II Directive, this module aims at quantifying the capital requirement needed to cope with the impact of a change in the structure of the yield curve (upwards or downwards) on the value of the balance sheet. This shock contains two inner shocks: an upward shock and a downward shock. Indeed, the variation of interest rates can be harmful in both directions. A rise in interest rates causes a loss in value of bonds already in the portfolio because their yield is lower than the yield of new bonds. A fall in interest rates is certainly an advantage compared to the yield of new bonds but constitutes a loss of real return when the assets are reinvested.
The shocks, up and down, apply to all financial products based on interest rates (simple rate products, derivatives, structured products, etc.). These shocks also change the associated discount rates.

The value of the shocks applied to each point of the curve depends on the maturity. The table below details the two stress scenarios.

Maturity	Upward shock	Downward shock
⩽ 1 year	70%	-75%
2	70%	-65%
3	64%	-56%
4	59%	-50%
5	55%	-46%
6	52%	-42%
7	49%	-39%
8	47%	-36%
9	44%	33%
10	42%	-31%
11	39%	-30%
12	37%	-29%
13	35%	-28%
14	34%	-28%
15	33%	-27%
16	31%	-28%
17	30%	-28%
18	29%	-28%
19	27%	-29%
20	26%	-29%
⩾ 90 years	20%	-20%

Table 1.3: Shocked rate curves

The application of these shocks to assets alone leads to an approximation of the real impact of the shock. In fact, a change in the interest rate curve usually has a global impact: both on assets and on liabilities. Technical provisions are sensitive to fluctuations in interest rates since they are used as discount factors. It is therefore important to consider the fluctuation in liabilities in the impact of the interest rate shock for each line.
The two shocked rate curves obtained on 31st December 2019 are presented below.

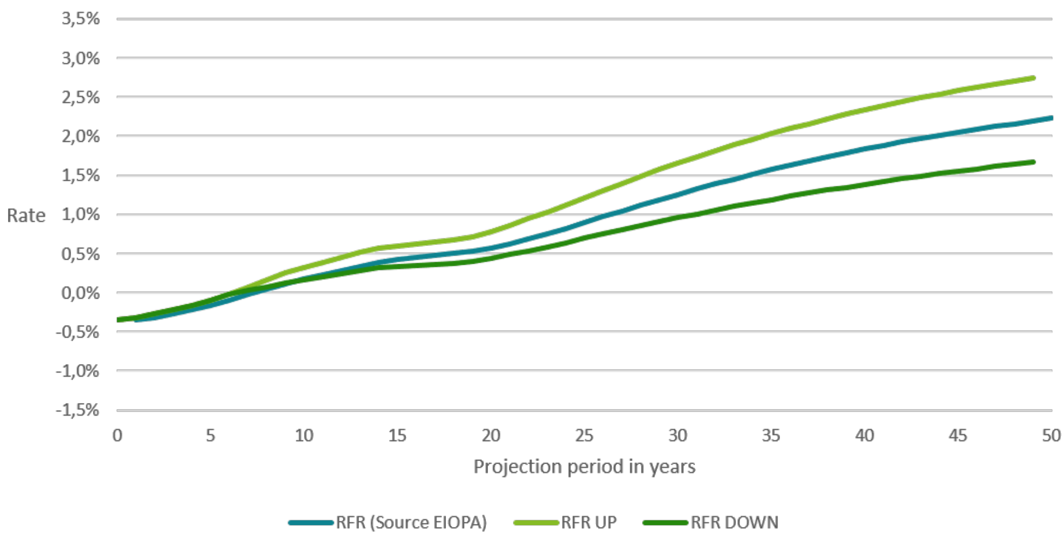


Figure 1.4: Interest rates curves

As negative rates aren't shocked in SII, the rate curve isn't shocked before the 7^{th} year of projection.

1.4.2 Equity risk

Article 105.5.b of the Solvency II Directive defines equity risk as the risk arising from "the sensitivity of the value of assets, liabilities and financial instruments to changes in the level or volatility of the market value of shares".
There are two sub-modules for this risk which correspond to the differentiation of two categories of equity:

– Global equities (type 1): corresponding to equities listed in the European Union or in an OECD member country.

– Other equities (type 2): other listed non-OECD equities, unlisted equities, hedge funds, commodities and other assets not classified in a market risk module. As this category is composed of less secure equities, the shocks applied will obviously be stronger.

The shock values provided by EIOPA are 39% for type 1 equities and 49% for type 2 equities. However, there is also the symmetric adjustment to consider.
The symmetrical adjustment (SA) allows the level of equity markets to be considered in the calculation of the SCR. It can increase or decrease the level of applied shocks. The aim is that higher are the markets relatively to the economic cycle, stronger is the shock, as the risk in case of a sudden drop is then greater.
The adjustment is calculated according to the following formula:

$$SA = 0,5 \text{ x } (\frac{CI - AI}{AI} - 8\%)$$

– CI: current level of the share index

– AI: daily weighted average over the last 3 years of the share index

The aggregation of the two sub-risks is done as follows, where 0.75 is the correlation between the two categories:

$$SCR_{equity} = \sqrt{0,75 \text{ x } SCR_1 \text{ x } SCR_2 \text{ x } 2 + SCR_1^2 + SCR_2^2}$$

1.4.3 Property risk

The property risk module aims at quantifying the impact of the decrease in value of property on assets and liabilities. The shock advocated by the Solvency II Directive consists in instantly reducing the value of property assets by 25%.
It covers all positions related to the ownership of land, buildings or property rights as well as investment in real estate structures that can be directly used by the insurance company. However, investments in companies managing real estate assets or engaged in real estate development projects must be included in the equity risk module.
Below is a summary of the methods used to calculate the property SCR (if applicable) according to the type of property investment.

1.4.4 Spread risk

Spread risk corresponds to the portion of risk arising from the volatility of the spread in connection with the risk-free rate (difference between the actuarial rate of an instrument and the risk-free rate of the security). The capital requirement for spread risk on bonds and loans is equal to the loss of core capital that would result from a sudden relative decrease in the value of each bond or loan in the portfolio. The three sources of variations for the spread are defined as credit risk, liquidity risk and international risk aversion.

This sub-module concerns all fixed-rate products such as sovereign and corporate bonds, secured or unsecured, subordinated debt, securitization products, credit derivatives and instruments backed by a fixed-rate product (e.g. forward and future). Because of the diversity of instruments subject to spread risk, the regulations have proposed an approach that breaks down SCR into three categories of assets: bond assets, securitization products and credit derivatives.

$$SCR_{spread} = SCR_{spread,bonds} + SCR_{spread,titrisation} + SCR_{spread,\ credit\ derivatives}$$

Shocks are then applied depending on the risk sub-module, the maturity and the financial rating of the security's issuer. The values to apply for some ordinary bonds are in table 1.6 and for higher ratings, the shock is defined as in table 1.5.

duration$_i$ (years)	F^{up}
up to 5	3.0 %. *duration$_i$*
More than 5 and up to 10	15.%+1.7%.(*duration$_i$* -5)
More than 10 and up to 20	23.5%+1.2%.(*duration$_i$* -10)
More than 20	Min(35.5% + 0.5 %.(*duration$_i$* -20);1)

Figure 1.5: Spread shock for higher-ratings bonds

credit quality step / *duration$_i$ (years)*	0	1	2	3	4	5	6
up to 5	0.9 %. *duration$_i$*	1.1 %. *duration$_i$*	1.4 %. *duration$_i$*	2.5 %. *duration$_i$*	4.5 %. *duration$_i$*	7.5 %. *duration$_i$*	7.5 %. *duration$_i$*
More than 5 and up to 10	4.5% + 0.5 %.(*duration$_i$* - 5)	5.5% + 0.6%.(*duration$_i$* - 5)	7.0% + 0.7%.(*duration$_i$* - 5)	12.5% + 1.5%.(*duration$_i$* -5)	22.5% + 2.5%.(*duration$_i$* -5)	37.5% + 4.2%.(*duration$_i$* -5)	37.5% + 4.2%.(*duration$_i$* -5)
More than 10 and up to 15	7.2% + 0.5 %.(*duration$_i$* - 10)	8.4% + 0.5 %.(*duration$_i$* - 10)	10.5% + 0.5 %.(*duration$_i$* - 10)	20.0% + 1.0 %.(*duration$_i$* -10)	35.% + 1.8 %.(*duration$_i$* -10)	58.5% + 0.5 %.(*duration$_i$* -10)	58.5% + 0.5 %.(*duration$_i$* -10)
More than 15 and up to 20	9.7% + 0.5 %.(*duration$_i$* - 15)	10.9% + 0.5 %.(*duration$_i$* - 15)	13.0% + 0.5 %.(*duration$_i$* - 15)	25.0% + 1.0 %.(*duration$_i$* -15)	44.0% + 0.5 %.(*duration$_i$* -15)	61.0% + 0.5 %.(*duration$_i$* -15)	61.0% + 0.5 %.(*duration$_i$* -15)
More than 20	12.2% + 0.5 %.(*duration$_i$* - 20)	13.4% + 0.5 %.(*duration$_i$* - 20)	15.5% + 0.5 %.(*duration$_i$* - 20)	30.0% + 0.5 %.(*duration$_i$* -20)	46.6% + 0.5 %.(*duration$_i$* -20)	63.5% + 0.5 %.(*duration$_i$* -20)	63.5% + 0.5 %.(*duration$_i$* -20)

Figure 1.6: Spread shock for ordinary bonds

1.4.5 Concentration risk

The concentration SCR sub-module aims at covering the insurer's portfolio against excessive concentration in selected counterparties. Indeed, the regulation seeks through this module to encourage insurers to diversify their portfolio positions in order to avoid being too sensitive to the different shocks that may impact a specific counterparty.

1.4.6 Currency risk

The currency risk module aims at quantifying [4] the capital requirement corresponding to the loss generated by the foreign exchange effect on the value of foreign currency assets. The measurement of this risk concerns all securities denominated in foreign currencies and foreign exchange derivatives (options, futures, swaps, forwards). The calculation principle is to associate a specific exchange rate stress to each currency and apply it to the portion of the portfolio exposed to that currency.

Devise		Choc
DKK	Danish krone	0,39%
BGN	Bulgarian lev	1,81%
XOF	West-african CFA Franc	2,18%
XAF	Central Africa CFA Franc	1,96%
KMF	Franc of the Comos	2,00%

Figure 1.7: Example of currency shocks

[4] Art. 105 (4) (e) de la Directive 2009/138/EC) The sensitivity of the values of assets, liabilities and financial instruments to changes in the level or in the volatility of currency exchange rates.

Chapter 2

Current economic context and French insurance market

In this chapter we introduce the current economic context and the French market for life insurance.

In section 2.1, we present the characteristics of major French life-insurance products. In section 2.2, we detail the situation in 2019 of the life-insurance French market focusing on the Euro-fund products. In section 2.3, we detail the context of low interest rate environment and describe the impacts and the challenges resulting for insurers. Finally, the last section gives insights on levers available for insurers to better understand and control the risks they are facing.

2.1 Savings products in life-insurance

Among all products on the market, savings products are the most popular.
The Insurance Code refers to savings products as "a contract committing the insurer towards the policyholders to cover a category of risks determined by the contract, in exchange for the payment of a sum of money known as the insurance premium".

There are different types of savings products, each with varying degrees of risks for the insurer. The remuneration of these products depends on the fund chosen.

A fund corresponds to a method of investment of the contributions paid by the policyholder. The most common types of products are the mono funds products: Euro fund products or Unit-Linked (UL) fund products. There are also “multi funds” products which are a mix of the two previous types and finally there is a new type of contract: the Eurocroissance product which will be detailed below.

2.1.1 Euro fund products

Among life insurance products, approximately 78% [9] of the mathematical provisions on the French market is denominated in euro fund products (“multi funds” & mono funds) according to the French Insurance Federation (FFA). This product is the largest investment in terms of amounts in French people’s savings.
In exchange for his investment, the insured receives a remuneration each year according to his contract. The contract is secure as the amounts paid by the insured are guaranteed. The sums paid by the insured are revalued each year according to a contractual guaranteed minimum rate plus the profit-sharing. The revaluation depends on the insurer’s financial and technical results but cannot be lower than the guaranteed minimum rate. Funds in euros are mainly invested in bonds without or with limited credit risk.

2.1.2 Unit-Linked (UL) fund products

Under a unit-linked contract, the capital invested corresponds to investment fund units. For this type of contract only the number of units corresponding to the contract are guaranteed. The amount of the redemption value depends on the net asset value of the financial assets in which it is invested, which may rise or fall.

These products are often less risky for the insurer because the insured bears all or part of the market risk. However, the insured may be offered additional guarantees against capital losses. These are called floor guarantees and can take several forms, such as the ratchet guarantee: the amount paid at the end of the contract will be equal to the highest historical level during the contract's life.

2.1.3 EuroCroissance products

EuroCroissance products are a type of diversified euro fund product guarantying at the end of the contract, and not over the entire term, part or all of the net initial payment (or payments). The policyholder benefices from an assumed higher expected return than traditional euro products. When the guarantee of capital at the end of the contract term is nil, Eurocroissance products can be identified with unit-linked products.

Another novelty brought by EuroCroissance product is the level of commitment. Indeed, it is up to policyholders to choose the percentage they want to be guaranteed at the end of the term. He can choose to have 100% of the capital invested guaranteed or less.

2.1.4 Guarantees

In the case of euro fund savings products, the capital invested on funds is guaranteed by the insurance companies. Each year, this capital is revalued through two mechanisms: the minimum guaranteed rate and profit sharing.

Minimum Guaranteed Rate (MGR)

This is the minimum rate of revaluation of the policyholder's savings, it's contractual and must be paid regardless of the insurance company's financial performance.
Since a certain number of years, MGRs have generally been equal to zero. They are subject to regulations detailed in the Article A132-3 of the Insurance Code. It should be noted that euro-fund products systematically offer a guaranteed capital.

Profit-Sharing (PS)

According to Article A331-4 of the Insurance Code, an insurance company must pay to its policyholder at least 85% of the financial profits and 90% of the technical profits for the year.
The insurer owns some freedom to distribute the PS. In particular, the PS may not be distributed equally between products and may be deferred over time. For the insurer to defer the distribution of profit-sharing, a provision, the bonus reserve (BR) is endowed. This provision and the interest earned on it remain the property of the insured. The sums deposited on the bonus reserve cannot be kept for more than 8 years (Article A132-1 of the Insurance Code). After this period, they must be redistributed to the insured.

2.2 French insurance market

In France, life insurance plays an important role in the economy through the amount of assets under management and also by the response it provides French people who want to save money. For some, it is a source of security, while for others it responds to a desire of performance on the financial markets.

Mathematical reserves increased by 5.1% to reach €1,722.1 billion at December 31, 2019[1]. Unit-linked products accounted for 23% of these provisions (€391.0bn) and increased of 14.6% over the year, benefiting from the strong performance of stock markets. The bonus reserve rose sharply, reaching €60.6 billion (+12,5%) at the end of 2019.

There has been a positive net inflow of €20.4 billion in 2019, slightly higher than the €20.1 billion in 2018. This net inflow was mainly oriented towards euro funds: €15.3 billion compared to € 5,1 billion for net inflows from unit-linked denominated funds. There has been a revival of euro funds.

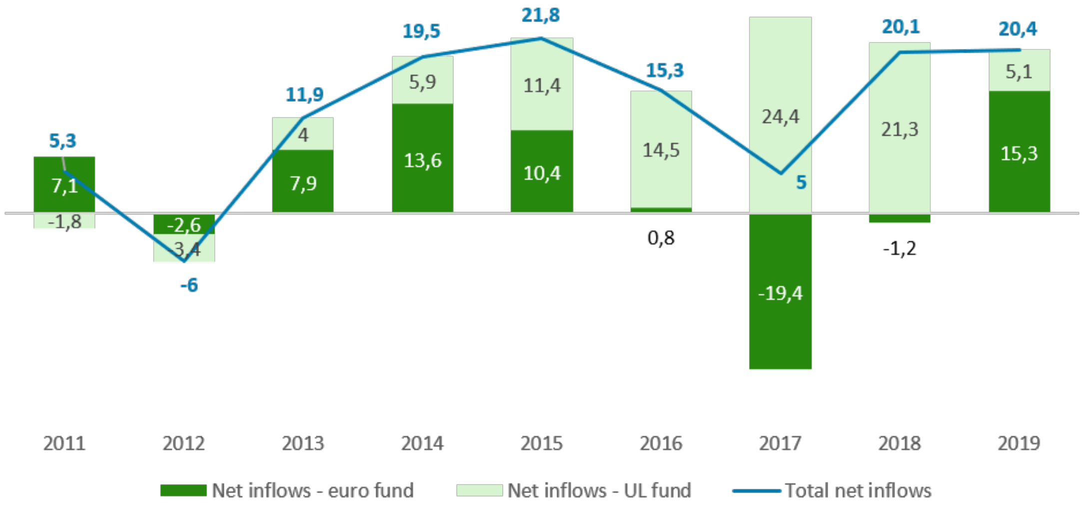


Figure 2.1: Annual net inflows since 2011 (in billion€) [Source: Banque de France, ACPR]

Despite low rates, life insurers' results and inflows increased in 2019. For life and mixed companies, net profit increased to €7.3 billion, being 2.7% down compared to 2018, according to the 2019 report of the French Insurance Federation (FFA)[9].

Nevertheless, this should not hide the difficulties which is currently facing the life insurance sector in France and more widely in Europe due to the environment of low rates. In fact, a significant portion of the insurers' asset portfolio is invested in government bonds meaning that when bonds are to be bought in the current context, their return will be low. In addition, due to Solvency II regulation insurers are driven to reduce the proportion of assets invested in equities to reduce the amount of capital to immobilize. Investment returns may thus decrease, making it difficult for insurers to serve the rates advertised to their policyholders.

Faced with very low or even negative rates, which reduce the revalued net assets of life insurance companies, a ministerial order was published on December 28, 2019 authorizing life insurers to include part of the bonus reserve in the calculation of their required capital in order to improve their solvency. The decree provides that "by exemption of the allocations provided in the first paragraph of Article A. 132-16 and in exceptional situations, the bonus reserve may be reversed after authorization by the ACPR[1]."

[1] Autorité de Contrôle Prudentiel et de Résolution

2.3 Current context

2.3.1 Interest rates steady decline

The second quarter of 2019 saw a sharp fall in interest rates, pushing the 10-year French OAT (Obligations Assimilables au Trésor) bond rate into negative territory for the first time. The economic environment has been characterized for several years by low bond interest rates, as a consequence of the financial crisis in 2008, the Eurozone sovereign debt crisis in 2010 and the policy of the Central Banks. In order to boost the economy, they have chosen to promote credit rather than savings and have therefore drastically and sustainability reduced their reference interest rates.
For more than three decades, interest rates have been on a very sharp downward trend and almost uninterrupted. A slight upward trend was observed in 2017 but did not continue in 2018.

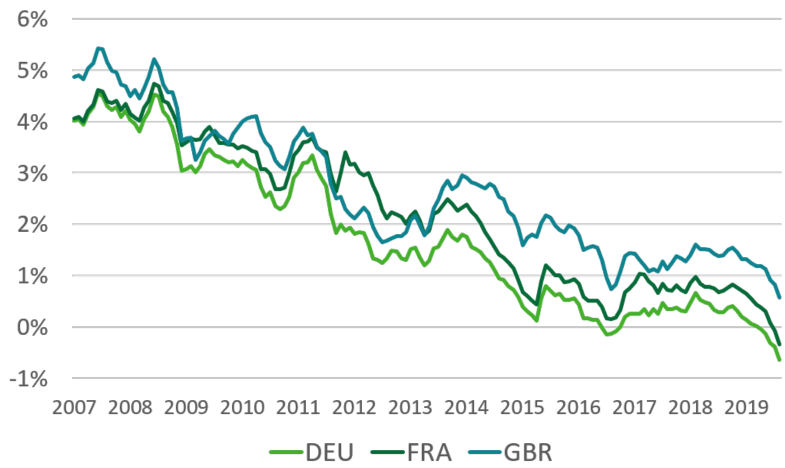


Figure 2.2: 10-year OAT rate curves

French OATs are securities representing a share of French government bonds issued for different maturities. The coupon rate of OATs with a maturity of ten years is an economic indicator which is particularly monitored by institutional players.
Although 10-years maturity bond is not representative of the bonds portfolio's completeness of a life insurer, it is a reference as its maturity is close to the maturity of the usual duration of insurer's assets.

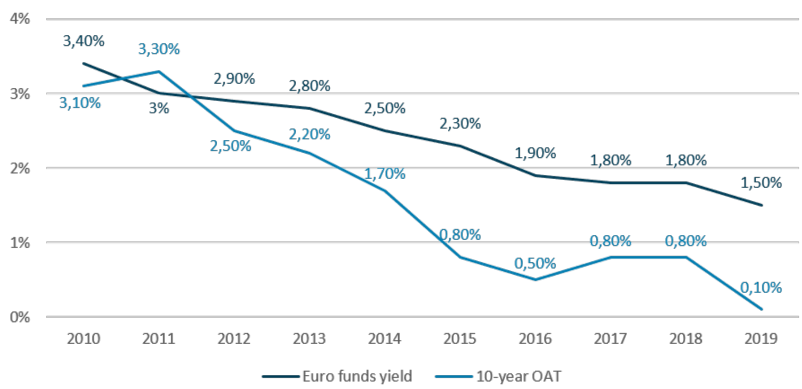


Figure 2.3: Euro fund products yield (net of management loadings and gross of social and financial debits) [Source: Banque de France, FFA]

The fall in rates was initially favorable to the life insurance business by enabling insurance companies to offer attractive returns on their previous investments, which were becoming much higher than those observed on the markets.

In 2019, the rate of return for euro funds products was estimated at 1.5%. In a context where interest rates have reached new historic lows, the yield has fallen over the year (-30 bps) after the stability observed in 2018 and the almost continuous decline over the last 5 years. Insurers are thus suffering from the rate environment which is driving down the income from the bonds they hold. The fall in bond rates weighed on the returns of insurers' assets and was reflected in a downward trend in the served rates.

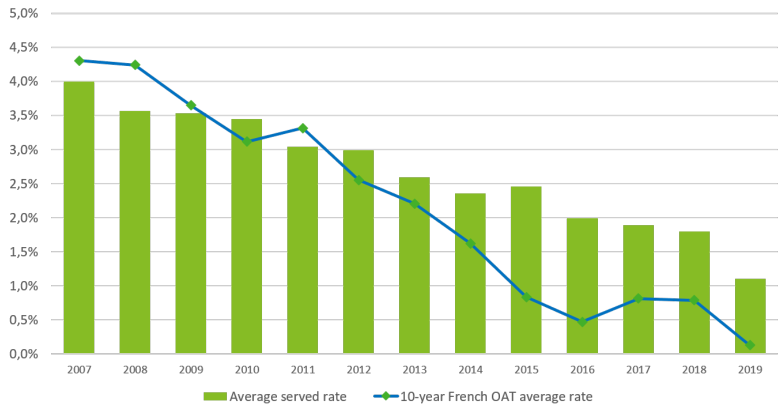


Figure 2.4: Average distributed-rate variation between 2007 and 2018 for French insurance companies [Source: Deloitte]

Trends in rates have consequences for the insurer. The level of the interest rates might impact policyholders' behaviour.
Indeed, if rates rise, the insurer's portfolio might underperform the market with a lower return. The policyholders can surrender their contract because the rate paid will not be high enough compared to competitors. If too many policyholders want to surrender their policy at the same time, the insurer will lack liquidity to meet the outflows, and will have to sell assets.
On the other hand, if rates fall, the insurer's portfolio might have a higher return than the market and might pay higher income to policyholders. Policyholders will therefore tend to stay longer. However, when assets meet their maturity, the new assets purchased will have lower returns than the previous ones. In case where minimum guaranteed rates are high, the insurer must pay a high rate while the portfolio yield is low. The fall in interest rates, in this case, can be qualified as a long-term risk if the downward lasts.

2.4 Thesis' approach

In this context without precedent of low rates, particularly because of its extended duration, it is essential for insurers to better understand the issues affiliated with this period.
This may involve a commercial change in the insurance products promoted as well as an improvement in the management of the insurer's available financial tools.
Within the framework of this report, our study will focus on the projection of an insurer's balance sheet under Solvency II regulation. The idea is that it is possible for an insurer to limit its sensitivity to risks due to low rates, on the one hand by optimizing the composition of its portfolio and on the other hand by improving asset management within the projection model itself.

First, under defined assumptions, an ALM study will be conducted to find the portfolio optimizing the return yield of the insurer's portfolio while mastering the underlying risk. Then we will focus on the ALM models where without any surprise, ALM allocation strategies can be improved to take better account of the economic context in which they are evolving. This part will consist in developing a new asset allocation strategy and analysing the effects on the insurer's solvency and profitability.

Chapter 3

Asset and liability modelling

Insurance companies, particularly life insurers, are subject to specific risks arising from a particular interaction between assets and liabilities. It is therefore necessary to allow for such interactions in the projection model by projecting both assets and liabilities and modelling their interactions (e.g. profit sharing, options and guarantees offered to policyholders...)

The Asset and Liability management (ALM) model has multiple uses such as:

- Producing results for financial communication (i.e. future profitability)
- Calculating prudential Solvency II metrics
- Realizing studies: for the ORSA report, Business Plan analysis or even ALM studies to improve the modelling

ALM can be used to satisfy various purposes such as:

- Structuring contractual guarantees
- Defining strategic assets allocations
- Driving strategies: unrealized wealth management, technical provisions (profit-sharing, capitalization reserve) management

Steering the ALM consists in identifying and implementing actions that will enable the control of the insurer's trajectories within various life insurance frameworks such as Solvency II or even IFRS frameworks.

In a low rates environment, the insurer is in a weaker position mainly due to his assets generating less return. This encourages him to seek higher returns on the financial market. The primary purpose of an ALM study is to make strategic investment decisions but it can also be used for risk assessment. This study will focus on the optimisation of the return ratios in the real-world universe for the insurer while keeping in mind the adjacent risk.

3.1 Robust model: analysis of leakage sources

Before realizing the studies, we first started by ensuring the resilience of the ALM model used. To do so, we had to realize some tests. Most of them were not satisfying, therefore, a lot of time was spent to correct the model and ensure that the results were robust enough to be used and presented in the context of this thesis. The tests we have performed are the following:

1. Accounting leak proof test: the objective of this test is to ensure that the balance sheet is balanced at each year of the projection and for each scenario used. Therefore we ensured, trough tests that the model respected the equality between the liabilities' book value and the assets' book value at each time step.

2. Martingale test: martingale test is performed on the risk neutral stochastic scenarios used. The results of this test are presented in this chapter.

3. Economic leak proof test: the objective of this test is to ensure that the leak of the model is within a tolerable range. Its results are detailed below.

3.2 Economic Scenarios Generator

The Economic Scenarios Generator allows us to build multiple economic scenarios which will then be used as inputs to the ALM model.
An economic scenario corresponds to the projection of financial and/or economic variables over a given horizon. The scenarios generated can be stochastic with multiple paths simulated or deterministic with only one path simulated. For both types of scenarios, the same properties are respected and will be explained afterwards.

The long-term projection of financial assets is a non-mandatory requirement in the implementation of an asset-liability management model for a company offering life insurance policies because of the duration of these products and their strong dependence on the economic environment. The scenarios generated by the tool are used for the valuation of the company, in the calculation of the SCR and more generally, in the projection of the company's balance sheet.

The different models implemented were calibrated using data from 12/31/2019, which is the most recent closing date while writing this report.
Correlation between assets in the ESG is displayed in the following matrix:

	Equity	Real Estate	10y risk-free interest rate
Equity	100%	32%	-20%
Real Estate	32%	100%	-16%
10y risk-free interest rate	-20%	-16%	100%

Table 3.1: Correlation in the ESG

The percentages were calibrated on historical data by another team and the methods used are not treated in this paper.

The economic scenarios generator provides several stochastic simulations. Four diffusion models were implemented:

- diffusion of interest rates;
- diffusion of a stock index;
- diffusion of a real estate index;

- diffusion of inflation.

The generation of risk-neutral economic scenarios is performed in three stages.

1. The first stage is the calibration of the parameters minimizing the difference between the prices seen on the market and the prices outputted by the model.

2. In the second step, the scenarios are generated once all the parameters have been estimated.

3. Finally, martingale tests are performed and the replication of the prices of the derivative products used during model calibration is checked (market consistency test). The regulation gives insights of the expected tests to be performed in a recent publication published on 7th December 2020[2].

The data required for calibration depends on the choice of model. Interest rate models are calibrated on swaptions, more precisely using implied volatilities (normal in our case) and an initial yield curve. Equity models are calibrated on the implied volatilities of equities (Eurostoxx50, etc.).
This market data can be obtained through information providers such as Bloomberg. Each model used for the modelling of the different assets classes is detailed in the next sections.

3.2.1 Interest Rate model

In this study, we used the LMM-DDSV[1] model, which corresponds to the best practice in the industry, as it allows the modelling of negative interest rates.

$$df(t,T) = a(t,T)dt + \sigma(t,T)dW_t$$

Where $f(t,T)$ is the forward rate of maturity T, seen from t, $a(t,T)$ the drift and $\sigma(t,T)$ the volatility. Both depend on time and can depend on interest rates and the history of W_t the attached Brownian Motion.

Based on the classic Libor Market Model, this model is a market model meaning that rates (even negative rates) observed on the market, such as the Euribor, are modelled. In addition, there is a desire to limit explosive rates, which is one of the problems of the "classic" Libor Market Model. This is due to the fact that it is a log-normal model (explosive rates being a characteristic of this type of model). The dynamics of the LMM-DDSV are different ("mixed" log-normal), which limits this undesirable effect.
The interest rate yield curve with Volatility Adjustment was chosen to calibrate the model.

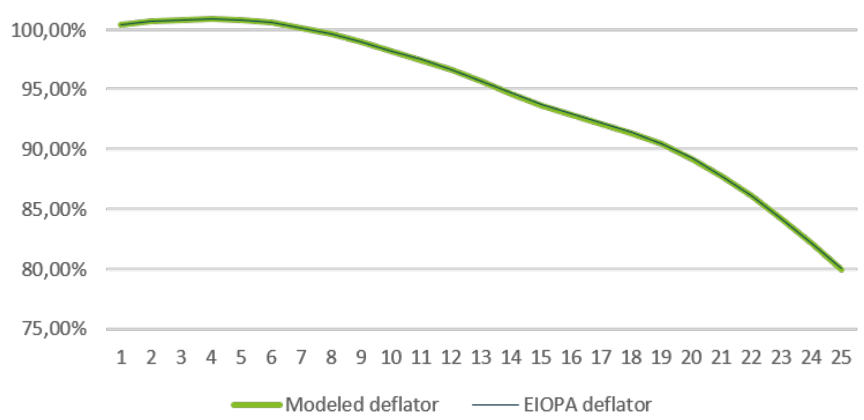


Figure 3.1: Comparison of the deflator curve from the ESG with the deflator curve obtained from the EIOPA

In addition, the projection of interest rates has a significant impact on the mechanisms linked to the valuation of a company's liabilities. It is therefore particularly important to focus on modelling this rate in the economic scenarios generator.

[1] LMM-DDSV or Libor Market Model with displaced diffusion and stochastic volatility

3.2.2 Equity model

Equity was modelled using the Heston model. A well-known property of asset return is that their volatility varies over time. Usually, stochastic volatility is used to capture this feature and that's what led us to choose the Heston model for equity.

The price of equity share (S) is given by the following dynamic process:

$$dS_t = \mu S_t dt + \sqrt{\nu}_t S_t dW_t^S$$

with the following volatility process

$$d\nu_t = \alpha(S_t, \nu_t, t)dt + \sigma\beta(S_t, \nu_t, t)\sqrt{\nu_t}dW_t^\nu$$

W_t^S, W_t^ν are Brownian Motions.

- μ: drift parameter
- function $\alpha()$ and $\beta()$ can be replaced by different functional forms
- σ: volatility of volatility

3.2.3 Real estate model

Black & Scholes standard model was used to model real estate distribution. The associated dynamic for the price S_t of the real estate is reminded here:

$$\frac{dS_t}{S_t} = r_t dt + \sigma dW_t$$

with:

- r_t is the risk-neutral rate obtained with the model detailed previously
- σ the volatility parameter
- W_t the Brownian Motion

This model allows property prices to be estimated using closed formulas which makes it simple to use.

3.2.4 Inflation model

Inflation was modelled using the Hull-White one-factor model. The associated dynamic is described below:

$$dr_t^{infl} = (b_t - ar_t^{infl})dt + \sigma^{infl}dW_t$$

with:

- a is a positive constant corresponding to the speed of return to the average
- b_t is a deterministic function controlling the long term average and enabling a calibration on the actual
- σ^{Infl} is the volatility
- W_t is a Brownian Motion

This model is convenient for the modelling of inflation because it is Gaussian and leads to tractable computation.

3.2.5 ESG validation

The diffusion laws of the ESG indexes are expressed under the risk-neutral probability. A property linked to this universe of projection is the market consistency of the assets. It means that the expected return for any asset must be equal to the central risk-free rate return. This property was approved for our results.

Another way to approve the ESG is the martingale test which is performed to verify that the expectation under the neutral risk probability of the present value of each asset is equal to the initial value of the asset. This test is performed for all the assets, as well as for the deflator. Usually, if the difference between the initial value of the asset and its discounted value with the risk-free rate for each maturity is less than a defined threshold, then the test is approved. In practice, the threshold is higher for volatile assets than for deflators, but it is possible to set an average threshold. In this study, we chose 5% as the average threshold to validate our ESG results.

We generated 1000 simulations and the observed distance to 1 is always inferior to 5% which validates the test.

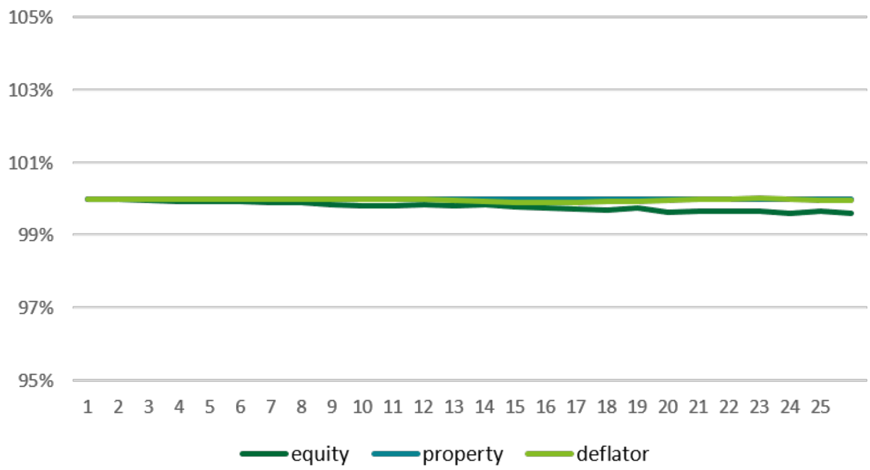


Figure 3.2: Martingale test

3.3 Model architecture

Generally speaking, the projection process covers four main phases:

- Definition of the assumptions and construction of the input tables
- Loading of the various assumptions and input data required for the projection into the ALM model
- Projection: projection of the balance sheet over the lifetime of the contract
- Output of selected variables to be analysed

Focusing on the projection, the calculation step is annual, meaning each year multiple calculi are performed. It was assumed that the benefits occur in the middle of the year and that the calculation of the revaluation is done at the end of the year. The details of these steps are shown in the diagram below:

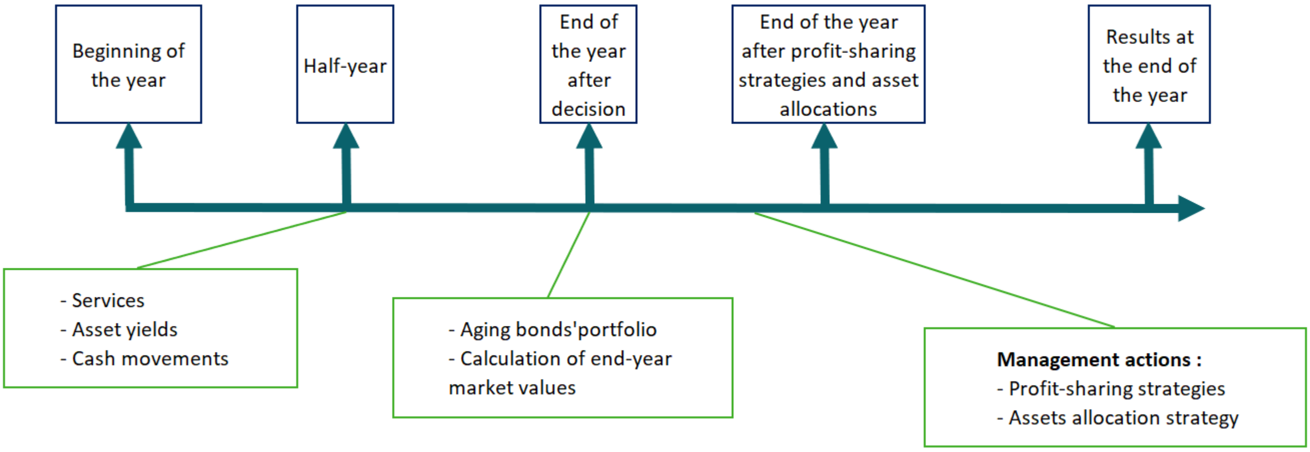


Figure 3.3: Main calculation steps over the projection year

The asset purchase/sale policy of the ALM model allows the management of the main characteristics of the projected asset portfolio. Movements of asset purchases or sales are made during the projection process, in particular to:

- Realign the assets allocation:
  - after payment and collection of liability flows during the year
  - and before the profit-sharing strategy,
- Realize the unrealized capital gains in connection with the profit-sharing strategy.

The strategy set to manage the allocation of the assets will be detailed in section 3.5, after the description of the assets modelling in the following section.

3.4 Asset Modelling

The model used allows the projection of the following asset classes:

1. fixed rate bonds
2. equity
3. property
4. cash

This section details their modelling.

3.4.1 Fixed-rate bonds

For a fixed-rate bond of maturity T and nominal N, its price will be calculated by the following formula:

$$Bond_N(t) = C_T \text{ x } N \sum_{i=1}^{T-t} \delta_t(t+i) + N\delta_t(T)$$

Where:

- $\delta_t(t+i)$ is the deflator between date t and t+i, read in the ESG.
- C_T the coupon rate of the T-maturity bond

Market prices for zero-coupon bonds are obtained from the economic scenarios generator and bonds prices are calculated using the discount factors read in the ESG.

The notion of discount factor needs to be clarified as it is essential for the calculations we are going to make. Under Solvency II, the calculations must be carried out in a "market consistent" model (i.e. consistent with market values). The deflator is used to go from prices expressed under the risk-neutral probability in the zero-coupon forward numeraire to financial markets prices.

$$Deflator(t) = \frac{1}{ZCB(t,T)}$$

Note: Credit risk will not be treated in the following studies.

3.4.2 Equity

In the ALM model, equities' market values are calculated as the sum of the equity market value at the previous time step and the global profits realized.

$$\text{Price}_t = \text{Price}_{t-1} + \text{global gains}_t$$

where global $\text{gains}_t = \text{dividends}_t + \text{financial performance}$

The financial performance corresponds to the evolution of the share price on financial markets. The model computes annual dividends that are paid each half year. The amount of dividend paid is equal to the mid-year market value of the equities multiplied by the dividend rate.

Here, we supposed that dividends were null. Indeed, in the low interest rate environment and a low financial performance, it is necessary to realise unrealized gains for the insurer to meet the MGR guaranteed to policyholders. However, when there are dividends, these unrealized gains have already been included.
Due to the rebalancing of the assets and a regular turnover, there is an impact which is quite limited and does not materially change the results in the low interest rate environment.
If financial performance was highly above the target rate, there may be an impact in the absence of management of unrealized gains and dividends.

The financial performance is obtained via the equity index generated by the ESG.

3.4.3 Property

Real estate investments are modelled as the equities in the model. Rents are assimilated to dividends. The price of real estate evolves according to the same principle as the equity price. Its total return index is calculated using the rent index generated by the ESG.

3.4.4 Cash

In the model, cash is invested in one-year-zero-coupon bonds. It is used in the middle of the year to pay services, expenses, corporate taxes and social contributions calculated on the previous year's result.

The cash yield at date t corresponds to the one-year-zero-coupon rate at this same date. It is therefore calculated as follows:

$$\begin{aligned} \text{Cash yield}_t &= \frac{1}{ZCB(t,t+1)} - 1 & (3.1)\\ &= \delta_t(t+1) - 1 & (3.2)\\ & & (3.3) \end{aligned}$$

where:

- $ZCB(t, t+1)$ is the zero-coupon price of maturity t+1 seen for date t.
- $\delta_t(t+1)$ is the deflator between date t and t+1, read in the ESG.

At the end year t, the value of the cash is:

$$\text{Cash}_{\text{end year t}} = Cash_{\text{half year t}}(1 + \text{cash yield}_t)^{0.5}$$

3.5 Asset allocation strategy

After each year of projection, the ALM model aims at balancing the insurers' portfolio after the different movements caused by the assets' purchases or sales made during the projection process.
Asset allocation can be fixed or dynamic. A dynamic allocation strategy evolves according to the evolution of market prices. This type of strategy enables the optimization of the strategy as the information goes along. On the other hand, fixed strategies can use part of the information arriving in the future, but will not evolve depending on it.
Some strategies include cash flow matching, fixed-mix, etc. The report of FALEH A. [16] gives a number of examples of fixed and dynamic allocation strategies.
In the initial model, a fixed strategy is used. It consists, for each asset class modelled, in meeting every year a predefined share regardless of the economic situation and liability constraints. The asset allocation is defined as a percentage of the fund in market value. Moreover, the algorithm isn't optimal in asset managing as detailed below.

3.5.1 Description of the existing algorithm

The following diagram outlines the operation of the fixed strategy.

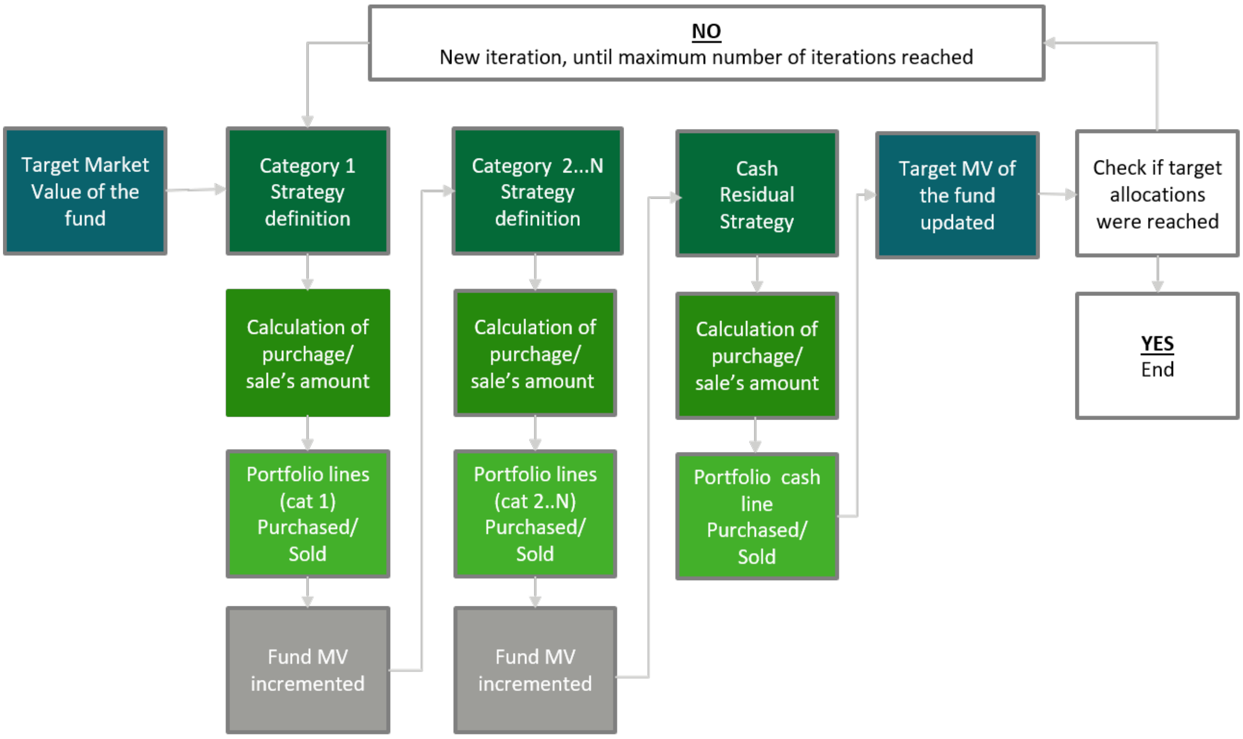


Figure 3.4: Diagram of the fixed ALM strategy

For each asset class to meet their target percentage, the algorithm performs a defined number of iterations (i.e. 25), until the target allocation for each asset class is reached.

The iteration loop is used because the process is built on a strategy where the strategy applied to a category is immediately performed before moving to the next category. This implementation may not work from the first time and needs to be performed several times.

If the proportion of the asset being looked at in the portfolio corresponds to the terms of the strategy, the algorithm moves to the next category. Otherwise, it calculates the amount to be bought or sold in order to rebalance the proportion of the asset in the portfolio. In the unbalanced case, the purchase or sale of the asset is carried out on each segment on an equally weighted basis.
For the next asset category, its share is calculated based on the new asset value of the portfolio calculated as above (and not the initial value of the portfolio). The cash is the last category to be treated.

The allocation strategy is supposed to replicate as closely as possible the actions an asset manager would perform in real life and the existing algorithm prevented the possibility to manage the strategy at asset product level.
The strategy only considers asset category levels which is inefficient considering the range of possibilities for an asset manager to interfere at product level (taking into account the unrealized gains and losses of each product, the duration of liabilities, or even deciding to keep a particular product throughout the projection).

The following figures show the behaviour of the assets. We can easily point out that the assets always make it to the allocation fixed target. This can be explained by the fact that the chosen portfolio is simplified enough for the algorithm to meet the target within the number of defined iterations. However, this wouldn't have been the case for an insurer with a portfolio composed of more than 3 assets categories, each constituted of multiple assets segments.

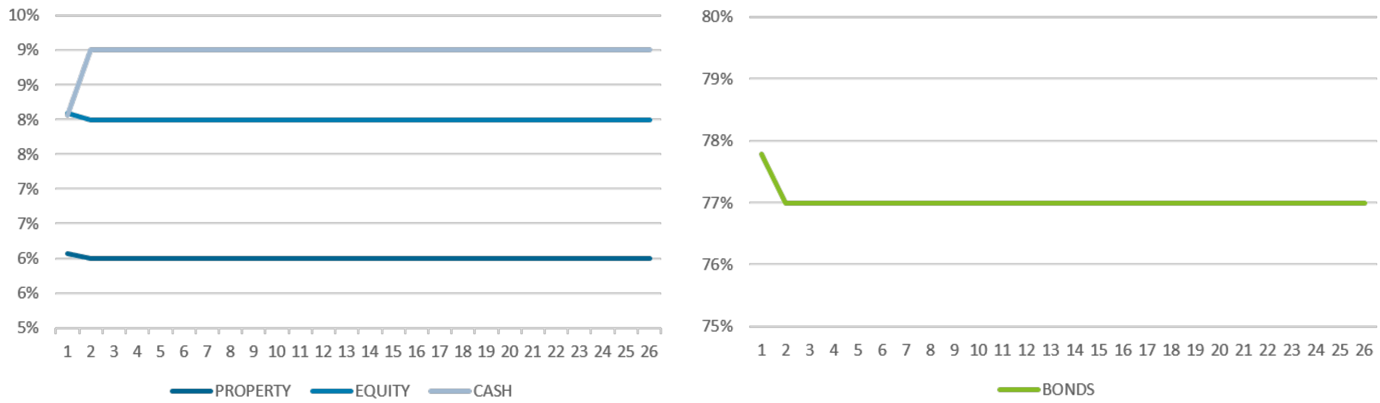


Figure 3.5: Share of each asset class for each projection step

3.5.2 Weaknesses of the algorithm

We looked at the weaknesses and limitations of this algorithm in order to suggest improvements. The major points are listed below.

- This algorithm does not ensure convergence towards a viable solution. Indeed, the allocation may not be optimal. The strategy is fixed and invariant in time. Weights given to each share of the portfolio should be adjusted considering economic conditions at the time of each allocation in order to adopt a more efficient management. In addition, during an iteration of the loop, we can carry out two transactions in opposite directions on the same asset class. For example, we may have to sell shares to repurchase bond units and then buy back shares to rebalance its share in the portfolio. This is not efficient coming to consuming unrealized capital gains.

- The use of a loop does not allow management actions to be carried out. Indeed, a management action performed in iteration i will be destroyed in i+1. The scheduling of categories also has an impact on the result of the algorithm. We have seen that the current algorithm uses the following category to adjust the share of the current category.

- The purchase and sale of each segment within the categories is carried out in an equally weighted way. Thus, many parameters are ignored: unrealized capital gains and losses, durations, bond ratings, current economic scenario, internal rate of return.

We will suggest ideas for improvement in chapter 5 by taking into account the different criteria listed above to optimize the portfolio management.

3.6 Provisions management

The Insurance Code compels insurers to protect policyholders against a certain number of risks by setting aside various provisions each year. In this chapter, we present the main provisions relating to savings products and modelled in the ALM model.

3.6.1 Capitalization Reserve

As defined in the Article R3433 of the French Insurance Code, the capitalization reserve is a reserve intended to cover the depreciation of values included in the company's assets and the reduction in their income.
The capitalization reserve is supplied with capital gains realised on the sale of bonds and is reversed in the event of realised capital losses. Therefore, there is no impact on the amount of the financial products, meaning that the purchase/sale of bonds do not participate in the profit. Note, however, that variable rate bonds are not included in the capitalization reserve.
Its purpose is to smooth out variations in bond rates and to encourage insurers to hold these securities until maturity. Its mechanism limits the interest of insurers to sell bonds.

The allocation to the capitalization reserve therefore leads to:

– preventing the distribution of the capital gain to policyholders, shareholders or to the State;

– increasing Own Funds, and thus the solvency margin;

– reducing the minimum share of financial products to be distributed to policyholders.

Example of capitalization reserve operating:

line of bonds	market value	net value	gain/loss realized by the sell	capitalization reserve movement
bond 1	110	90	+20	+20
bond 2	50	60	-10	-10

After this event, the capitalization reserve value is:

$$\text{Capitalization reserve}_t = \text{Capitalization reserve}_{t-1} + 10$$

3.6.2 Liquidity risk reserve

The provision for liquidity risk[2] is established when the investments, mentioned in Article R. 343-10[3], are in a position of overall net unrealised loss. An overall net unrealised loss is recognisable when the net book value of these investments is higher than the overall market value of these same investments. This reserve is intended for the insurer to meet his commitments in the event of a decrease in the value of all the assets mentioned in Article R. 343-10. The provision to be established is valued in accordance with the conditions defined in Article R. 343-5.

In our model, the liquidity risk reserve endowment is equal to one third[4] of the overall net unrealised loss. The formula used is detailed below.

$$LRR_t = min((AV_t - MV_t)^+; LRR_{t-1} + \frac{1}{3}(AV_t - MV_t)^+)$$

where

– AV_t: acquisition value in t of the assets concerned by the reserve

[2] Provision pour Risque d'Exigibilité (PRE) in French
[3] except for the depreciable securities the insurer has the capacity and intention to hold until maturity
[4] In respect with the Article343-5 of the Insurance Code

– MV_t: market value in t of the assets concerned by this reserve

Example of Liquidity Risk Reserve use:

line of asset	market value	net value	unrealized gain/loss	liquidity risk reserve
equity 1	80	90	-10	
equity 2	50	70	-20	0
property	90	50	+40	

The portfolio observed isn't in overall net unrealised loss which justifies the absence of movement for the liquidity risk reserve. However, if the property line had an unrealised loss instead of an unrealized gain, the reserve would have been endowed.

3.6.3 Bonus reserve

The insurer has the choice of either directly revaluing the policyholders' contract though profit-sharing, or investing the capital in a provision, known as the bonus reserve. If the insurer endows the bonus reserve with a certain amount on a given date, this amount must be redistributed at most 8 years after.
This provision enables the smoothing of products yields and thus ensures stable remuneration or can also be used for a year marked by a decline to offset the results.

The insurer's behaviour is guided by the management of the rate served to policyholders and the generated financial margin.
The insurer is guided in its profit-sharing strategy by three values:

– The Minimum Guaranteed Rate, defined in chapter 2

– The regulatory profit-sharing

– The target rate

$$\text{Profit-sharing}_{\text{regulatory}} = 85\% \text{ financial profits} + 90\% \text{ technical profits}$$

Financial profits correspond to the financial profits of the assets against the insurer's liabilities. In the model, the distribution of profit-sharing, the revaluation rates and the target return are managed while considering the specific features of the accounts, the types of contracts and/or products, by arbitrating:

– The bonus reserve

– The disposal of investments

– The financial margin.

In our case, the target rate corresponds to 90% of the average of the risk-free 10-year rate [5].

When the financial production is greater than the required one necessary to serve the target rate, the surplus is endowed in the bonus reserve. When the financial production is lower than the target amount, the following process is applied:

[5] French: TME, Taux Moyen d'emprunt d'État

Algorithm 1: Bonus reserve endowment

```
if financial production ≥ target amount then
|  endowment of the bonus reserve
else
|  realization of unrealized gains on Global and Other equities and real estate;
|  if financial production ≠ target amount then
|  |  if financial production < minimum guaranteed amount then
|  |  |  adjustment of the profit-sharing rate and the loadings
|  |  end
|  |  recover of the bonus reserve
|  |  if financial production < target amount then
|  |  |  if financial production ≥ minimum guaranteed amount then
|  |  |  |  adjustment of the profit-sharing and loadings
|  |  |  else
|  |  |  |  adjustment of the profit-sharing (considering its configured thresholds) and the
|  |  |  |    loadings
|  |  |  |  if financial production < minimum guaranteed amount then
|  |  |  |  |  adding of capital
|  |  |  |  end
|  |  |  end
|  |  end
|  end
end
```

The MGR is null in this study. Serving the MGR is therefore equivalent to guaranteeing the insured's capital in the event of low financial results. In extreme scenarios, the target rate may fall below the MGR. In this case, the target rate becomes the MGR.

3.7 Study-specific assumptions

3.7.1 Liability portfolio

The modelled portfolio is composed of 10,000 individuals with the same single premium Euro fund policy. The chosen technical rate is 0% because most products currently offered on the market have a zero minimum guaranteed rate, which is mainly due to the context of low rates. It was assumed, for simplification, that all policyholders were 40 years old at the date of subscription of the contract and that they all paid an initial premium of 115,000€.

Modelled expenses and loadings

Administrative expenses were considered dependent on the number of policies in portfolio. They were modelled as a unique cost of 5€ per contract with an annual inflation rate of 2%. To cover the various management costs of the activity, and for the insurer to maintain a certain profitability, the rate of management loadings is 0.3% of mathematical provisions and the commissions' amount is set to 40% of management loadings. The profit-sharing rate is set to 90% of the financial production.

The consistency of the results was verified using the roll-forward test and was positive:

$$PM(t+1) - PM(t) = \text{credited interests(t+1)} - \sum_i \text{services}_i(t+1)$$

where:

- $PM(t)$: mathematical provision of the portfolio observed for the tth year of projection
- credited interests(t): incremented PM resulting from the rate served by the insurer

- services(t): resulting from the exit of individuals from the portfolio and the payment of the associated services along the year (ie. exit for death, maturity of the contract, total and dynamic surrenders, additional expenses)

3.7.2 Asset portfolio

The asset portfolio is based on a global analysis of the composition of investments published by main actors on life-insurance French market through SII SFCR (Solvency and Financial Situation Report) in 2019. The choice made concerning the allocation of the initial portfolio for the study was based on an analysis of these observed distributions. The aim was also to have a non-negligible share of each asset class in order to highlight the effects different asset allocations would have.

The assets in the initial balance sheet have nevertheless been simplified compared to those of the insurers. In fact, our portfolio will only be composed of four assets categories: sovereign bonds, equity (including listed and unlisted equity, as well as OPCVM), property and cash.

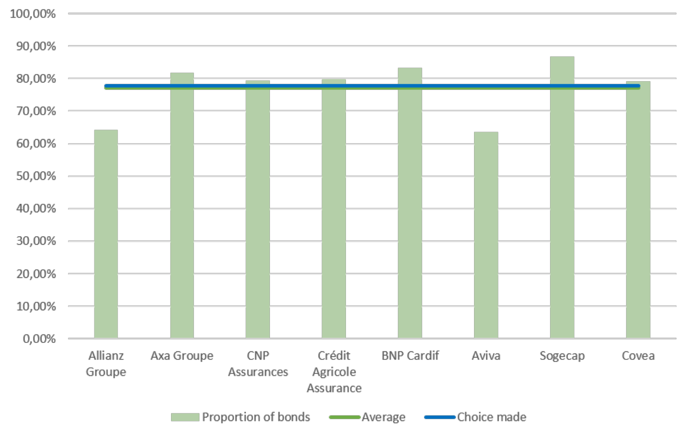


Figure 3.6: Bonds proportion based on insurers SFCR

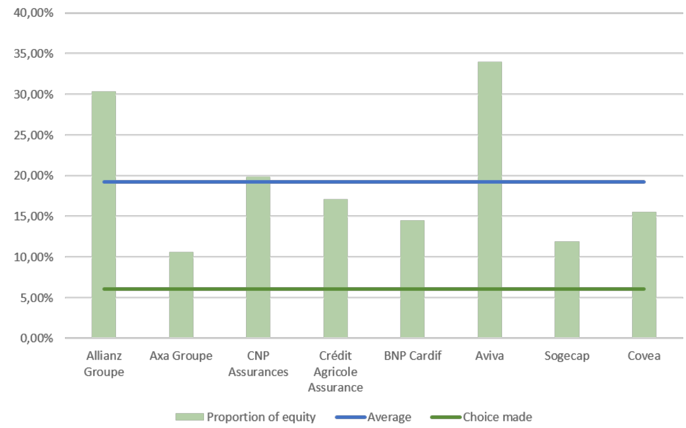


Figure 3.7: Equity proportion based on insurers SFCR

The initial allocation of the portfolio, with shares in percentage of the global market value, is depicted below:

Asset	Share
Bonds	78%
Equity	8%
Property	6%
Cash	8%

Table 3.2: Initial study-portfolio allocation

Remark: The modelled portfolio used in the following studies is only composed of government bonds as a matter of modelling simplification.

The balanced accounting balance sheet at t= 0 is displayed below.

Assets		Liabilities	
Government bonds	82 377	Capital Reserve	1 600
Equity	12 064	Mathematical Provisions	115 518
Property	9 047		
Cash	13 629		
Total	117 118		117 118

Figure 3.8: Accounting balance sheet at t=0

The portfolio was projected over 25 years. Usually, insurance companies make ALM model projections over a period of 40 to 50 years. The projection time has been shortened to reduce computation time which has been a constraint for modelling. In fact, numerous stochastic simulations had to be launched to produce the ALM study. Also, the choice was consistent with the observed end-of-projection materiality estimate. Indeed, at the end of the projection the mathematical provision represents less than 14% of the initial mathematical provision in the run-off model.

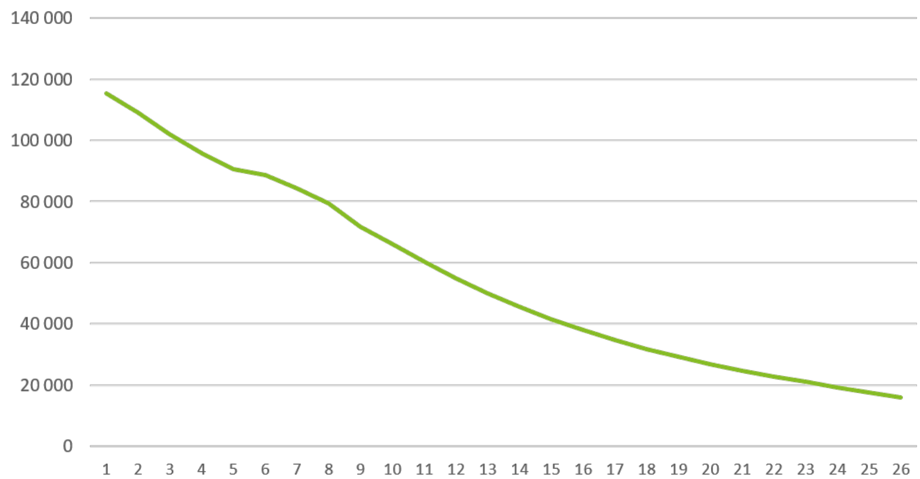


Figure 3.9: Decrease of the mathematical provision within the projection

There are no Own Funds because annual profits come from the portfolio.
The assets modelled only correspond to the modelling of the mathematical provisions, the capitalization reserve, and the liquidity risk reserve. This has a slight impact on the profit-sharing to be distributed. It is related to the difference in return between the assets corresponding to the Own Funds and the assets related to the products. Generally, this impact is not material, especially when everything is in a general fund, as it is the case for us. Therefore, no distinction was made.

3.7.3 Model and portfolio validation

To approve the model, we checked the model's consistency and reliability for a central scenario with the economic leak proof test and by looking at the values of the income statement.

The economic leak proof test consists in confirming that during the projection, there has been neither creation nor loss of value, such that:

$$MV_{\text{initial}} = VIF + BEL + DeferredTaxes$$

In our case, the model is in "Run-off", meaning there are no new contracts taken into consideration during the projection.

We can thus compare the initial value of the assets with the discounted flows values obtained with the model for each simulation. The model leak then corresponds to the average of the deviations obtained. If the deviation is considerable, the model will not be able to be correctly interpreted and therefore unusable.

For the deterministic projection, the obtained leak was of 0%, which was expected as no dynamic comes from this one simulation projection.

ASSETS	
Market Value of Assets	150 792
Bonds MV	116 051
Cash MV	13 629
Equity + Property MV	21 112
Ok	0

LIABILITIES	
Net Asset Value (NAV)	2 561
Discounted gross results	3 015
Taxes	1 111
Dicounted net results	1 904
Insurer's final P&L	-454
BEL	148 239
BE Services	137 643
Discounted services	117 118
Annuities	0
Deaths	3 147
Maturity	0
Surrenders	62 813
Partial surrenders	51 158
Ok	0
Funds (after tax and transfers) end projection	20 526
Final P&L distributed to insured	0
BE Premiums	-
BE Costs	10 596
Frais actualisés	5 485
Commissions	5 111

Sharing % for final P&L	90%
Final P&L	-568
TOTASS_FAV_FL(1)	22 846
TOTASS_MV_FL(1)	22 278
Final discount rate	79,95%
% BEL	
% BEL	98,3%
% PVFP	1,7%

Convergence deviation	-8 360	-0,0055%

Figure 3.10: End of projection balance sheet stochastic scenario

For the stochastic projection, we obtained a leak of 0.0055%. The deviation is acceptable given the acceptance threshold set at 0.5%.

The balance sheet displayed above, enables us to check the share of PVFP on the liabilities side.

$$\%PVFP = \frac{NAV}{\text{Market value of Assets}}$$

This value is between 1% and 2% which is a market standard.

The model and portfolio succeeded the market-standard validation tests and are convenient for the studies detailed in the next chapters.

Chapter 4

ALM study: solvency and profitability measurements

4.1 Framework and purpose of the study

In the current context, insurers are facing several challenges. One of these is to continue serving competitive income for their clients while remaining solvent and more specifically limiting their exposure to market risks.
Strategic asset allocation aims either at confirming that the existing asset structure is optimal or to propose an optimal asset structure that allows the company to achieve a certain financial performance target while meeting its commitments with a given level of confidence.
The obtaining of the most optimal asset allocation can be done from a risk/return perspective, i.e. the one that gives the best return, while minimising risk. This study will therefore be based on the Markowitz's Modern Portfolio Theory, which outlines how rational investors use diversification in order to optimise their portfolio and what should be the price of an asset given its risk compared to the average market risk.

The Markowitz theory is based on two assumptions:

- financial asset markets are efficient. From an objective point of view, assets' prices and returns are expected to reflect all information available concerning these assets.
- investors are risk-averse: they are unwilling to take more risk in exchange for a higher yield.

Real world projection is used in ALM studies, while risk-neural projection is privileged for regulatory requirements. Indeed, to be able to make investment decisions, it is essential to appreciate financial markets considering the inner performance of each investment.

ALM studies are conducted for various purposes (financial strategy improvement, internal communication, appreciation of the current situation of the insurer) using different indicators. Therefore some of the indicators available for the insurer are listed below:

- solvency indicators
- profitability indicators
- cashflows for policyholders
- taxes
- underwriting business and fees
- expenses
- ...

In this study, the chosen indicators were the profitability and the solvency. For the expected return the VIF is chosen because this yield is closely kept under surveillance by the insurers to follow the good wealth/profitability of their portfolio.
The SCR was the privileged indicator to explain the exposure to risk as this study was performed under Solvency II regulation. Closer attention was given to the interest rate module due to the economic environment.

The profitability is chosen as the ordinate and the SCR as the abscissa. This choice was made because the aim of the ALM study is to obtain an analysis displaying for each allocation tested the expected return in function of the risk taken.
The results of the study will give all the major information needed for the asset manager to make a decision about the insurer's portfolio.

There were three main steps to this study, each will be further detailed in the next sections:

1. Establishing the panel of allocations for the study
2. Producing a complete SCR for each allocation in a risk-neutral environment
3. Calculating the expected return for each allocation in a real-world environment

Like in the previous section, the portfolio studied is a simplified version of what can be found on the market.
The model was run for 32 different allocations. This low number is due to the non-automated process necessary to generate all the inputs for the model in the real-world and risk-neutral universes.

Asset category	Tested interval	Average step
Bonds	[60% , 80%]	1.5%
Equity	[0% , 16%]	1%
Property	[0% , 14%]	1%
Cash	[2% , 10%]	1.5%

Table 4.1: Framework of the study

The ranges were chosen to map the main possibilities insurers would have with their portfolio.

4.1.1 Projection universes

1. **Real world environment**
   This environment is mainly used for the strategic and financial management of the company. Assets are projected as faithfully as possible into the real world under historical probability. These models aim at imitating the behaviour of financial assets in the real world by capturing their main characteristics. The risks taken are considered and remunerated by a risk premium (premiums are different from one asset to another). Therefore, models are calibrated on past data.

   We determined two risk premiums, one for the equity assets and one for the property assets. The risk premium represents the difference between the required rate of return on the equity/real estate market and the risk-free rate.
   Equities were calibrated on the commonly used CAC40 index (performance of the 40 most important companies listed on the French stock exchange) and property on the IPDUFROR (a property index based on the average performance of property price evolution in France). We took the 10-year French OAT as the risk-free-rate on the market.

To get an estimation of the risk premium, we subtracted the average return of the 10-year OAT French bond to the chosen market indicator average annual return.
One limit of our approximation is that the IPDUFROR isn't publicly accessible and was calibrated on the 2006-2014 data period. The results are reported below:

Equity risk-premium	Property risk-premium
4.40%	2.5%

Table 4.2: Risk premiums

As expected, the risk premium assessed on the CAC40 is higher than the one assessed on the property index.

These averages where used to calibrate the ESG for the equity and property indexes and generate real-world scenarios that were used afterwards for the ALM study.

2. **Risk-neutral world**
Contrary to the previous universe, the risk-neutral universe is a theoretical universe where models are calibrated on observed market prices at the time of calibration.
The expectation of gains for an option is equal to the price of that option Scenarios are only understood on average, unlike scenarios in the real-world universe. This universe is part of a market-consistent valuation objective. Therefore, it will be used to evaluate the Best Estimate or NAV but also to estimate the value of a life insurance company (MCEV).

In this study, we studied the SCR based on a simplified calculation using NAV values[1].

4.2 Calculation of the SCR

4.2.1 Assumptions and limits of the study

As explained previously, the process enabling the ALM study was not automated. To spare time, the calculation of the SCR wasn't performed for every allocation through the asset and liability model but was estimated using a replicating Excel interface associated with Python code that was developed internally. Therefore, a part of the SCR was considered stable over the allocations tested.

4.2.2 Life SCR calculation

For the purpose of this study, we decided to freeze the Life SCR for all tested allocations. For recall, life risk calculation, as specified in the Article 136 of the Delegated Acts [7], is composed of:

- a mortality risk[2]
- a longevity risk[3]
- a disability-morbidity risk[4], not modelled
- a life-expense[5]
- a revision risk[6], not modelled
- a lapse risk[7]

[1] Cf. Chapter 1 SCR Calculation: standard formula
[2] Commission delegated Acts - Article 137
[3] Commission delegated Acts - Article 138
[4] Commission delegated Acts - Article 139
[5] Commission delegated Acts - Article 140
[6] Commission delegated Acts - Article 141
[7] Commission delegated Acts - Article 142

- a life-catastrophe risk[8]

These risks are aggregated using the correlation matrix given by the EIOPA for this risk. We obtained the following Life SCR:

Life SCR	**12 644 713 €**
Mortality risk	-
Longevity risk	14 507 €
Life-expense	84 901 €
Lapse risk	12 447 727 €
Life-catastrophe risk	10 238 €

Table 4.3: Life SCR module

We noticed that our life risk was majorly composed of lapse risk.
Lapse SCR is calculated using the following formula:

$$\text{Lapse risk} = \max\left(0, \text{decrease lapse risk, increase lapse risk, massive lapse risk}\right)$$

Our portfolio was only sensitive to lapses' decrease which is detailed in the following paragraph.
We gathered further information about our exposure to the decrease lapse risk and particularly on the different flows' interactions during the projection. This sensitivity to lapse risk can be explained by the economic environment and the composition of the portfolio.

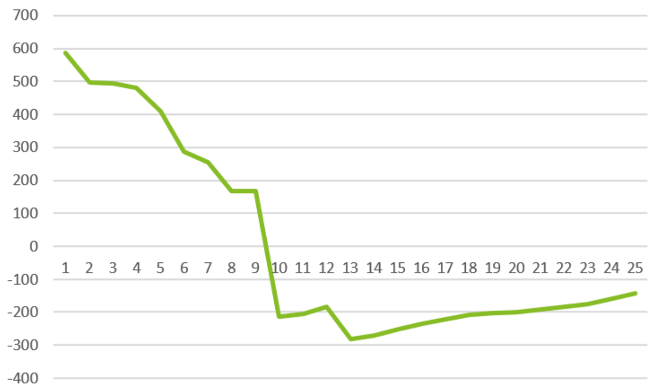


Figure 4.1: Financial result

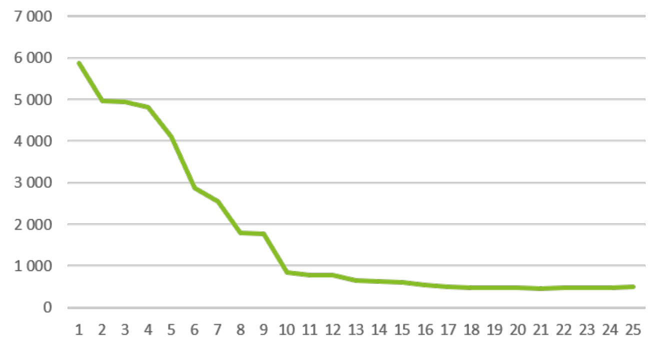


Figure 4.2: Financial production

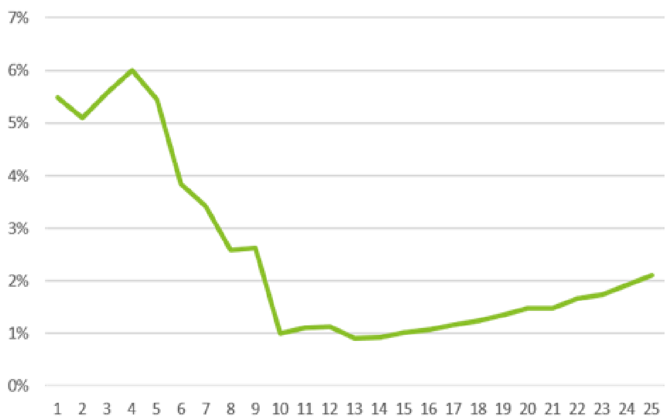


Figure 4.3: Yield of the bonds in the portfolio

[8] Commission delegated Acts - Article 143

The financial result is an interesting indicator to explain this situation. Indeed, from the beginning of the projection and more intensely closer to the 10^{th} year of projection, the result is declining and even becoming negative in the 11^{th} year of projection.
First, it can be explained by the low financial return observed from year 10, which is due to the lack of return on the invested bonds.
This abrupt decline is due to the portfolio composition. In year 10, all bonds constituting the initial portfolio will have reached their maturity and new bonds are bought. However, even if bonds are bought in the model, they have a low income as they are bought at the money in a low interest rates environment. Due to the risk-neutral projection used for the calculation of the SCR, the return and the proportion of the equities and properties in portfolio can't improve the situation.

The slight increase occurring from year 14 is due to the increase in the bonds' yield, due to the increase of the spot rate curve and therefore the slight increase of return for the bought bonds.

Despite this unfavourable situation, the insurer must respect his commitments towards his policyholders. Therefore, whatever the number of contracts in portfolio is, he still has to revalue the capital of his policyholders on the base of the guaranteed minimum rate but also to ensure the viability of his portfolio. Another factor to keep in mind is the management loading which is proportional to the mathematical provision (increase in the case of a decrease of the lapse rate) deducted from the financial production because it is already counted in the technical result.

This leads to a lower financial result when the number of lapses decrease as presented below.

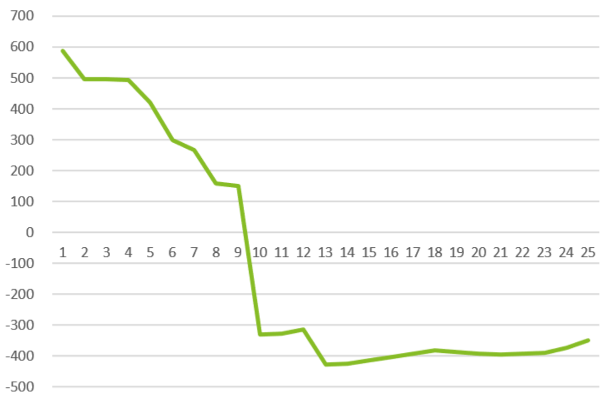


Figure 4.4: Financial result for the lapse risk scenario

Bigger is the number of policyholders in the portfolio, weaker is the insurer's situation, and therefore it justifies why our portfolio is sensitive to the decrease lapse risk and not to the others.

The following chart represents the evolution of the credited and the target rates for the central scenario and the lapse risk scenario.
The rate served in the shocked scenario is lower due to the higher number of policyholders and therefore the amount of mathematical provisions. After year 15 the insurer's situation enables him to meet his target rate which expresses the distress of his situation. The sharp increase of the served rate around year 5 is due to an import release of Bonus Reserve in favour of policyholders.

Figure 4.5: Evolution of the target and served rate over the projection for the central scenario

Figure 4.6: Evolution of the target and served rate over the projection for the lapse shock scenario

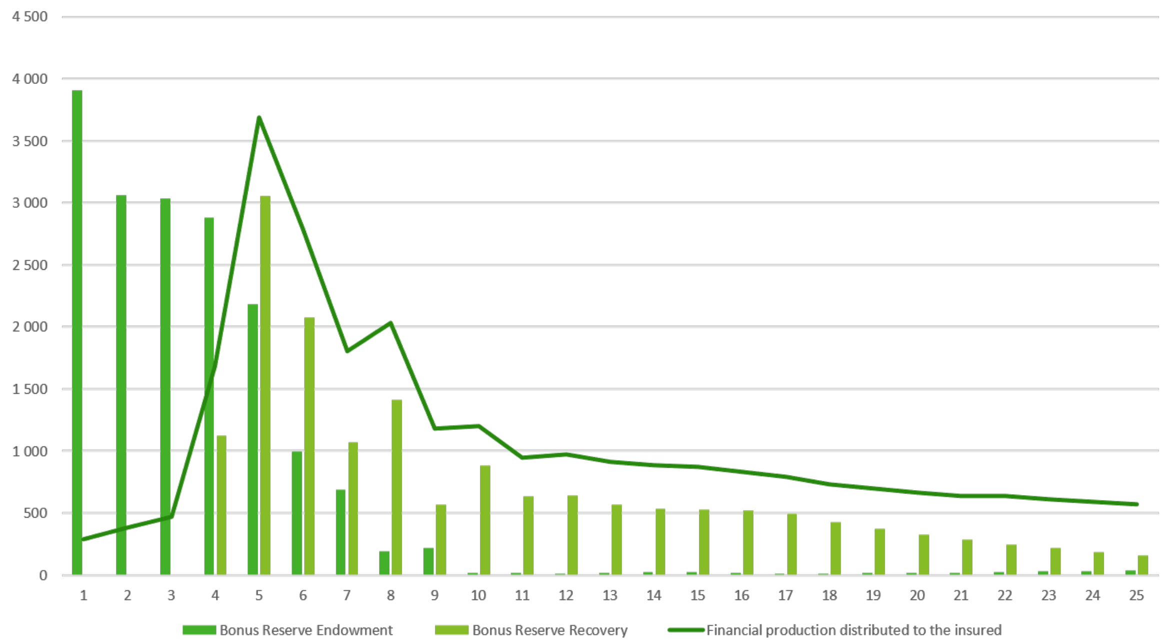


Figure 4.7: Interactions around the profit-sharing -central scenario

The Bonus Reserve is highly endowed at the beginning of the projection and releases a large amount around year 5. The algorithm of the Bonus Reserve management doesn't seem optimal and should be smoothed over time for a better management of the resources. However, this subject won't be treated in this report.

In this study, we decided to focus on the market risk. The improvement required to correct the use of the Bonus Reserve wasn't the purpose and won't be explored.

Disclaimer: It should be noted that this simplified modelled bonus reserve and the management of the profit-sharing strategy draw a limit for the following studies. Indeed, a smoother and better driven release of the bonus reserve would have enabled the insurer, in the case of unfavourable events, for example of equity losses, to recover part of the bonus reserve (belonging to the insured) to serve the target rate and therefore improve his result and profit (i.e. VIF). It should be kept in mind when reading the conclusion of the ALM study.

4.2.3 Market SCR calculation

Market risk for the initial portfolio was assessed through the asset and liability model. Our portfolio is only composed of Euro economy valuated assets, implying a null risk of change.
For simplification, we suppose that our portfolio is diversified and therefore not subject to the concentration risk, neither the spread risk.

We decided to make the following assumption: the equity shock applied for this study and the following one (Chapter 5) will be equal to 49% regardless of the type of equity in portfolio. This assumption is strong and has to be kept in mind when analysing the results.

This position was taken as a conservative approach: we will analyse worst situations than those observed with the correct shock rates.

Market SCR	**4 695 408 €**
Interest Up risk risk	-
Interest Down risk	2 025 031 €
Equity risk	2 614 081 €
Property risk	855 536 €

Table 4.4: Market SCR results for the initial portfolio

For recall, the portfolio has the allocation displayed below.

Asset	Share
Bonds	78%
Equity	8%
Property	6%
Cash	8%

Table 4.5: Initial study-portfolio allocation

The value for the equity risk is high, due to the high value of shock applied to this type of asset. Moreover, we notice that our portfolio isn't sensitive to the increase of interest rates but to its decrease. The increase of the interest rates isn't a risk because dynamic lapses are not triggered despite a rise in interest rates as rates are initially very low. Overall a rise in interest rates only comes into play when dynamic redemptions are triggered. On the other hand, the interest rates decrease risk is the riskier in our situation.

For all the allocations tested, we decided to use an Excel/ Python tool replicating all market risks shocks to assess the market SCR.
As a matter of simplification, the interest rate shock is calculated using the sensitivity approach. This approach consists in projecting the shock on the market exposure adjusted with the sensitivity to interest rate risk, most commonly known as the modified duration. The interest rate exposure is obtained with the following formula:

$$SCR_{\text{interest rate}} = \text{Exposure x } (\Delta \text{ r x Modified Duration})$$

- By Modified Duration, we refer to the average life (duration between the current date and the payment date) of the cash flows weighted by their likely discounted value. A good approximation of the duration can be obtained by the sensitivity.

$$\text{Modified Duration} = \frac{\sum_{i=0}^{\text{maturity}} \frac{\text{i x Flows}_i}{(1+r_i)^i}}{\sum_{i=0}^{\text{maturity}} \frac{\text{Flows}_i}{(1+r_i)^i}}$$

- The exposure is the market value of the asset and Δ r is estimated for each maturity.

Both equity and property shocks were applied to the initial market prices of the assets.

In this tool, the interaction between the asset and liability isn't modelled, we used the assumption that this interaction could be approximated by applying the loss-absorption rate to the SCR value obtained via the tool. The loss-absorption capacity process of the insurer is described in section 1.3.1. This capacity is primordial in the ALM model and even if we chose to approximate the model, not taking it into consideration wouldn't have been consistent with the study we were going to do.

The loss-absorption capacity is used to calculate the net SCR after obtaining the gross SCR such that:

$$SCR_{net} = SCR_{gross} \text{ x loss-absorption capacity} = SCR_{gross}(1\text{- absorption rate})$$

It can be easily obtained for each risk module as:

$$\text{loss-absorption capacity} = \frac{(BEL_{shocked} BEL_{central})}{(VM_{shocked} VM_{central})}$$

For each shock, the loss-absorption capacity was, first, obtained using the ALM model:

Equity	Property	Interest (shock down)
55.78%	62.17%	1869%

Table 4.6: Absorption for each risk

For the interest rate risk, the unusual loss-absorption capacity is justified by the composition of the portfolio and the way the interest rate shock is applied following the EIOPA's Standard Formula guidelines.
Indeed, the interest rate shock can't be applied to negatives rates, and on the 12/31/2019, rates were negative until year 7 of projection (refer to section 1.4.1). According to the method of calculation of bonds' prices, the deflating rates used are the same until the 7^{th} year, so bonds with a duration inferior to 7 years will have the same market value in the central and shocked projections.

As a matter of fact, our portfolio has an average duration of 3.6 years. This duration is short but not unusual in insurers' asset portfolios, on the other hand our liability portfolio has an 8.3 years duration.

Remark: An additional study is conducted concerning the observed duration gap in section *Appendix A: In-depth study of the bonds allocation strategy.*

This situation implies that the delta between the market values of the central and shocked scenarios are so close that the loss-absorption capacity exploded.

in 10^3€	Central	Interest (shock down)
market value	150 792	150 906
NAV	2 553	527

Table 4.7: Comparison between the central scenario and interest rate shock down

Still in our search for an optimized process, we decided to study the loss-absorption capacity's behaviour in the case of the interest shock down. To do so, we ran 7 allocations, by varying the proportion of bonds and cash in the portfolio. Each allocation was used in the ALM model to run a central scenario and an interest rate down shock scenario to be able to calculate the corresponding SCR. Simultaneously, we used these allocations to calculate the SCR via an Excel/Python Tool before absorption by the liabilities.

We obtained the following results and calculated the loss-absorption capacity as:

$$1 - \frac{SCR_{\text{ALM model}}}{\text{SCR before absorption}_{\text{estimated}}}$$

	Initial allocation	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6
Bonds proportion	77.8%	60%	63%	67%	70%	73%	80%
Prophet IRdown SCR	2 025	1 613	1 645	1 667	1 704	1 779	2 079
Excel IRdown SCR before absorption	9 265	9 272	9 271	9 270	9 268	9 267	9 264
Absorption capacity	78.14%	82.63%	82.25%	82.02%	81.61%	80.80%	77.56%

Table 4.8: Loss-absorption capacity calibration, results in million

The curve shows a decline in the loss-absorption capacity when the proportion of bonds increases.

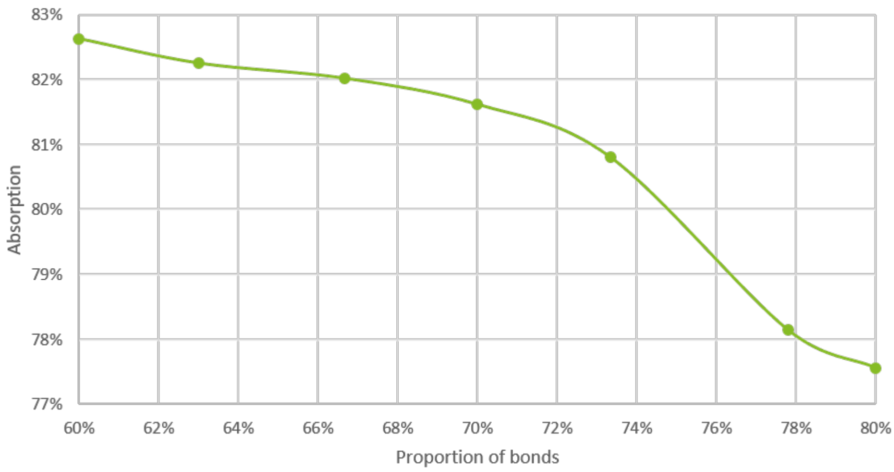


Figure 4.8: Absorption for the interest rate shock down explained by the proportion of bonds in portfolio

Since the gross SCR from the Excel tool is stable and the net SCR increases proportionally to the proportion of bonds in portfolio, it is not surprising to obtain an absorption capacity decreasing with the increase of bonds in the portfolio. This is mainly due to the method of calculation of the absorption capacity.

This curve was used to deduce the absorption for each bond allocation tested using interpolation techniques.

We ran a similar study for the loss-absorption capacities of the equity and property classes. 7 allocations were run for each class with varying proportions of equity (i.e. property) and cash, while the other classes remained fixed.
The absorption capacities were obtained using the outputs of the ALM model.

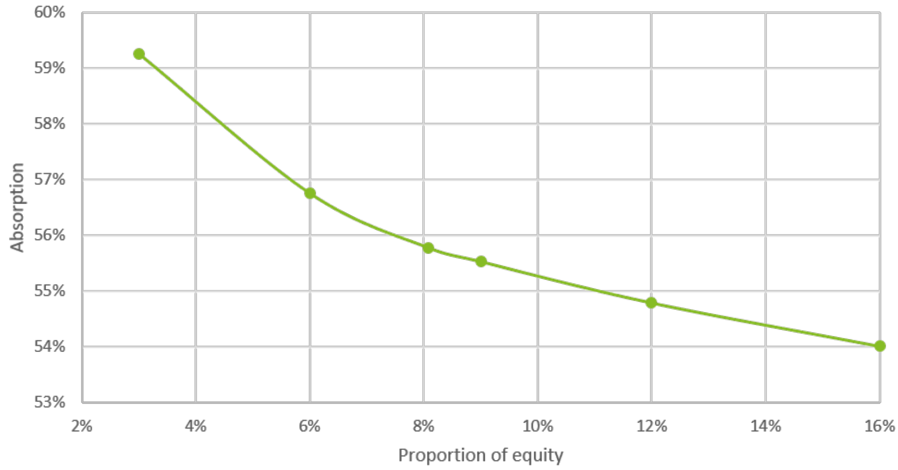


Figure 4.9: Absorption for the equity shock explained by the proportion of equities in portfolio

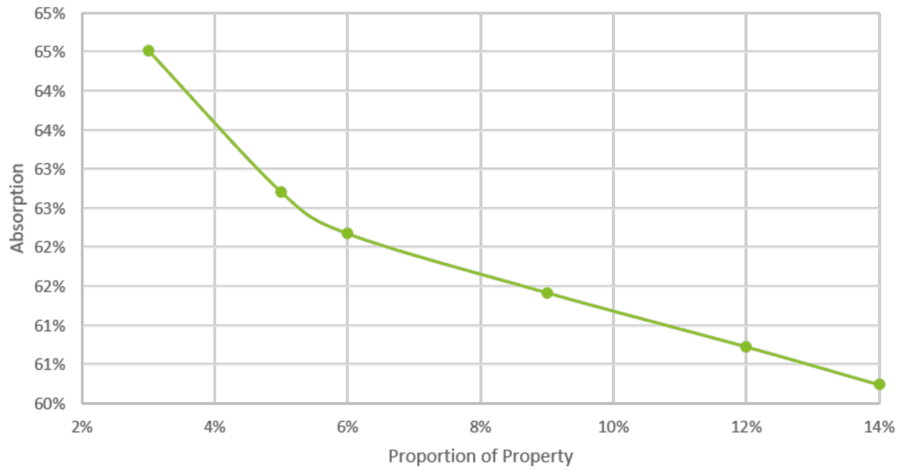


Figure 4.10: Absorption for the property shock explained by the proportion of properties in portfolio

The chart shows flat curves for both shocks, so we decided to keep a fixed capacity absorption for these shocks equal to the ones obtained for the initial portfolio composition.

4.3 Results and analysis

For each allocation tested, the SCR was obtained through the Excel/Python tool and a real-world central scenario was run with the projection model to obtain the indicators for the study. The VIF was obtained by running real-world central scenarios in the ALM model.

Each allocation was placed on a two dimensions chart and then interpreted depending on the allocation of each class of assets. The graphs are displayed below.

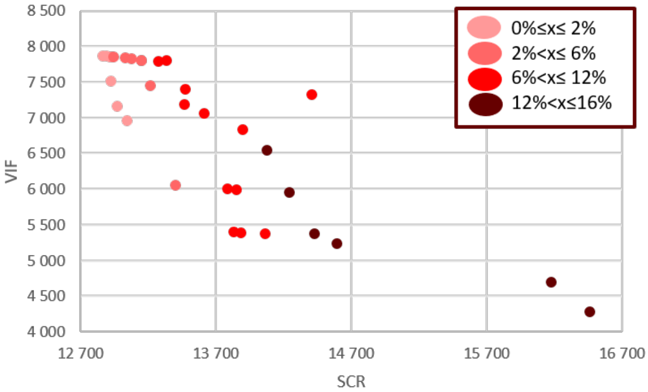


Figure 4.11: VIF/SCR based on the proportion of equity

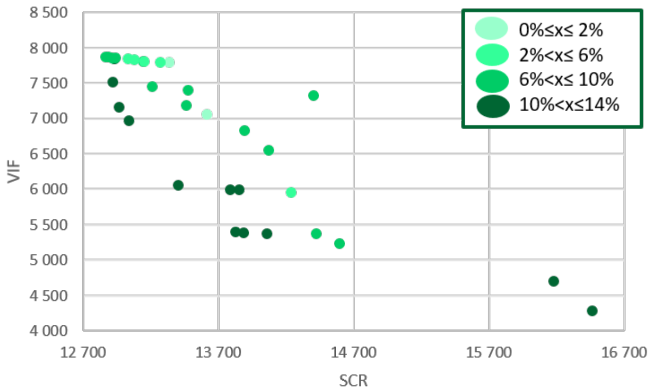


Figure 4.12: VIF/SCR based on the proportion of property

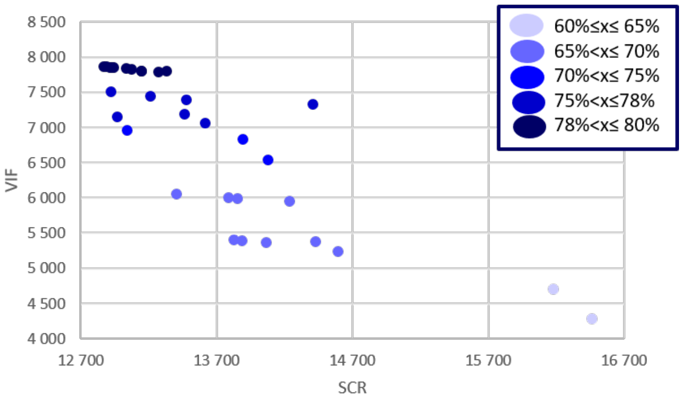


Figure 4.13: VIF/SCR based on the proportion of bonds

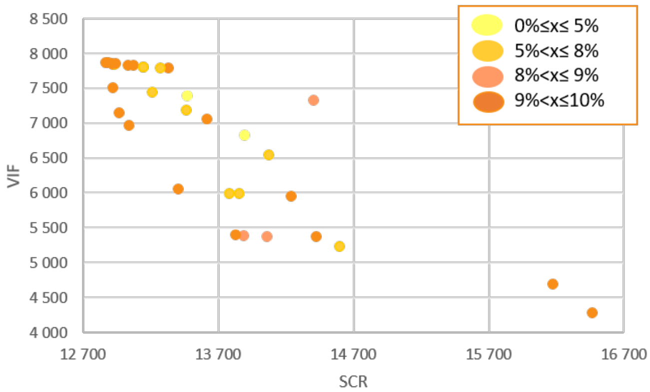


Figure 4.14: VIF/SCR based on the proportion of cash

Given the stakes and regulatory changes, ALM processes must evolve aiming at improving risk management, optimising SCR and limiting its volatility.

The linear aspect of the graphics can be explained by the inherent low TVOG (i.e. TVOG for the initial portfolio allocation is equal to less than 3.5% of the deterministic BEL).
TVOG comes from the dynamic resulting from the policyholder's behaviour in stochastic projections (profit-sharing, dynamic lapses). Indeed, all insurers are facing a difficult situation due to the economic context and low rates environment, therefore surrenders are rarely triggered. In addition, due to our Minimum Guaranteed Rate being equal to 0% and the percentage profit-sharing to 90%, our financial result is close to being deterministic. It justifies why our chart is more or less a straight line.

The allocation optimizing the profitability while mastering the encountered risk is included in the following intervals.

Bonds	Equity	Property	Cash
[78% ; 80%]	[0% ; 2%]	[6% ; 10%]	[9% ; 10%]

Table 4.9: Allocation optimizing the risk/return couple

Disclaimer: It should be noted that the modelled bond portfolio only includes government bonds and therefore omits a share of corporate bonds which could imply credit risk and a correlation with equity risk not taken into account here. The simplification of the modelled financial assets and the high share of bonds may have led to an imbalance in the results obtained at the cost of other asset classes.
Furthermore, under Solvency II, equities represent a significant capital charge on the SCR due to

their high volatility, which encourages insurers to drive their investment strategy towards the bond market. In addition, the low interest rate environment is leading investors to consider corporate bonds to seek a return which cannot be obtained with sovereign bonds. The interesting study conducted by SABOURIN E. depicts the implementation of a credit risk model in an ALM model[22].

Analysis of the optimal portfolio

First, the recommended allocation for equity is the consequence of the Solvency 2 regulation which penalises equity more than other asset classes. This phenomenon is amplified in our study due to the assumptions we chose. The method of determining the SCR presented above is based on approximations overestimating the required capital. However, this overestimation of risk generates a surplus of capital to be immobilize and therefore a higher cost for the insurer. Other SCR alternatives are not explored in this report, a study can be found in EK O-D report[15].

Property and bonds are slightly higher than the original allocation. This ensures a certain level of income for the portfolio while still securing the encountered risk.

Finally, the large amount of cash represents less risk and therefore assures a controlled risk.
If we could sum the analysis, it would be performed as in the chart below.

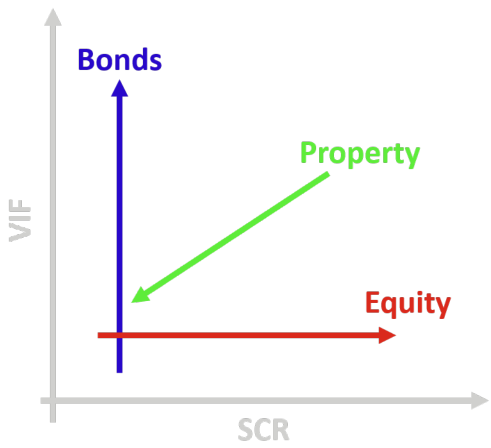


Figure 4.15: ALM study results simplification

To conclude, this ALM study showed that the initial allocation of the portfolio is well-balanced compared to the tested allocations. It provides a medium exposure to risks while ensuring a well-placed return. The optimal solution gives insights on how the portfolio allocation could be more efficient. However, the assumption made for the equity biased the result concerning this category. The consideration of the real equity shock may allow to increase the share of equity in the portfolio.

Moreover, because negative rates aren't shocked in the Standard formula, the SCR might not be the best indicator to use. Indeed, currently rates are negatives and the SCR is limited in the presence of negative rates which distorts the results of the ALM studies. We can therefore imagine a risk indicator where negative rates would be shocked. This would be more realistic and would attest to the resilience of insurers in the current economic context and not a better version of it.

To perform ALM studies monitoring indicators such as the SCR or the VIF, it is also possible to develop dynamic allocation strategies which evolution depends on the behavior of the chosen indicators. This would have the benefit of optimizing the result for each allocation tested and sharpening the results for the insurer. This kind of dynamic allocation is fully detailed in the report of SARR A.[23].

Chapter 5

Improvement of the allocation strategy

This section presents the impacts of developing a new allocation strategy in an existing ALM model. In our case, the existing strategies were the ones supplied with the original version of Prophet and consequently could be improved to replicate at best the actions an asset and liability manager would do, especially in a low interest rates context. This study aims at showing the possibility for an insurer to easily improve his model of asset and liability management even in a simplified version of an ALM model and be more aligned with real practices.

In our model, the allocation strategy is performed once a year. This step occurs after maturing the assets and before taking into account the profit-sharing strategy.

The initial allocation strategy was a fixed percentage strategy. It consisted in defining a percentage for each asset category. The algorithm was meant to meet it at the end of the projection year. These actions occurred after integrating the New Money.
In a run-off model, the New Money is the integration of periodical premiums and deposited services.

$$\text{New Money} = \text{Cashflows in} - \text{Cashflows out}$$

Our portfolio didn't generate periodical premiums which lead, most of the time, to a negative new money.

For the assets' dynamic to better match reality, we chose to use a Drift strategy, enabling the assets to evolve more dynamically between thresholds configured for each asset category. With this strategy, it is possible to adjust the position of the assets between the thresholds depending on the economic situation.

For recall, our portfolio is composed of 4 asset categories: equities, properties, bonds and cash. The allocation strategy studied will only concern these 4 but can be easily extended to a larger number of categories.
In the algorithm, each category is treated in a predefined order. We defined the order as follows: first equities are treated, then property, after are the bonds and finally the cash.

After studying the existing strategy and its limits, the development of the drift strategy will be detailed. Finally, studies have been conducted on how to optimize the choice of thresholds and associated parameters when watching risk indicators.

5.1 Implementation of the threshold strategy

The gain in flexibility with the new algorithm is obtained by distinguishing the main processes at:

- Strategies level
  All three classes are treated separately in the code.
- Reporting level
  The processing of all categories takes place before the purchases and sales of all segments.

Within the framework of this study, we based our work of implementation on the methodology suggested in 2013 by Pierrick PIASER [20] in his thesis on measuring the impact of the financial strategy on PVFP and regulatory capital.

5.1.1 Step 1: category and segment processing split

This alteration is structuring for the algorithm. In the existing algorithm the loop performing purchases and sells on all segments was nested in the loop on all categories (cf. Figure 3.4 in section 3.5.1). The change consists in taking the processing of the segments out of the loop processing categories. Once the purchases/sales of all categories are calculated, the processing of buying/selling at segment level takes place in a second loop.

For each segment the category is identified to allow the portion of the purchase/sale amount to be allocated. The existing processing at segment level is not changed.

This modification increases the flexibility of the algorithm as the strategy can be carried out taking into account all categories before any purchases/sales are made. In addition, each segment in the existing algorithm was crossed as many times as the number of categories within an iteration. The alteration allows each segment to be browsed only once per iteration.

5.1.2 Step 2: Introducing a cash pivot method

One of the main problems of the algorithm was that it might not reach the optimal solution, and by the same time the wanted allocation. This was mainly due to the fact that each asset category was depending on the market value after allocation of the previous asset category, there was no exchanges between categories.
To avoid this inconvenience, we decided to fix all needed amounts for each category, previously to acknowledging any decision.
However, to make it possible, one category had to be loosened, for the allocation to stay flexible. As in the existing algorithm, the choice was made to keep the cash as a residual category, as it is most commonly practiced on the market.

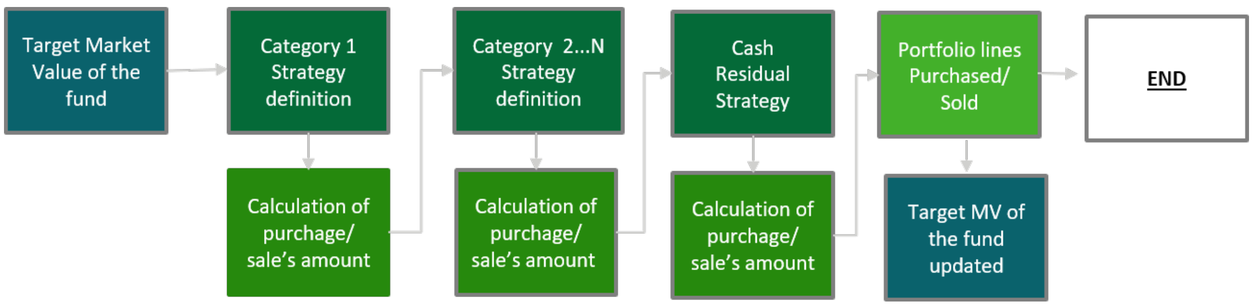


Figure 5.1: New algorithm implemented

5.1.3 Threshold Management

The threshold management, included in the drift strategy, is the introduction of predefined limits for each asset category. Unlike in the original strategy the asset isn't restricted to meeting one fixed target but is enabled to be between two boundaries, depending on the economic situation.
Each time an asset is above (or below) its threshold, the algorithm will bring it to the predefined threshold by selling (or buying) the exact amount of asset needed.

For example, let's take the equity category for which the limits are defined as below:

Allocation bottom limit	2%
Allocation upper limit	7%

Table 5.1: Configuration of the thresholds strategy

The algorithm will respond as detailed below:

	Example 1	Example 2	Example 3
Proportion of the Market Value before allocation	5%	12%	1,5%
Assessment	Proportion between boundaries	Proportion above the maximum boundary	Proportion under the minimum boundary
Proportion of the Market Value after allocation	5%	7%	2%

Table 5.2: Results of the algorithm for different situations

For the modelled portfolio, we obtained the following results before introducing thresholds to the cash category.

Figure 5.2: Proportion of (a) equity (b) property (c) bonds (d) cash every time step before cash management

Adding a threshold to the cash category enables the insurer to avoid negative cash as noticeable on the previous figure.
The cash was fixed between 0% and 11%.
To do so, some of the other assets have to be sold/bought. We decided that bonds are bought when there was too much cash and equities are sold when cash was missing. We made this choice under Solvency 2 regulation constraint. Indeed, equities are more capital requiring than bonds. It is important to note that if these two categories are not sufficient to balance cash, property can also be used.

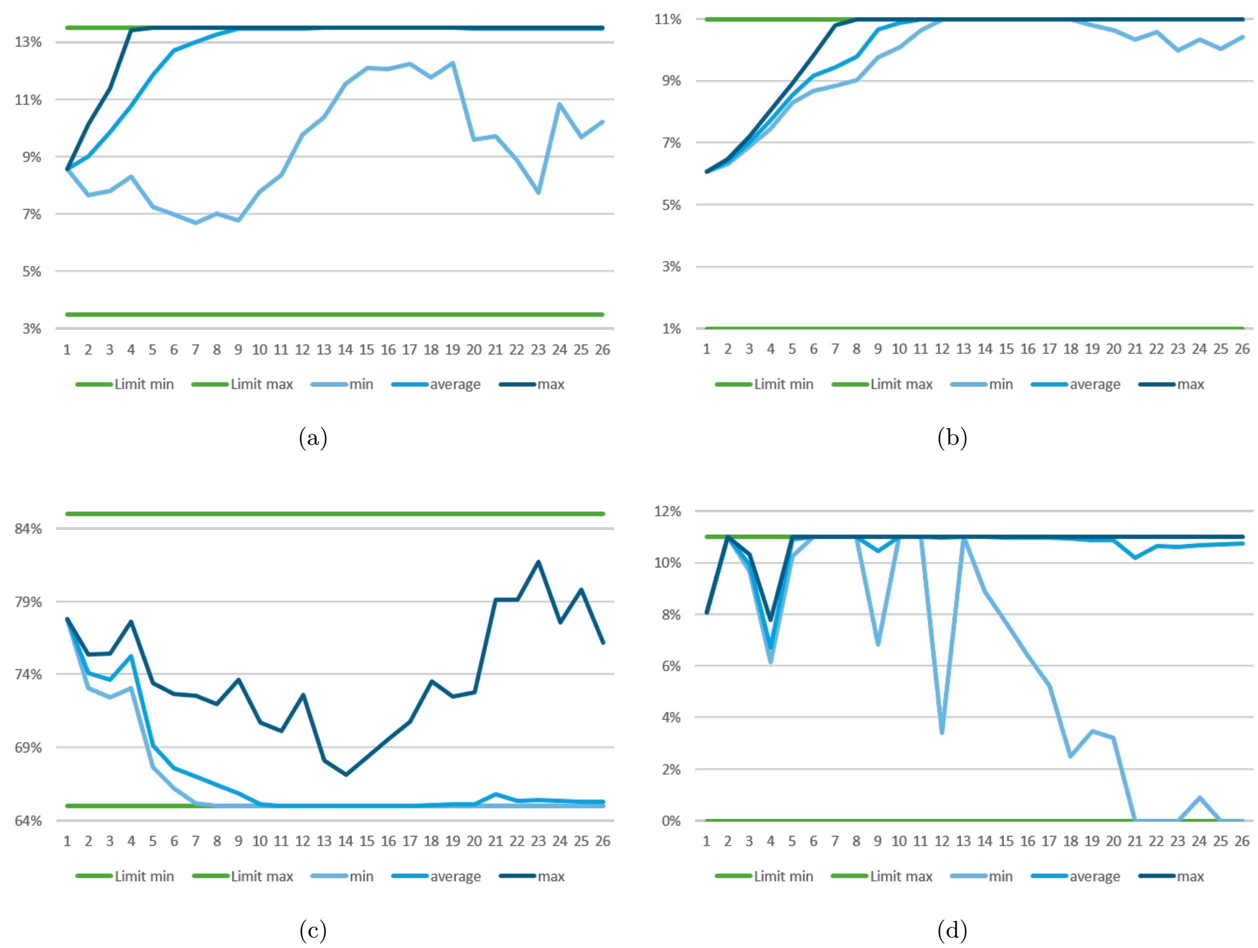


Figure 5.3: Proportion of (a) equity (b) property (c) bonds (d) cash every time step with cash management

Due to the wide intervals we chose for the thresholds, the impact on the result was small between step 2 and the threshold management. The delta between NAVs was less than 0.1%.

5.1.4 Introduction of dynamic targets

To amplify the possibilities for the insurer to master his portfolio allocations we decided to introduce dynamic targets. This target located in the middle of the limits previously determined, is coupled with a recall capacity.
For a recall capacity of 100%, the allocation will perfectly meet the target fixed in the middle of the thresholds. On the other hand, a recall capacity of 0% will lead to the results obtained in the previous section (matching the closest threshold). The recall capacity can be set for each year of the projection, meaning multiple functions can be implemented. The aim was to narrow the possibilities of these functions. Also, as the target depends on the limits, it is suitable to conduct a range of sensitivities to find the optimal settings. This process will be discussed in the next section.

It is also important to mention that a recall capacity of 100% has the same results as the asset-fixed strategy originally implemented. Indeed, that's what we confirmed by testing the following drift strategy and obtaining the same accounting result.

	Equity	Property	Bonds
Initial fixed strategy	8%	6%	77%
Thresholds tested with a 100%-recall	[6% ; 10%]	[4% ; 10%]	[76% ; 79%]

Table 5.3: Example of thresholds with a 100% recall capacity resulting in the same results as the fixed strategy

The SCR, calculated using the approximation of a 49% shock for all equities, was identical for both strategies and equal to the market SCR presented in section 4.2.3.

5.2 Threshold with dynamic target optimisation

For this study, we first did a number of tests starting from defined thresholds and then widening or narrowing the distance between the thresholds to measure the impact on the SCR. We decided to study the SCR because this risk indicator is commonly privileged by the insurers to drive their investment strategies.
We first stated a starting point for the intervals. This starting point was arbitrarily chosen but could be related to a management agreement for an insurer.

Our initial thresholds were defined as follows.

	Equity		Property		Bonds	
	Min	Max	Min	Max	Min	Max
Thresholds	4%	12%	6%	13%	72%	82%

Table 5.4: Initial thresholds configuration

The first attempt of this study was to measure the impact of the thresholds for a drift strategy with a 0%-recall capacity.
The thresholds parameters were tested with 20 tests displayed in the following table and the cash was enabled between 0% and 11% thresholds. These thresholds are wide enough to avoid major cash allocations.

		Equity		Property		Bonds	
	Delta tested	min	max	min	max	min	max
1	-3,50%	7,50%	8,50%	9,50%	9,50%	75,50%	78,50%
2	-3,00%	7,00%	9,00%	9,00%	10,00%	75,00%	79,00%
3	-2,50%	6,50%	9,50%	8,50%	10,50%	74,50%	79,50%
4	-2,00%	6,00%	10,00%	8,00%	11,00%	74,00%	80,00%
5	-1,50%	5,50%	10,50%	7,50%	11,50%	73,50%	80,50%
6	-1,00%	5,00%	11,00%	7,00%	12,00%	73,00%	81,00%
7	-0,75%	4,75%	11,25%	6,75%	12,25%	72,75%	81,25%
8	-0,50%	4,50%	11,50%	6,50%	12,50%	72,50%	81,50%
9	-0,25%	4,25%	11,75%	6,25%	12,75%	72,25%	81,75%
0	Initial settings	4%	12%	6%	13%	72%	82%
10	0,25%	3,75%	12,25%	5,75%	13,25%	71,75%	82,25%
11	0,50%	3,50%	12,50%	5,50%	13,50%	71,50%	82,50%
12	0,75%	3,25%	12,75%	5,25%	13,75%	71,25%	82,75%
13	1,00%	3,00%	13,00%	5,00%	14,00%	71,00%	83,00%
14	1,50%	2,50%	13,50%	4,50%	14,50%	70,50%	83,50%
15	2,00%	2,00%	14,00%	4,00%	15,00%	70,00%	84,00%
16	2,50%	1,50%	14,50%	3,50%	15,50%	69,50%	84,50%
17	3,00%	1,00%	15,00%	3,00%	16,00%	69,00%	85,00%
18	3,50%	0,50%	15,50%	2,50%	16,50%	68,50%	85,50%
19	4,00%	0,00%	16,00%	2,00%	17,00%	68,00%	86,00%

Figure 5.4: Value of thresholds tested for each asset class

					SCR share		
	Equity SCR	Property SCR	Rate SCR	Market SCR	Equity	Property	IR
1	3 044	795	2 031	5 625	54,1%	14,1%	36,1%
2	3 619	967	2 042	6 341	57,1%	15,3%	32,2%
3	4 040	1 168	2 051	6 924	58,4%	16,9%	29,6%
4	4 518	1 273	2 060	7 493	60,3%	17,0%	27,5%
5	4 885	1 399	2 068	7 965	61,3%	17,6%	26,0%
6	5 081	1 536	2 075	8 274	61,4%	18,6%	25,1%
7	5 192	1 573	2 077	8 417	61,7%	18,7%	24,7%
8	5 298	1 593	2 080	8 541	62,0%	18,7%	24,4%
9	5 340	1 615	2 083	8 603	62,1%	18,8%	24,2%
0	5 372	1 637	2 087	8 656	62,1%	18,9%	24,1%
10	5 401	1 661	2 090	8 707	62,0%	19,1%	24,0%
11	5 432	1 687	2 093	8 761	62,0%	19,3%	23,9%
12	5 467	1 712	2 097	8 819	62,0%	19,4%	23,8%
13	5 496	1 739	2 101	8 873	61,9%	19,6%	23,7%
14	5 545	1 758	2 108	8 943	62,0%	19,7%	23,6%
15	5 611	1 769	2 116	9 027	62,2%	19,6%	23,4%
16	5 659	1 780	2 123	9 089	62,3%	19,6%	23,4%
17	5 714	1 788	2 130	9 157	62,4%	19,5%	23,3%
18	5 713	1 796	2 133	9 166	62,3%	19,6%	23,3%
19	5 711	1 804	2 137	9 174	62,3%	19,7%	23,3%

Figure 5.5: SCR corresponding to each threshold tested

First, we notice that, even if the observed SCR varies, the contribution of each risk module stays stable.

Equity shock	Property shock	IR shock
61.1%	18.5%	25.5%

Table 5.5: Average of the contribution of each risk module to the market SCR

Another observation we can make when looking at these results is that the variations observed between the smallest observed SCR and the maximum SCR is bigger for the property and equity risks than for the interest rate shock.
This first statement comes from the application of the shock which is linear for the equity and property risks but not for the interest rate shock, especially due to the complexity of the relation between bonds and interest rates curves.
Also, the high value for the equity SCR is due to the chosen SCR assumptions, explained in chapter 4.

The second statement is about the situation of the portfolio in the central scenario. The PVFP obtained using the risk-neutral projection decreases with the widening of the thresholds as shown in the graph.

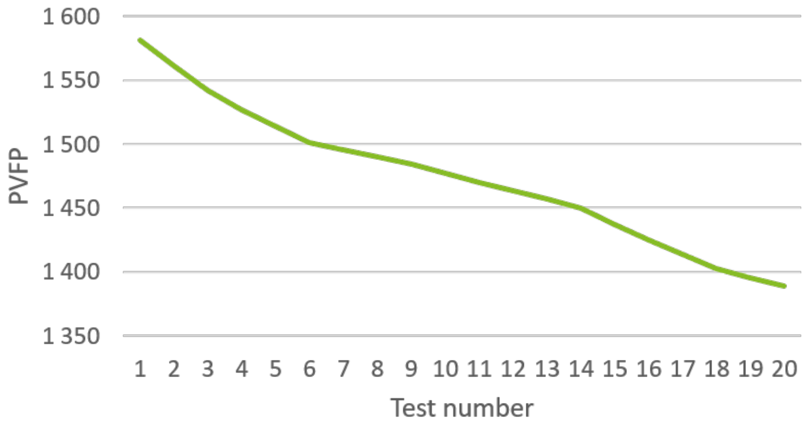


Figure 5.6: Evolution of the PVFP depending on the thresholds widening

The final statement made with these first results is that closer are the thresholds, minimum is the SCR.

To explain this phenomenon, we decided to look closer at the extreme points of the above curve. The thresholds and results studied are the following.

	Thresholds	Equity SCR	Property SCR	Interest rate SCR
Test 1 (close thresholds)	Equity:[7.5% ; 8.5%] Property:[9.5% ; 9.5%] Bonds:[75.5% ; 78.5%]	3 044	795	2 031
Test 19 (distant thresholds)	Equity:[0% ; 16%] Property:[2% ; 17%] Bonds:[68% ; 86%]	5 711	1 804	2 137

Table 5.6: Information of the studied tests

a) **Behaviour of the assets before shock**

To attest the allocation strategy implemented, we decided to use the deterministic scenario. The next group of graphics displays the proportion of the asset in portfolio before the intervention of the allocation strategy (noted BD – Before Decision) and after (noted BR – Before Reserving) for both tests.

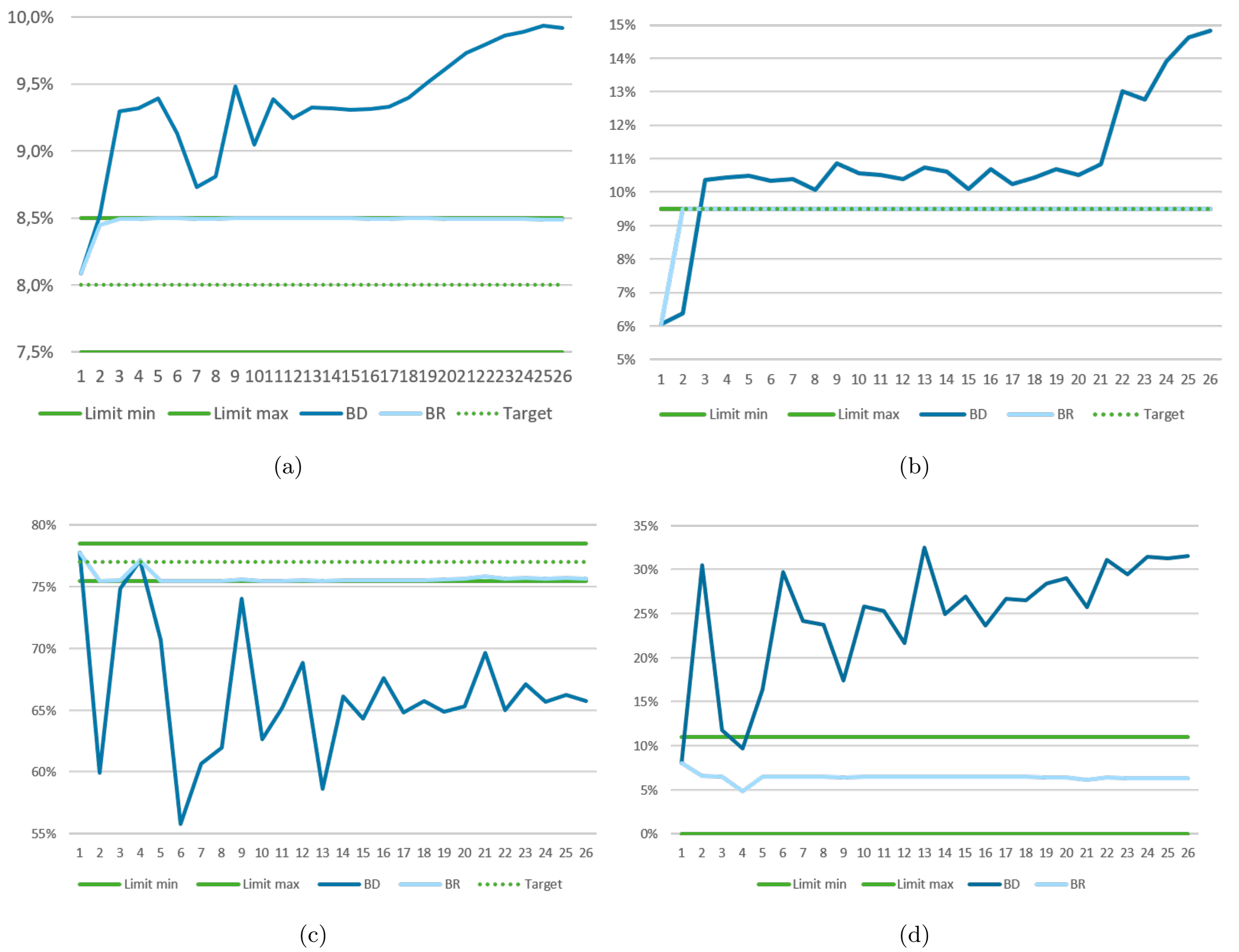


Figure 5.7: Proportion of (a) equity (b) property (c) bonds (d) cash every time step – Test 1

The situation where thresholds are tight leads to a fixed percentage strategy for the property asset class.

From the first years of projections, equities as well as properties are sold to reach the closer limit of their allocation range. However, this doesn't lead to a noticeable better situation of the insurer. From the first year of projection, bonds fall under the lower threshold. 22% of the bond portfolio is invested in one-year-maturity bonds leading to a substantial renewal in the composition of the portfolio from the first year of projection. This explain the peak, noticeable in both tests, to 60% of the proportion of bonds in portfolio. Also, the bonds bought to realign the portfolio are bought in a low interest rate environment leading to poor returns in the following year.

Therefore, the proportion of bonds decreases normally due to the run-off model and falls under the lower limit from the first year of projection. The model naturally realigns the bonds on their lower limit leading to the same situation the following year.

Figure 5.8: Proportion of (a) equity (b) property (c) bonds (d) cash every time step – Test 19

The graphics don't explicitly show the advantage of having close thresholds. However, this is justified because we looked at the average results of the projection. The global view doesn't give insights on the reason why the PVFP would be better.
However, looking at the Liquidity Risk Reserve reveals it is lower for the test with the lower PVFP. The question over this reserve will be detailed in the next section where its consideration will be more relevant to the reader.

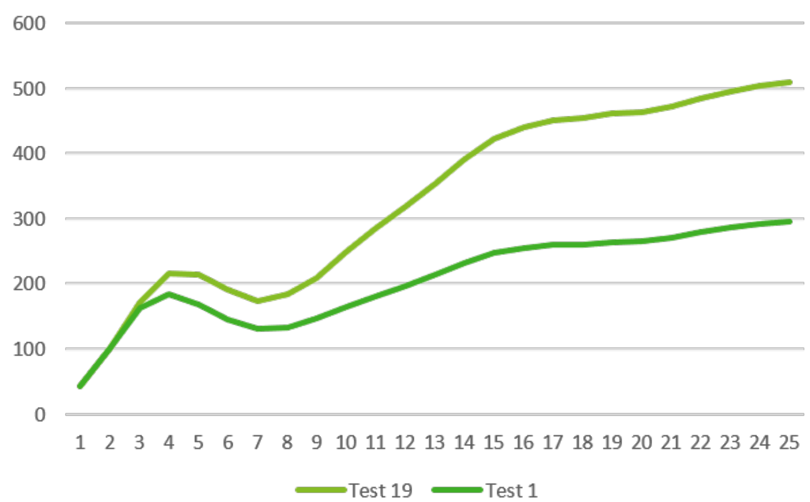


Figure 5.9: Liquidity Risk Reserve evolution for both tests

b) **Behaviour of the assets in the case of the equity shock**

The following results will be analysed on average of the stochastic scenarios when not specified.

The difference with the central scenario lies in the value of the equities in portfolio at the beginning of the projection. The market value of equities is divided in half due to the Solvency II shock.

Therefore, as our initial portfolio doesn't have unrealized gains and losses, this shock causes the apparition of large amounts of unrealized losses in the portfolio shown in the following graphic.

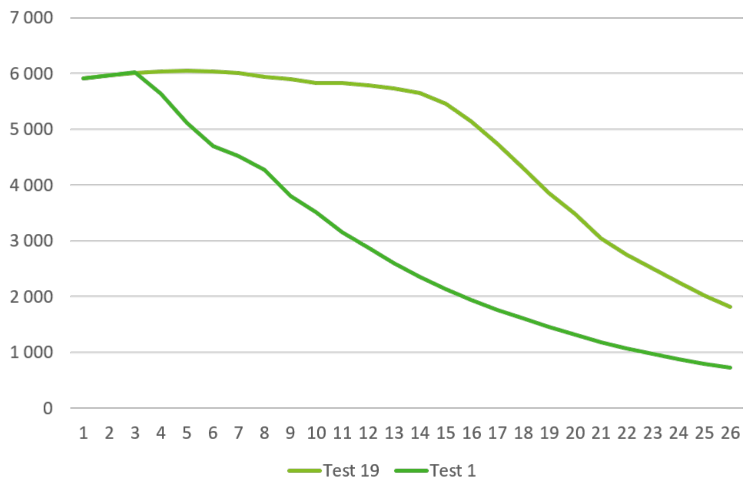


Figure 5.10: Amount of unrealized losses in the portfolio for both tests

Then, during the projection the evolution of this amount differs widely between the tests and results in endowing the liquidity risk reserve.

For recall, this reserve is endowed when the portfolio (equity and property) is in its whole in the situation of unrealized losses. It enables the insurer to set up a safety margin before selling equity and property assets at a loss in the event of an unrealised loss. Endowing this provision represents a cost for the insurer which reduces its profit.

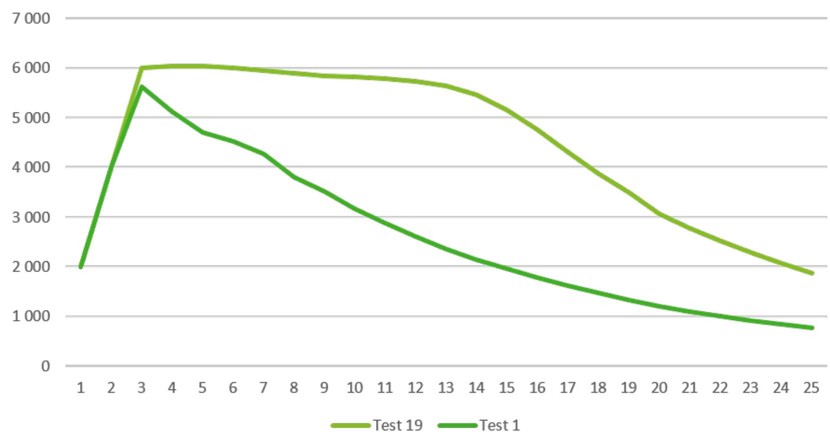


Figure 5.11: Amount of Liquidity Risk Reserve along the projection

The discounted profits corroborate this statement.

Therefore, the unfavourable situation stated for test 19 with less profit and a higher liquidity risk reserve forces the insurer to immobilize more capital to respond to SII requirements.
Now, let's have a closer look to the causes of this disparity resulting from the choice of thresholds

in the thresholds allocation strategy.

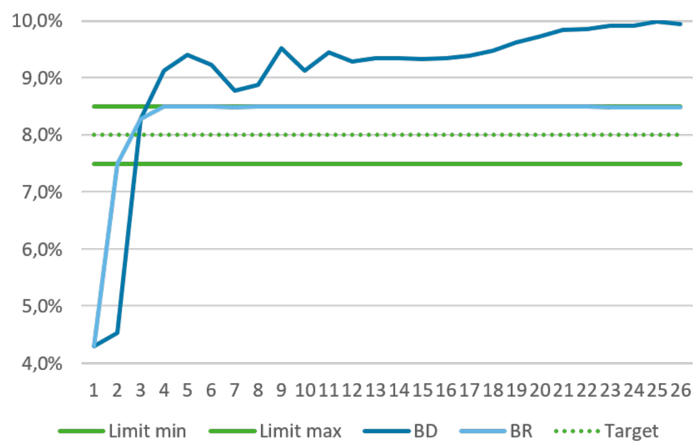


Figure 5.12: Evolution of the equity proportion during the projection - test 1 close thresholds

In the case of close thresholds, the equities quickly go above the maximum boundary assigned to their allocation. As planned, the model will therefore realign the equities from the second year of the projection by selling equities. This sell will result in realizing capital losses. This realization allows the release of part or the Liquidity Risk Reserve.
Then the same process will occur the following year.
These sells improve the insurer's situation and lead to better results in term of solvency.

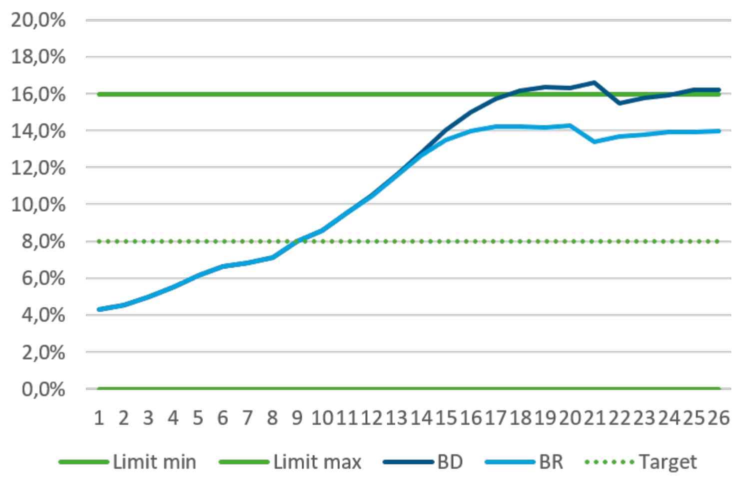


Figure 5.13: Evolution of the equity proportion during the projection - test 19 distant thresholds

On the other hand, when thresholds are wide, the allocation of equities stays within the thresholds for more years. Therefore, keeping the unrealized losses much longer which generates a high liquidity risk reserve and consequently penalising the insurer.

The same phenomenon is observed for properties.

The next attempt consisted in modifying the recall-capacity considering the remarks made on the previous test.

We aimed at changing the recall-capacity.
The percentage applied for each year of projection for all SCR sub modules and central scenarios can be found in the *Appendix B: Tested recall-capacities values.*.

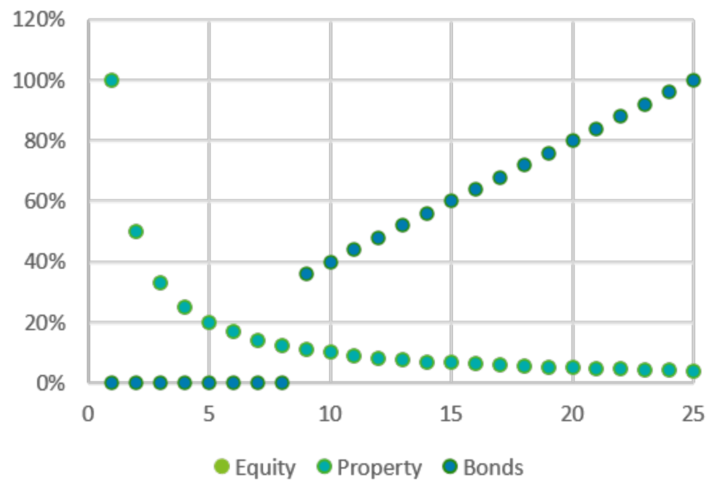


Figure 5.14: Display of the chosen recall-capacities values tested for the thresholds allocation strategy

To force the algorithm to sell equities from the beginning of the projection (in order to reduce the liquidity risk reserve as seen above), we chose to set the recall capacity to 100% and then let it decrease close to 4% for the last projection year. Indeed, it is easier to be confident about the first years of projection and to chose a strong position, on the other hand, the last years of projection are let free for more freedom of the algorithm. This is more aligned with how insurers are managing their assets in the current context.
As seen previously, the bonds were mostly under the lower threshold chosen. To avoid buying large amount of bonds in unfavorable interest rates environment we chose to only meet the closest borders until the 7^{th} year where the interest rate becomes positive. It should be outlined that the assumption of rates only rising in 7 years is a strong assumption which means that the return of economic growth is not expected until 2026. However it represents an interesting case to test the recall capacity algorithm. Afterward, we initiated the buying of more bonds from to ensure better returns on investment.

The recall-capacities were applied regardless of the applied shocked.

					SCR share		
	Equity SCR	Property SCR	Rate SCR	Market SCR	Equity	Property	IR
1	2 782	796	2 031	5 361	51,9%	14,9%	37,9%
2	2 964	869	2 039	5 606	52,9%	15,5%	36,4%
3	3 090	949	2 047	5 800	53,3%	16,4%	35,3%
4	3 201	985	2 055	5 945	53,8%	16,6%	34,6%
5	3 313	1 017	2 062	6 089	54,4%	16,7%	33,9%
6	3 372	1 051	2 068	6 179	54,6%	17,0%	33,5%
7	3 361	1 063	2 067	6 176	54,4%	17,2%	33,5%
8	3 353	1 072	2 066	6 174	54,3%	17,4%	33,5%
9	3 341	1 077	2 064	6 164	54,2%	17,5%	33,5%
0	3 329	1 079	2 063	6 153	54,1%	17,5%	33,5%
10	3 319	1 081	2 062	6 143	54,0%	17,6%	33,6%
11	3 314	1 082	2 062	6 138	54,0%	17,6%	33,6%
12	3 305	1 086	2 060	6 130	53,9%	17,7%	33,6%
13	3 298	1 086	2 059	6 122	53,9%	17,7%	33,6%
14	3 282	1 090	2 058	6 109	53,7%	17,8%	33,7%
15	3 271	1 091	2 057	6 097	53,7%	17,9%	33,7%
16	3 263	1 092	2 056	6 089	53,6%	17,9%	33,8%
17	3 246	1 095	2 056	6 074	53,4%	18,0%	33,9%
18	3 230	1 095	2 056	6 057	53,3%	18,1%	33,9%
19	3 213	1 095	2 056	6 041	53,2%	18,1%	34,0%

Figure 5.15: SCR resulting from the change in recall-capacities

Like in the previous study, we looked at the extreme values.

	Equity	Property	Bonds
Test 1	-8,61%	0%	0%
Test 19	-43,74%	-39,90%	-3,79%

Table 5.7: Variation of SCR values between the results with a 0% recall-capacity and a varying recall-capacity optimization

The analysis reveals that changing the recall capacity has a more significant impact on the results when thresholds are distant. The assessment is not surprising, indeed there is more freedom when thresholds are distant and therefore each action engaged can have a more significant impact.
The impact of the management of bonds is less significant on the interest rate SCR. However, the changes made were based on a simple analysis of the behaviour of bonds in central scenarios. If this study was pushed further, bonds should be studied more deeply, particularly by performing different sensitivity tests of the interest rate SCR to changes of in the recall-capacity function.

The use of wide intervals can be recommended. Indeed, only by changing a few parameters, we significantly improved the sensitivity of our portfolio to the equity and property risks. The management of thresholds can differ from one risk to another and even from the central situation resulting in multiple management choices, just as an asset manager would have.
Risks with more complex impacts require more time to be fully monitored.

5.3 Conclusion

In this chapter, we aimed at implementing a new asset allocation strategy. This strategy consisted in implementing an asset allocation which would better represent what's happening in reality, offering more dynamic to the assets during all the projection and therefore increasing the possibilities of management within the model.
This study aimed at optimizing the allocation strategy regarding the risk for the insurer. In this case the risk was measured under Solvency II regulation and therefore risk neutral projection was used to appreciate the impacts on the SCR. If the study was conducted in a real world environment for other purposes, the valuation of the allocation algorithm would be influenced by the economic scenarios chosen and therefore the conclusions could be different from the ones obtained here.

The results attest that the model is correctly working. However, simply implementing this algorithm doesn't immediately give better results. Indeed, the resulting capital requirements were higher than the ones obtained with the fixed strategy. Thresholds need further studies to reveal their true potential. In particular, if the strategy was tested in a context of positive rates the results would have been different implying that the configuration of the thresholds directly depends on the economic context and need to be continuously challenged.
The second test performed revealed the true potential of such an asset allocation strategy and the complexity it could bring to the ALM model. This new strategy needs further studies to be fully understood.
Also, the modelling of only one policyholder profile drew some limits to the leaded studies: indeed the studies do not capture the benefit of diversification between policyholders. In addition, the observed duration gap between assets and liabilities can lead the insurer to different situations according to the average duration of the portfolio. The allocation strategy should match the characteristic of the portfolio.

The following actions come naturally as a second step (not treated in this study because of a lack in timing) to improve the study:

- Perform multiple sensitivity tests to improve the results obtained with the threshold strategy and go further than manually changing the recall-capacity by implementing a dynamic recall function adapting itself to the portfolio and economic situations during the projection.
- Mutually optimize the allocation threshold strategy and the optimization of the liquidity risk reserve and other provisions.

Conclusion

The environment of low rates still represents a grey area for insurers due to its relatively new appearance. In an environment of negative rates and high equity volatility, as it is currently the case, the aim of this report was to show the necessity for insurers to try new allocation approaches in order to better control their risks while keeping in mind the desired profit.

We first studied the impact of a change in the asset portfolio of insurers in terms of expected profitability (through the Value of In Force) and risk aversion (through the Solvency Capital Requirement) and in terms of regulatory requirements in a real world scenario. The results of this study showed the limits of the Standard Formula in the context of low rates by omitting the shock of negative rates. Indeed, by not considering that negative rate situation might worsen, results from the ALM study are to invest massively on government bonds. This study showed therefore that the SCR while using the Standard Formula may not be a sufficient indicator for risk assessment in a Real-World environment. In addition, the results can be narrowed by using the exact Solvency II shock for the equity SCR.

In the second section, we tried to measure the impacts of a change in the asset allocation strategy to be more aligned with reality and give insights on how to optimize the new threshold strategy. For its study, we decided to increase and decrease the interval between thresholds. We noticed that increasing the intervals made it possible to delay the crossing of boundaries for the equity class and therefore increasing the value of unrealized gains and losses in portfolio. On the other hand, when reducing the intervals, the model had to realign equities from the first years of projection. In the case of unfavourable situations, such as in a low interest rate context, several unrealized losses appear in the portfolio and therefore not realizing them isn't beneficial for the insurer, in particular due to the endowment of the Liquidity Risk Reserve.
The results obtained can still be challenged and can lead to new optimizations to find the most suitable configuration for the thresholds strategy in line with reality. Another improvement would be to automate the recall-capacity and make it depend on the economic situation for each time step during the projection.
To push further the study of the allocation strategy algorithm, it should be conducted in a real-world environment as it is commonly done on the market when changes are made to the allocation strategy.

Finally, the model used still needs improvements to ensure the reliability and performance of the results. These improvements consist in a smoothing of the bonus reserve used or the implementation of an effective renewal of the assets during the projection.

This report conclusions show some limits mainly due to the following elements:

- The modelled liability portfolio: only one profile of insured was tested therefore no case of positive MGR was tested and the dynamic resulting from having different contracts in the portfolio couldn't be observed.
- The composition of the asset portfolio: due to modelling simplifications, the asset portfolio only contains 4 types of assets with a high part of government bonds, moving aside the spread risk.

- The model limits: it occurred during the researches and implementation for this report that the available model needed further implementations to obtain more qualitative results. For example, the insurer wealth could be more accurately smoothen over time while the investment strategy on bonds should be dynamically calibrated and not fixed.

To conclude, the analysis carried out shows that there are still challenges to come for the insurers which will undoubtedly motivate new researches for the improvement of ALM strategies and models.

Note de synthèse

Mots-clefs : assurance-vie ; contrat d'épargne Euro ; taux bas ; Solvabilité II ; SCR de Marché ; stratégie financière ; allocation d'actifs ; modélisation ALM ; VIF ; couple Rendement/Risque.

Ce mémoire s'inscrit dans le contexte des taux bas et de la réglementation Solvabilité II.
La baisse des taux a d'abord été favorable à l'assurance vie en permettant aux compagnies d'assurance d'offrir des rendements attractifs liés à leurs investissements antérieurs. Ces rendements devenaient beaucoup plus élevés que ceux observés sur les marchés.
En 2019, le taux de rendement des contrats en Euro était estimé à 1,5% dans un contexte où les taux d'intérêt ont atteint de nouveaux planchers historiquement bas. Les assureurs souffrent de l'environnement des taux qui fait baisser les revenus obligataires qu'ils détiennent. La baisse des taux obligataires pèse sur le rendement des actifs et se traduit par une tendance à la baisse des taux servis. Cela pousse les assureurs à vouloir augmenter le rendement généré par leur actif en augmentant leur exposition sur les marchés actions. Toutefois cette hausse d'exposition semble limitée du fait des contraintes réglementaires de solvabilité.

La bonne gestion de l'actif pour un assureur est un enjeu majeur. Il est notamment important, du fait de la compléxité et de la durée des contrats d'assurance, de correctement modéliser les interactions actif et passif afin de représenter au mieux la réalité dans les modèles de projection. Il est à noter que l'environnement des taux bas génère des interactions et des résultats qu'il n'était pas habituel de constater jusqu'à présent.
L'ALM (Asset and Liability Management) répond au besoin des assureurs et permet de fournir des éléments de décision à une compagnie d'assurance afin qu'elle puisse obtenir le meilleur rendement possible tout en maitrisant le niveau de risque auquel elle s'expose.

Dans le cadre de ce mémoire, nous nous intéresserons en particulier à l'optimisation de la composition du portefeuille de l'assureur afin de limiter sa sensibilité aux risques. Dans un second temps, nous étudierons la stratégie de réalignement des actifs au cours de la projection. Ces deux études sont complémentaires. La première permet de statuer sur l'existant dans un modèle ALM et d'apporter des solutions sans modifier le modèle tandis que la deuxième vient directement agir sur l'implémentation du modèle.

Le portefeuille modélisé dans ce mémoire est fictif ; sa composition s'inspire de celle des portefeuilles des principaux acteurs du marché français. L'ensemble des contrats d'épargne modélisés sont des fonds Euro avec un taux minimum garanti à 0%. Un seul profil d'assuré a été modélisé. Le fonds Euro a été privilégié dans cette étude notamment parce le risque est porté par l'assureur et c'est ainsi dans son intérêt de mieux maitriser les risques associés au contexte des taux bas.
Le portefeuille d'actifs est simplifié et ne contient que quatre classes d'actifs : obligations souveraines, actions (quotées et non quotées, OPCVM), immobilier et monétaire. Le parti pris de ne pas modéliser d'obligations d'entreprises ne permet pas de capter le risque de crédit dans les études menées et omet toute forme d'intéraction avec le risque action.

Après avoir détaillé le fonctionnement du modèle ALM, nous décrirons les différentes études faites et les résultats obtenus. Nous détaillerons notamment la nouvelle stratégie financière implémentée dans le modèle.

■ **Etude ALM du portefeuille d'actifs**

Le modèle ALM est ici utilisé en run-off, donc sans nouveaux contrats, dans un univers risque-neutre pour les indicateurs réglementaires de risque, et dans un univers monde réel pour les indicateurs de rendement.
Différentes allocations de portefeuille seront testées pour cette étude.
L'indicateur de rendement retenu est la VIF[1] qui est un indicateur privilégié des assureurs vie pour suivre la profitabilité future de leur portefeuille.
Enfin, pour mesurer le risque qu'encourt l'assureur pour chaque allocation testée, nous avons privilégié le SCR obtenu via la formule standard. Une hypothèse forte de l'étude, sera pour le risque action, de choquer les actifs concernés à 49% quel que soit le type d'actions en portefeuille, c'est-à-dire en supposant qu'aucune action du portefeuille n'est éligible à des chocs à 39% [2] ou 22% [3]. Les résultats obtenus seront prudents.

- **Projection en univers risque-neutre (RN)**
  Cet univers s'inscrit dans un objectif d'évaluer la situation de l'assureur conformément au marché et sera utilisé pour évaluer le Best Estimate ainsi que la NAV [4].
  - L'univers risque-neutre est un univers théorique dans lequel les modèles sont calibrés sur les prix du marché observés au moment du calibrage.
  - Les scénarios ne sont analysés qu'en moyenne, contrairement aux scénarios de l'univers monde réel.
  - Les scénarios générés en RN respectent la propriété de « Market Consistency », c'est-à-dire que le rendement attendu de tout actif doit être égal au rendement du taux central sans risque.
- **Projection en univers monde réel**
  Cet environnement est principalement utilisé pour la gestion stratégique et financière de l'entreprise.
  Les actifs sont projetés aussi fidèlement que possible dans le monde réel selon une probabilité historique. La notion de risque est prise en compte dans cet univers via une prime de risque propre à chaque actif. Elle se calcule comme la différence entre le taux de rendement exigé sur le marché des actions ou de l'immobilier et le taux sans risque (ici l'OAT 10 ans français). La prime de risque action a été estimée sur le CAC40 tandis que celle pour l'immobilier l'a été sur l'IPDUFROR[5].

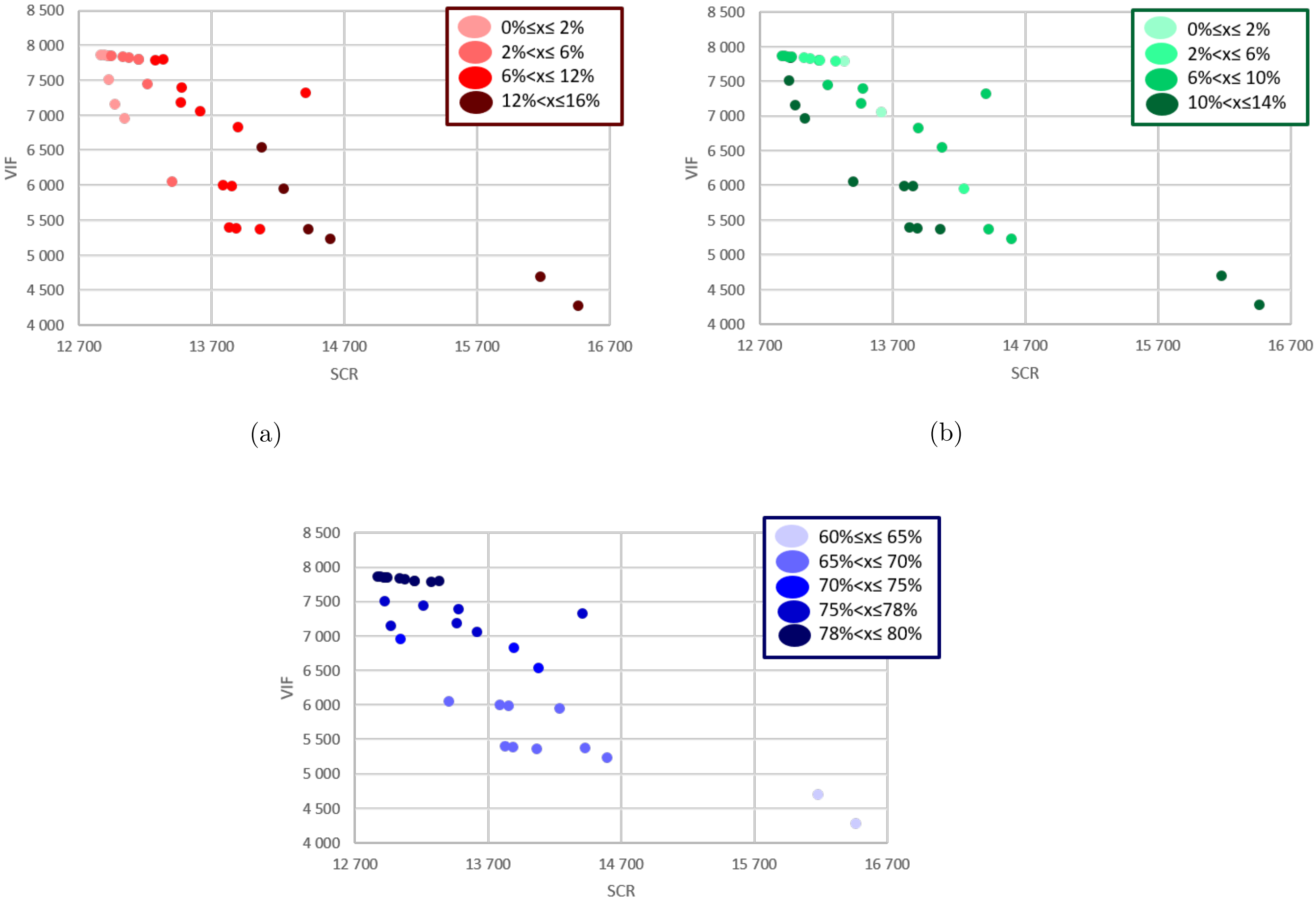
Cette étude montrera que parmi les allocations testées, celle permettant d'optimiser le rendement du portefeuille tout en maitrisant le risque encouru est la suivante :

Obligations	Action	Immobilier	Cash
[78% ; 80%]	[0% ; 2%]	[6% ; 10%]	[9% ; 10%]

Table 0.1: Portefeuille optimisant le couple rendement/risque pour l'étude ALM

L'allocation obtenue est, pour la classe action, influencée par le choix fait pour le choc action et par la pénalisation des actions, plus forte que les autres classes d'actifs, sous Solvabilité 2. De plus, la simplification du portefeuille d'actifs financiers modélisé, notamment avec l'absence d'obligations corporate et la forte part attribuée aux obligations souveraines expliquent l'équilibre du portefeuille obtenu.

[1] Value of In Force
[2] Cela équivaut à l'absence d'actions globales, dites de type 1, en portefeuille.
[3] Cela équivaut à l'absence d'investissements stratégiques actions en portefeuille.
[4] Valeur d'Actif Nette
[5] Indice de référence pour l'immobilier se basant sur l'évolution moyenne des prix de l'immobilier en France



(c)

Figure 0.1: Résultat des portefeuilles testés sur les axes rendement/risque selon la proportion (a) action (b) immoblier (c) obligataire du portefeuille

■ Stratégie de réalignement des actifs

A la fin de chaque année de projection, le modèle ALM vise à équilibrer le portefeuille des assureurs à la suite des différents mouvements causés par les achats ou les ventes d'actifs ayant eu lieu durant le processus de projection.

Le réalignement des actifs se fait initialement via une stratégie fixe où chaque classe d'actif modélisée doit respecter chaque année la même allocation exprimée comme un pourcentage de la valeur de marché du fonds.

Nous avons examiné les faiblesses et les limites de cet algorithme afin de proposer des améliorations.
Du fait de la structure de son implémentation, l'algorithme initial n'assure pas la convergence vers l'allocation voulue, la réallocation peut ne pas être optimale. De plus, il serait plus correct que les poids donnés à chaque classe d'actif du portefeuille soient ajustés en fonction des conditions économiques au moment de chaque réallocation afin d'adopter une gestion plus efficace. Enfin, l'implémentation initiale peut générer des transactions inverses sur une même classe d'actifs pour une même année de projection. Cette méthode n'est pas jugée efficace notamment concernant la gestion des plus-values non réalisées.

Nouvelle stratégie de réallocation

Afin d'être plus en accord avec la réalité, nous avons choisi d'utiliser une stratégie *Drift*, permettant aux actifs d'évoluer plus dynamiquement entre des bornes calibrées pour chaque catégorie d'actifs. L'évolution des actifs peut être orientée et affinée au sein de ces bornes pour répondre à un besoin de l'assureur.

Pour attester de l'impact de cette nouvelle stratégie et de sa calibration, les intervalles initiaux retenus et les SCR de marché associés sont les suivants :

	Bornes	SCR action	SCR immobilier	SCR taux
Situation initiale	Action : [4% ; 12%] Immobilier : [6% ; 13%] Obligations : [72% ; 82%]	5 372	1 637	8 656

Table 0.2: Situation initiale - calibration et SCR associés

L'algorithme fonctionne de la façon suivante : chaque fois qu'un actif est supérieur (ou inférieur) à sa borne, l'algorithme le ramène à celle-ci en vendant (ou en achetant) la quantité exacte d'actif nécessaire. La classe monétaire permet d'ajuster l'ensemble des actifs pour que l'ensemble des allocations soient respectées. C'est la stratégie "cash-pivot".

Nous avons pris en compte dans la nouvelle stratégie la gestion de la Trésorerie correspondant au solde des flux de passif. Comme le modèle est en run-off la Trésorerie est nécessairement négative. L'intégration de la Trésorerie revient à désinvestir de l'actif pour respecter l'équilibre du bilan. Nous avons décidé de désinvestir intégralement la Trésorerie sur le cash.

Pour cette étude, les bornes de départ ont été choisies en se basant sur l'allocation initiale, nous avons par la suite cherché à mesurer l'impact sur le SCR de la variation de la taille des intervalles (élargissement ou rétrécissement).
Dans les résultats de l'étude, on constate que l'élargissement des intervalles provoque une augmentation du SCR de marché et l'inverse pour leur rétrécissement.

Les résultats obtenus mettent en évidence que lorsque les bornes délimitant la stratégie d'allocation d'actifs sont proches, les SCR obtenus sont meilleurs que lorsque les bornes sont très éloignées.

	Bornes	SCR action	SCR immobilier	SCR taux
Test 1 (bornes proches)	Action : [7.5% ; 8.5%] Immobilier : [9.5% ; 9.5%] Obligations : [75.5% ; 78.5%]	3 044	795	2 031
Test 2 (bornes éloignées)	Action : [0% ; 16%] Immobilier : [2% ; 17%] Obligations : [68% ; 86%]	5 711	1 804	2 137

Table 0.3: Tests étudiés

Les actions, dont le SCR associé double quasiment d'un test à l'autre, permettent de se rendre compte de l'impact marqué qu'ont les moins-values sur le capital requis.
En effet, suite au choc action, la valeur des actions se retrouve en forte moins-value latente.
Dans le cas de bornes rapprochées, les actions sont systématiquement réallouées sur l'allocation cible, provoquant ainsi la réalisation de moins-value latente et le rachat d'action à la monnaie au prix de marché. Le fait de réaliser des MVL permettra de libérer la PRE [6] plus rapidement.

[6] Provision pour Risque d'Exigibilité

Dans le cas inverse, les actions initialement en moins-value latentes seront vendues tardivement ce qui viendra augmenter la PRE tout au long de la projection avant même la réalisation de moins-values.

La PRE est une provision permettant à l'assureur de mettre en place une marge de sécurité en amont de ventes à pertes des classes action et immobilier en situation de moins-value latente. Doter cette provision représente un coût pour l'assureur qui vient réduire son profit.

L'optimisation des bornes permet à l'assureur de reproduire plus fiablement la réalité mais nécessite un paramétrage conséquent en amont pour pourvoir être bénéfique et maitrisé.
Nous avons réalisé une étude dans ce sens prenant en compte le paramétrage d'une force de rappel vers une cible fixée au milieu des deux bornes. L'utilisation de cette force, s'avère donner de meilleurs résultats lorsque les bornes sont éloignées. La liberté possible entre les bornes laisse plus de place à l'intervention de la force de rappel et ainsi donne à l'assureur plus de liberté sur le pilotage de ses actifs suite à la calibration de cette force. Ce résultat est satisfaisant d'un point de vue pilotage et permet d'amener l'algorithme à une meilleure reproduction de la réalité.
Cette étude a été menée en environnement risque-neutre. Afin d'apprécier au mieux l'impact d'un changement de stratégie de réalloaction des actifs, il est envisagé de réaliser cette étude en contexte monde-réel. De plus, dans l'objectif d'une optimisation de la stratégie, une piste d'étude serait de rendre la force de rappel dépendante des conditions économiques à chaque pas de temps lors de la projection. Ceci permettrait d'affiner la sensibilité de la stratégie financière modulo l'appétence au risque de l'assureur.

L'objectif de ce mémoire aura été de montrer que dans un environnement de taux négatifs et de forte volatilité action, comme c'est le cas actuellement, il est nécessaire pour les assureurs de tester de nouvelles approches stratégiques et tactiques afin de mieux maitriser leurs risques tout en gardant à l'esprit le profit voulu.
De plus, les analyses réalisées permettent de mettre en avant que les études faites dans des contextes de taux bas peuvent mener à des positions très différentes de celles réalisées dans un contexte plus favorable. Il est donc nécessaire, notamment à des fins de pilotage, que le choix des indicateurs de performance et de risque soient adaptés au contexte.

Executive summary

Key words: life insurance; Euro fund products; low interest rates; Solvency II; market SCR; financial strategy; assets allocation; ALM modelling; VIF; risk/return relationship.

This report is written in the context of low interest rates and within the Solvency II framework.
The fall in rates was initially favourable to life insurance by allowing insurance companies to offer attractive returns on investment thanks to their previous investments. Therefore, these returns became much higher than those observed on the markets.
In 2019, the rate of return on Euro fund products was estimated at 1.5% in a context where interest rates reached new historic lows. Insurers are suffering from this interest rate environment because it is driving down the bond income they hold. The fall in rates weighs on assets' yields and is reflected in a downward trend in the rates served. This drives insurers to seek an increase in the return generated by their assets, especially by increasing their exposure on equity markets. However, their exposure's increase seems limited due to Solvency II regulatory constraints.

Asset management is a major issue for insurers. Due to the complexity and duration of insurance products, it is particularly important to correctly model the asset and liability interactions in order to represent at best the reality in projection models. It should be noted that the environment of low rates generates interactions and results which were unusual, until now.
ALM (Asset and Liability Management) responds to the insurers' needs and provides insurance companies with decision-making elements for them to obtain the best possible return while controlling the level of risk to which they are exposed.

In this report, we will especially focus on optimising the composition of the insurer's portfolio in order to limit their sensitivity to risks. In a second step, we will study the asset allocation strategy during the projection. These two studies are complementary. The first allows us to decide on the existing ALM model and to provide solutions without modifying the model, while the second directly affects the implementation of the model.

The portfolio modelled in this report is fictitious. Its composition is inspired by the portfolios of the main players on the French market. All savings products modelled are Euro funds with a minimum guaranteed rate set at 0%. Only one type of 45-year old insured was modelled. The Euro fund was privileged in this study because the risk is borne by the insurer and therefore it is in their interest to better control the risks associated with the low interest rate environment.
The asset portfolio is simple and is only composed of four asset classes: government bonds, equities (listed and unlisted, OPCVM), real estate and cash. The decision not to model corporate bonds does not allow credit risk to be captured in the studies and omits any form of interaction with equity risk.

After having detailed the functioning of the ALM model, we will describe the different studies carried out and the results. In particular, we will detail the new financial strategy implemented in the model.

■ **Portfolio ALM study**

The ALM model is used in run-off with an absence of new contracts in a risk-neutral universe (for regulatory risk indicators) and in a real-world universe (for performance indicators).
Different portfolio allocations will be tested for this study.
The performance indicator selected is the VIF[1], which is commonly used by insurers to monitor the future profitability of their portfolio.
Finally, in order to measure the risk incurred by the insurer for each allocation tested, we used the SCR obtained via the standard formula. A strong hypothesis of the study will be for the equity risk to shock the market values of equities at 49% whatever the type of equity in the portfolio is, i.e. meaning no equities in the portfolio are eligible for shocks at 39%[2] or 22%[3]. The results obtained will therefore be conservative.

- **Risk-neutral projection universe**
  This universe is intended to assess the insurer's situation in line with the market and will be used to evaluate the Best Estimate as well as the NAV[4].
  - The risk-neutral universe is a theoretical universe where models are calibrated on observed market prices at the time of calibration.
  - Scenarios are only understood on average, unlike scenarios in the real-world universe.
  - This universe is part of a market-consistent valuation objective i.e. the expected return on any asset must be equal to the return on the risk-free central rate.
- **Real-world projection universe**
  This environment is mainly used for the strategic and financial management of the company. Assets are projected as faithfully as possible in the real world under historical probability. The risks taken are considered and remunerated by a risk premium specific to each asset. It represents the difference between the required rate of return on the equity or property market and the risk-free rate (here the French 10-year OAT[5]).
  The equity risk premium was calibrated using the CAC40 while the property risk premium was estimated on the IPDUFROR. [6].

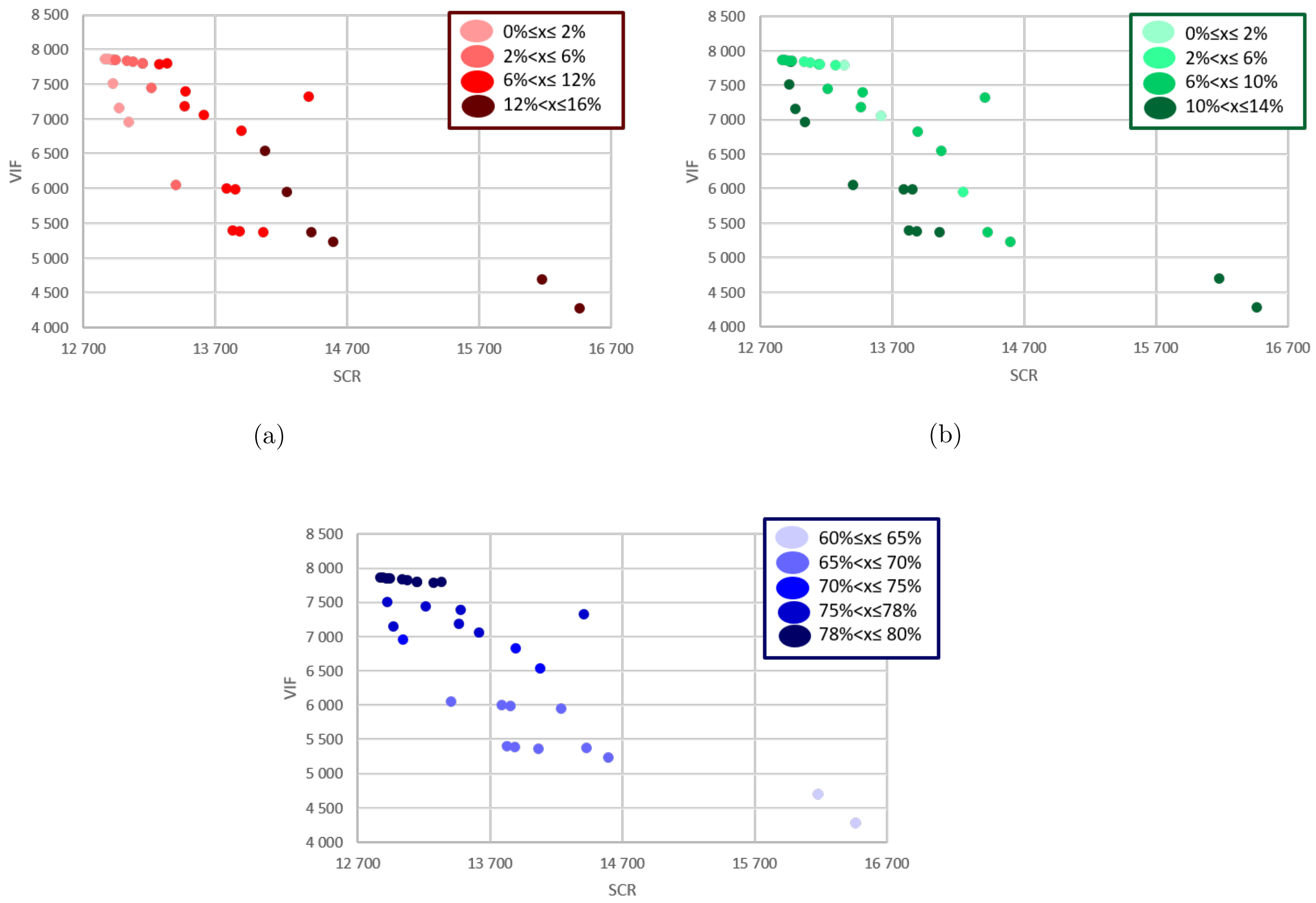
Among the allocations tested, this study will show that the one optimizing the portfolio's return while controlling the risk incurred is as follows:

Bonds	Equity	Property	Cash
[78% ; 80%]	[0% ; 2%]	[6% ; 10%]	[9% ; 10%]

Table 0.1: Allocation optimizing the risk/return relationship for the ALM study

The recommended allocation for the equity class is influenced by the choice made for the equity shock and by the penalisation of equities which is higher than the one of other asset classes under Solvency 2. Moreover, the simplification of the modelled financial asset portfolio, i.e. with the lack of corporate bonds and the high share allocated to sovereign bonds, explains the imbalance of the obtained portfolio.

[1] Value of In Force
[2] Therefore there are no global equity / type 1 investments in portfolio.
[3] Therefore there are no equity strategic investment in portfolio.
[4] Net Asset Value
[5] Obligations Assimilables au Trésor
[6] Reference index for real estate based on the average evolution of real estate prices in France



(a) (b)

(c)

Figure 0.1: Impact of the different allocation tested on the risk/return relationship for each asset class (a) equity (b) property (c) bonds

■ Assets allocation strategy

At the end of each projection year, the ALM model aims at balancing the insurer's portfolio after the different movements caused by the assets' purchases or sales made during the projection process.

The initial allocation strategy was a fixed strategy where each modelled asset class was meant to respect each year the same allocation expressed as a percentage of the market value of the fund.

We examined the weaknesses and limitations of this algorithm in order to suggest improvements. Due to the structure of its implementation, the initial algorithm doesn't ensure convergence towards a viable solution, and therefore the allocation may not be optimal. Furthermore, it would be more relevant if the weights given to each asset class in the portfolio were to be adjusted considering the economic conditions at the time of each allocation in order to adopt a more efficient management. Finally, the initial implementation may generate opposite transactions on the same asset class for the same projection year. This method is not considered effective, particularly regarding the management of unrealised capital gains.

New allocation strategy

In order to increase the reality of the dynamic of the assets' allocation in the model, we chose to use a threshold/Drift strategy, enabling the assets to evolve more dynamically between thresholds calibrated for each asset category. With this strategy, the evolution of assets can be managed to meet at best the insurer's needs.

To demonstrate the impact of this new strategy and its calibration, the initial intervals used and the associated market SCRs are as follows:

	Thresholds	Equity SCR	Property SCR	Interest rate SCR
Initial situation	Equity: [4% ; 12%] Property: [6% ; 13%] Bonds: [72% ; 82%]	5 372	1 637	8 656

Table 0.2: Configuration and SCR of the initial situation

The algorithm works as follows: whenever an asset is higher (or lower) than its threshold, the algorithm brings it back to the closest threshold by selling (or buying) the exact amount of asset needed. The cash allows the adjustment of all the assets so that all the allocations are respected. This is the cash-pivot strategy.

In the new strategy, we considered the new money management, which corresponds to the balance of liability flows. As the model is in run-off, the new money position is necessarily negative. Integrating the new money position means disinvesting assets to maintain a balanced balance sheet. We have decided to disinvest all the new money on cash.

For this study, the starting thresholds were chosen based on the initial allocation. We then sought to measure the impact on the SCR of the variation in the size of the intervals (widening or narrowing). In the results of the study, we find that widening the intervals causes an increase in market SCR and the opposite for narrowing them.

The results show that when the boundaries of the asset allocation strategy are close, the SCRs obtained are better than when the boundaries are far apart.

	Thresholds	Equity SCR	Property SCR	Interest rate SCR
Test 1 (close thresholds)	Equity: [7.5% ; 8.5%] Property: [9.5% ; 9.5%] Bonds: [75.5% ; 78.5%]	3 044	795	2 031
Test 2 (distant thresholds)	Equity: [0% ; 16%] Property: [2% ; 17%] Bonds: [68% ; 86%]	5 711	1 804	2 137

Table 0.3: Studied tests

The shares for which the associated SCRs almost doubled from one test to the other, allow the reader to consider the noticeable impact that unrealized gains have on the required capital.
Indeed, resulting from the equity shock, the value of the equities is in a position of strong unrealised losses.
In the case of close boundaries, the equities are systematically allocated to the target allocation, thus causing the realisation of unrealised losses and the repurchase of equities at market price. The realisation of unrealized losses will allow the quick recovery of the liquidity risk reserve.

On the other hand, when boundaries are distant, equities with initial unrealised losses will be sold later which will increase the liquidity risk reserve throughout the projection even before the realisation of the unrealized losses.

The liquidity risk reserve is a provision allowing the insurer to set up a safety margin prior to the sale at a loss of equity and property classes in the situation of unrealised losses. Endowing this provision represents a cost for the insurer which reduces its profit.

The optimisation of the thresholds enables the insurer to reproduce reality more reliably but requires a significant amount of configuration upstream to be beneficial and controlled. We carried out a study in this way, considering and configuring a recall-capacity towards a target set in the middle of the two thresholds. The use of this recall capacity, turns out to give better results when thresholds are distant. The interval between thresholds enables the intervention of the recall-capacity and thus gives the insurer more freedom in the management of their assets. This result is satisfying from a steering point of view and brings the algorithm to a better reproduction of reality.
This study was conducted in a risk-neutral environment. In order to better appreciate the impact of a change in the asset allocation strategy, the carry-out of this study should to be renewed in a real-world environment. In addition, aiming at optimising the strategy, one lead of study would be to implement a dynamic recall-capacity depending on the economic environment at each projection time-step. This would enable the sharpening of the insurer's financial strategy while considering its risk appetite.

The aim of this report is to show that in an environment of negative rates and high equity volatility, as it is currently the case, it is necessary for insurers to test new strategic and tactical approaches in order to better control their risks while keeping in mind the desired profit.

The analyses carried out makes it possible to highlight that studies carried out in a low interest rates context can lead to very different results from those achieved in a more favourable environment. It is therefore necessary, particularly for steering purposes, to adapt the choice of indicators for performance and risk to the context.

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Appendix A: In-depth study of the bonds allocation strategy

The purpose of this appendix is to discuss the impact of a strategy aiming at reducing the initial duration gap on the studies' results.

The duration gap between the modelled asset and liability is wide ($\sim$ 4 years). Generally insurers tend to reduce this duration gap however in this paper the duration gap is kept steady throughout the projection, i.e the investment on bonds respects the initial allocation of maturities in the bond portfolio. This lead to the financial results displayed in section 4.2.2.

To reduce the duration gap and therefore tend to reproduce a more common market practice, the allocation strategy was changed to extend the duration of the asset portfolio, resulting in a portfolio half invested on 10-year bonds in the allocation strategy.
Note that, in the simplified model used here, no duration matching strategy is implemented. This study only measures the impact of an allocation strategy aiming at buying 10-year bonds throughout the projection.

The different investment strategies are displayed below.

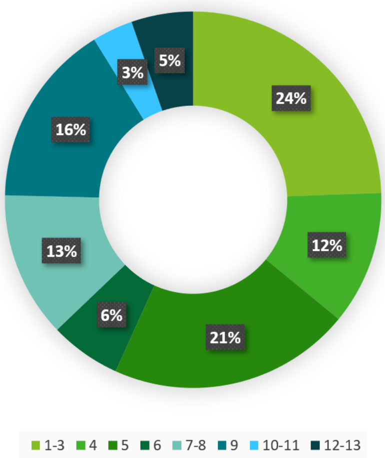

Figure 5.2: Previous investment distribution

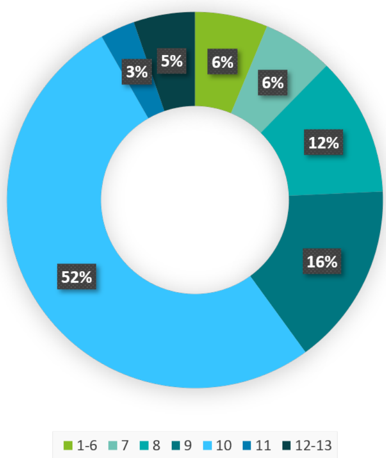

Figure 5.3: Tested investment distribution

Coherently with the conducted studies, the impacts of the different allocation strategies were studied for the SCR indicator.

	New investment strategy	Δ w/ previous strategy
Life SCR	**11 842 180 €**	- 6,3%
Mortality risk	-	-
Longevity risk	13 272 €	-9 %
Life-expense	84 079 €	- 1 %
Lapse risk	11 794 398	- 6 %
Life-catastrophe risk	8 685 €	-15 %

Table 5.4: Life SCR module

	New investment strategy	Δ w/ previous strategy
Market SCR	**4 695 991 €**	0 %
Interest Up risk risk	-	-
Interest Down risk	1 961 896 €	-3 %
Equity risk	2 657 633 €	+2 %
Property risk	869 261 €	+2 %

Table 5.5: Market SCR results for the initial portfolio

First, we notice that the insurer's risk profile is steady.
The market SCR wasn't improved by the change of strategy. However, the life SCR gains from the decrease of exposition to rates resulting in a lower volatility of the BEL due to a reduction in duration gap. Therefore, while keeping the same risk profile, the new bond allocation strategy had a positive effect by leading to a global decrease of the SCR amount and therefore improving the solvency of the insurer. It also has to be kept in mind that the proportion of bonds in the modelled portfolio is deliberately high and explains the high sensitivity to rates.
Moreover, the following financial indicators (initial allocation strategy in green and new allocation strategy in blue) show no significant change.

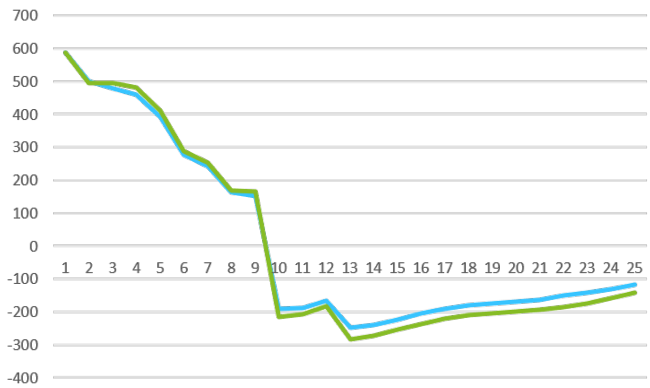

Figure 5.4: Financial result

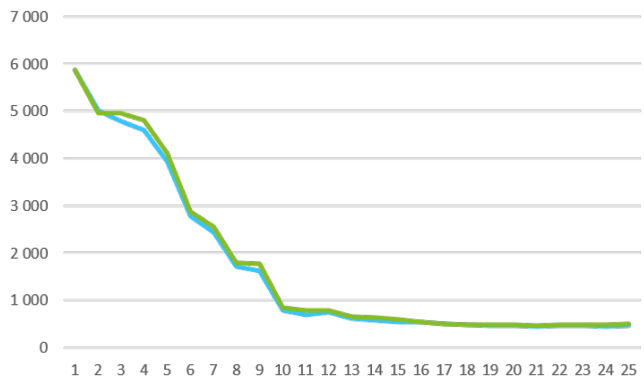

Figure 5.5: Financial production

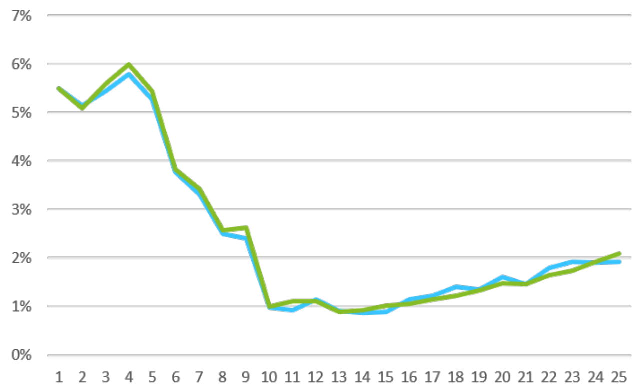

Figure 5.6: Yield of the bonds in the portfolio

Due to the initial allocation and composition of the asset portfolio, from the 5^{th} year of projection, half of the original portfolio is sold. However the investment made to match the target asset allocation, whether it is on 5-year maturity bonds or 10-year maturity bonds doesn't change the fact that it is done in a negative rate environment, meaning the rate of return on bonds is not favourable.
Another characteristic of the initial portfolio is outlined in the figure 5.5: during the 9th year, bonds with noteworthy returns reached their maturity explaining the sharp decline of the financial production held by bonds which couldn't be counterbalanced by the bonds bought during the previous years.

To conclude, due to the specifics of the initial portfolio and the economic environment, changing the allocation strategy on bonds depending on their maturities, wouldn't, in the case of this model, have changed the results of the studies conducted in this report.
However, as depicted during a conference displayed during CAP Actuariat [5], investing on longer maturities enables the insurer to be more resilient to movement in rates, which in a context of negative rates, seems a worthy strategy.

Appendix B: Tested recall-capacities values

Year	Equity	Property	Bonds
2020	100%	100%	0%
2021	50%	50%	0%
2022	33%	33%	0%
2023	25%	25%	0%
2024	20%	20%	0%
2025	17%	17%	0%
2026	14%	14%	0%
2027	12.5%	12.5%	0%
2028	11%	11%	36%
2029	10%	10%	40%
2030	9%	9%	44%
2031	8%	8%	48%
2032	7.6%	7.6%	52%
2033	7%	7%	56%
2034	6.7%	6.7%	60%
2035	6.2%	6.2%	64%
2036	5.8%	5.8%	68%
2037	5.6%	5.6%	72%
2038	5.2%	5.2%	76%
2039	5%	5%	80%
2040	4.7%	4.7%	84%
2041	4.5%	4.5%	88%
2042	4.3%	4.3%	92%
2043	4.17%	4.17%	96%
2044	4%	4%	100%

Table 5.6: Recall-capacities values tested for the thresholds allocation strategy