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Titre : Dynamics of Counterparty Risk and Market Risk Capital Charge in Volatile Markets

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Résumé

L'objectif de ce mémoire est d'évaluer la sensibilité des exigences de fonds propres pour le risque de marché et le risque de contrepartie aux fluctuations des marchés financiers.
La recherche s'appuie sur des documents de Bâle pour le calcul des exigences de fonds propres, et l'application est effectuée sur les marchés américain, turc et européen.
La première partie du mémoire aborde les différentes approches de cartographie des flux de trésorerie permettant d'évaluer la sensibilité du portefeuille de placements aux facteurs de risque, par exemple: taux de change, taux d'intérêt, volatilités, etc.
Deuxièmement, en utilisant les règles de Bâle III, un résumé des calculs des charges de fonds propres est présenté, mettant l'accent sur les risques de contrepartie et de marché. Enfin, un test de résistance sur les facteurs de risque est effectué pour évaluer les fluctuations des charges de capital et leur impact sur les ratios de suffisance du capital.
Nos résultats montrent des modifications importantes des ratios de capital et de suffisance du capital pour les portefeuilles de spéculation et de ventes.

Mots clés: Produits dérivés, portefeuille, négociation, vente, évaluation du marché, sensibilité, charge financière, Bâle III, test de résistance, choc, risque de marché, risque de contrepartie, cartographie des flux de trésorerie, options, swaps, taux d'intérêt, change, VaR, volatilité.

Abstract

The purpose of the thesis is to assess the sensitivity of market risk and counterparty risk capital charge to financial market fluctuations. The research relies on Basel documents for the computation of capital requirements, and the application is done on the American, Turkish and European markets.
The first part of the thesis discusses the different cash flow mapping approaches to assess the sensitivity of the investment portfolio to risk factors e.g. exchange rates, interest rates, volatilities, etc.
Secondly, using Basel rules, a summary of capital charge calculations is provided with an emphasis on counterparty and market risk.
Last but not least, a stress test on risk factors is performed to evaluate the fluctuations in capital charges and its impact on capital adequacy ratios.
Our results show significant changes in capital charge and capital adequacy ratio for trading and back-to-back portfolios.

Keywords: Derivatives, Portfolio, Trading, Sales, Back-to-Back, Mark-to-Market, Sensitivity, Capital charge, Basel III, Stress test, Shock, Market risk, Counterparty risk, Mapping, Cash flow mapping, Options, Swaps, Interest rate, Foreign exchange, VaR, Volatility.

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Abbreviations

- **IR:** Interest Rate
- **FX:** Foreign Exchange
- **MtM:** Mark-to-Market
- **P&L:** Profit and Loss
- **CCR:** Counterparty Credit Risk
- **CVA:** Credit Valuation Adjustment
- **SA:** Standardized Approach
- **SbM:** Sensitivities based Method
- **R-SbM:** Reduced Sensitivities based Method
- **CTP:** Correlation Trading Portfolio
- **VaR:** Value-at-Risk
- **ES:** Expected Shortfall
- **JTD:** Jump To Default
- **DRC:** Default Risk Charge

Introduction

In order to grow, everyone must take risks, and that includes banks and financial institutions who wish to expand their capital.
On one hand, investing in financial markets has proven to be incredibly rewarding. On the other hand, it has also proven to be incredibly risky, and substantially damaging if done wrong, just as exhibited in the financial crisis of 2008.
To mitigate risks accompanying high reward investments, banks have been subject to Basel regulations in order to keep them solvable towards their clients and counterparts in cases of crisis. The history of banking regulations started in 1988 and continued to evolve until today (cf. Hull, J. (2018)).
The subprime mortgage crisis of 2008 arose right after Basel II was put in place. It showed the weaknesses of Basel II and illustrated the need for stronger regulations taking into account, among other factors, two nuances of counterparty risk: counterparty credit risk which is the risk of default (cf. Basel III, The Standardised Approach for Measuring Counterparty Credit Risk Exposures, (2014)) and credit valuation adjustment which is the risk of decrease in the credit quality of the counterpart (cf. Basel III, Review of the Credit Valuation Adjustment Risk Framework, (2015)). It also illustrated the need for more severe sensitivity-based precautions against market risk (cf. Basel III, Minimum Capital Requirements for Market Risk, (2016)).

Judging by the fact that actual drivers of market fluctuations are political events, and that there are tensions between the world's two largest economies in a trade war, there is nothing pushing away the possibility of a crisis happening again - this time for political reasons.
This instigates the following question:

What would happen to the Basel III capital charge, and, as a result, the capital adequacy ratio, if anything happened and triggered disturbing fluctuations in the market even for backed positions?

In order to answer the above question, a sensitivity-based approach is adopted to study the changes of the capital charge and the capital adequacy ratio under three possible scenarios: mild, severe and severely adverse. This method proves efficient for large portfolios requiring a lot of time and effort in reevaluation. The process to calculate Basel III capital requirements is also long and complicated, especially with exotic and complex derivatives.
So as not to reevaluate instruments and recalculate the capital charge every time, the cash-flow mapping technique (cf. Alexander C. (2008)) makes it easier to stress test portfolios and their key metrics, such as the VaR.
In this study, both counterparty risk capital charges CCR and CVA and the market risk capital charge are calculated for two portfolios: a back-to-back portfolio (also referred to as a sales portfolio) in which each position is backed by the exact opposite position with another counterpart, and a trading portfolio which is based solely on speculating in the market and maximizing P&L. The change in P&L per portfolio for each of the scenarios above is calculated using the cash-flow mapping technique and incorporated into the capital charge calculation in order to determine how it's affected.

Part I

Cash Flow Mapping

Chapter 1

Introduction to Cash Flow Mapping

Risk management systems are based on models that describe potential changes in the factors affecting portfolio value. These **risk factors** are the building blocks for all pricing functions. In general, the factors driving the prices of financial securities are equity prices, foreign exchange rates, commodity prices, and interest rates. By generating **future scenarios** (in order to perform **stress testing**) for each risk factor, we can infer changes in portfolio value and reprice the portfolio accordingly for different 'states of the world'.

It can be complex and time-consuming to stress test a portfolio when it's composed of different financial instruments and derivatives. That is why we 'translate' the portfolio into a set of risk factors which makes it easier to manipulate and stress test.

Let's take the case of interest rate sensitive instruments, for example. These instruments could be FRNs, FRAs, vanilla IRS, bonds, loans, etc. All these instruments could be expressed or 'translated' into cash flows.
When a portfolio is large enough, and the number of maturity dates (thus the number of cash flows) increases, the payout of the portfolio is approximately continuous.
That's why it's easier to map these cash flows to standard maturity dates (which we will call ***vertices***), which are usually predefined, finite and correspond to a set of discount rates. Discount rates obviously affect the cash flows and the present value of a portfolio, so the set of discount rates becomes a set of risk factors.

Introductory example

We'll illustrate with an example of a simple portfolio consisting of a long position in a single Treasury bond with a principal of $1 million maturing in 0.8 years. The purpose is to calculate the VaR (Value at Risk) of the portfolio (cf. Hull, J. Technical Note).

We'll suppose the bond provides an annual coupon of 10% payable semiannually. This means that the cash flows are $50,000 in 0.3 years and $1,050,000 in 0.8 years (last coupon payment + principal).

We can **break down** the bond by saying it's equivalent to a 0.3-year zero-coupon bond with a principal of $50,000 and a 0.8-year zero-coupon bond with a principal of $1,050,000.

We then consider these two bonds and break them down again into bonds with standardized maturities (3-month, 6-month, n-years, etc.); the vertices. The position in the 0.3-year zero-coupon bond can be mapped into an equivalent position in 3-month and 6-month zero-coupon bonds (with payouts which are to be calculated).
The position in the 0.8-year bond can be mapped into an equivalent position in 6-month and 1-year zero-coupon bonds. The result is that the position in the 0.8-year coupon-bearing bond is translated

into three positions in different zero-coupon bonds having maturities of three months, six months, and one year.

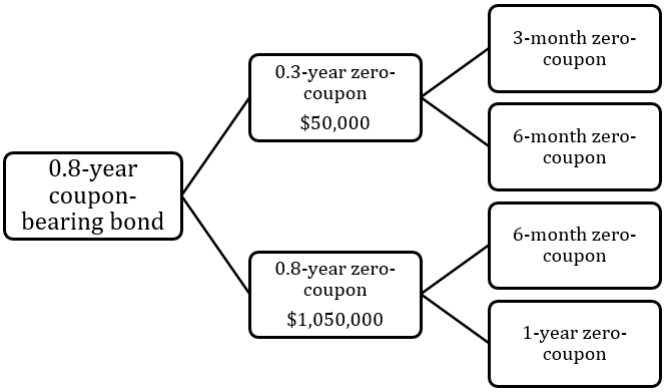

Figure 1.1: Breaking down initial bond example

What's missing is the payouts from each coupon at the end of the tree. That is what we call the **mapping procedure**. When choosing vertices, we already have the discounting factors (resulting from zero rates) corresponding to these maturities. They are extracted from yield curves. We also already have the bond price volatilities corresponding to each vertex. This means we interpolate the discounting factor as well as the volatility corresponding to the maturity of the original bond (0.8 years in this case).

In a more general case where we have more than one bond or type of cash flow, we consider the problem of mapping all the cash flows to the key rates (vertices).

Maturity	3-month	6-month	***0.8 year***	1-year
Zero rate (% with annual compounding)	5.50	6.00	***6.60***	7.00
Bond price volatility (% per day)	0.06	0.10	***0.16***	0.20

Correlation between daily returns	3-month bond	6-month bond	1-year bond
3-month bond	1.0	0.9	0.6
6-month bond	0.9	1.0	0.7
1-year bond	0.6	0.7	1.0

Table 1.1: Data to illustrate mapping procedure

The values emphasized in the table have been interpolated from the values corresponding to the chosen vertices.

The present value of the \$1,050,000 cash flow to be received in 0.8 years is $\frac{1{,}050{,}000}{1.066^{0.8}} = 997,662$.
Suppose we allocate α of the present value to the 6-month bond and $1-\alpha$ of the present value to the 1-year bond.
Matching variances, we obtain:

$$0.0016^2 = 0.001\alpha^2 + 0.002\,(1-\alpha)^2 + 2 * 0.7 * 0.001 * 0.002\alpha\,(1-\alpha)$$

This quadratic equation has been derived from the principle of *volatility invariance*. This means that the volatility of the 0.8-year bond should be the same as the volatility of the portfolio containing two bonds with volatilities equal to 0.001 and 0.002 and a correlation of 0.7.
Solving this equation gives $\alpha = 0.320337$. This means that 32.0337% of the value should be allocated

to a 6-month zero-coupon bond and 97.9663% should be allocated to a 1-year zero-coupon bond.

The 0.8-year bond worth \$997,662 is in fact **replaced** by a 6-month bond worth $\$997,662 * 0.320997 = \$319,589$ and a 1-year bond worth $\$997,662 * 0.679663 = \$678,074$.

The cash-flow mapping scheme not only preserves the variance, but also the value of the bond.

When we interpolate the rate and the volatility the same way for the 0.3-year (3.6-month) zero-coupon bond, we obtain that the present value cash flow is \$49,189 mapped into a position worth \$37,397 in a 3-month bond and a position worth \$11,793 in a six-month bond.

When we sum up the results **grouped by vertices**, we get the results summarized in the following table:

	\$50,000 received in 0.3 years	\$1,050,000 received in 0.8 years	Total
Position in 3-month bond (\$)	37,397		37,397
Position in 6-month bond (\$)	11,793	319,589	331,382
Position in 1-year bond (\$)		678,074	678,074

Since the purpose of this specific cash-flow mapping was to calculate the Value at Risk, we calculate it based on the Total column of table (1), which represents the **mapped** cash flows of the portfolio at the chosen vertices. If we had a larger portfolio, the number of data points available would be bigger.

The variance of the mapped cash flows is given by the portfolio variance formula and using the correlation matrix given in table 1.1.

$$Variance_{portfolio} = 2,628,518$$

$$\sigma_{portfolio} = \sqrt{2,628,518} = 1,621.3$$

We use the parametric VaR formula for our calculation:

$$Var^{99\%} = 1,621.3 * 2.33 = 3,777.6$$

The 10-day VaR is given by:

$$\sqrt{10} * VaR^{99\%} = 11,946$$

or about \$11,950.

Chapter 2

Types of cash-flow mapping

The previous section presented a concrete example on cash-flow mapping and one of its applications.

We will now look at a general idea of cash-flow mapping.

As specified in the previous section, we choose vertices and map the cash flows accordingly. However, there are several ways of going about it to obtain the mapped amount. There are several types of cash-flow mapping[1]. The most commonly used type is the simple elementary map. We can choose to preserve the present value, or the duration, or the PV01, or the volatility, or all at once (the latter would be called complex cash flow mapping) (cf. Alexander C. (2008)).

2.1 Present value and Duration Invariant Cash-Flow Map

Suppose an original cash flow with maturity T. We choose the vertices as being T_1 and T_2. Suppose the original cash flow has a present value of \$1. Let's note x_1 (resp. x_2) as the mapped cash flow to the T_1-maturity (resp.T_2-maturity) interest rate.

Assuming we want the **present value** to be preserved by the mapping, then

$$x_1 + x_2 = 1 \tag{2.1}$$

Here are the Macaulay durations[2] for each cash flow:

Cash flow	**Macaulay duration**
Original	T
Mapped	$\frac{x_1T_1+x_2T_2}{x_1+x_2}$

Now, assuming we want the **duration** to be preserved, then

$$x_1T_1 + x_2T_2 = (x_1 + x_2)T \tag{2.2}$$

Example 2.1.
In this example, we will consider that the issue is to map a cash flow in 1 year and 55 days with present value \$1 million. The chosen vertices are 12 months and 18 months. (We assume the day count convention in actual/360).
We take $x_1 = x$ and $x_2 = 1 - x$ so that equation (2.1) is satisfied.
Equation (2.2) becomes $x = \frac{T_2-T}{T_2-T_1}$.

[1] There are six types of mapping: cf. Henrard M., Comparisons of Cashflow Maps for Value-at-Risk

[2] $Macaulay\ duration = \frac{PV_{coupons}+PV_{principal}}{Currentbondprice}$

Since T_1 is the 12-month vertex, $T_1 = 360$ and by that $T = 415$ and $T_2 = 540$.
Hence,

$$x = \frac{540 - 415}{180} = 0.694444.$$

This means \$1 million is broken down into two: \$694,444 mapped to the 12-month vertex and \$305,556 mapped to the 18-month vertex in present value terms.

2.2 PV01 Invariant Cash-Flow Map

PV01 denotes the price change in response to a 1 bp fall in market interest rates all along the yield curve (parallel shift in the yield curve). It is used for fixed income securities such as bonds (mostly), swaps, preferred stock, etc.

There are two ways of calculating the PV01 based on whether we are using discrete or continuous compounding of interest.
We consider the same variables as before; a single cash flow at time T with a present value of \$1. Let r_T and R_T denote the zero coupon rate at maturity T under continuous and discrete compounding, respectively. Normally, T is not a standard maturity chosen among the vertices. It is a maturity situated between two adjacent chosen vertices. That is why we deduct r_T and R_T using the zero curve interpolation technique.

Below is a table regrouping the different mapping formulas under present value and PV01 invariant conditions.

	Discrete Compounding		Continuous Compounding	
	Original	Mapped to vertex	Original	Mapped to vertex
Present Value	1	x_i	1	x_i
Non-discounted value	$(1+R)^T$	$x_i\,(1+R)^{T_i}$	$\exp(rT)$	$x_i \exp(rT_i)$
PV01	$T\,(1+R)^{-1}$	$x_i T_i\,(1+R_i)^{-1}$	T	xT_i

Table 2.1: PV01 Invariant Cash flow mapping formulas

It follows from this table that the PV01 invariant condition which we were looking for for a cash flow map under *continuous* compounding is

$$\sum_{i=1}^{n} x_i T_i = T, \tag{2.3}$$

and under *discrete* compounding is

$$\sum_{i=1}^{n} \frac{x_i T_i}{(1+R_i)} = \frac{T}{(1+R)}. \tag{2.4}$$

The present value and duration invariant mapping always satisfies the PV01 invariant mapping condition (2.3) under continuous compounding but does not always satisfy (2.4) under discrete compounding.

The advantage of using the continuous compounding condition is that zero coupon rates at the vertices of the cash flow map do not need to be known for PV01 invariant mapping. However, they do need to be known for discretely compounded PV01 invariant mapping.
That's why it is ideal to work with continuous compounding when performing present value and duration invariant mapping to preserve the PV01 at once.

2.3 Volatility Invariant Cash-Flow Map

Suppose we wish to map a cash flow with \$1 present value to two standard vertices with maturities T_1 and T_2. Let the volatilities of the interest rates of the maturities T_1, T and T_2 be σ_1, σ and $\sigma 2$ respectively.
Let ρ denote the correlation between the changes in interest rates of maturities T_1 and T_2.
σ is linearly interpolated from the volatilities σ_1 and σ_2, just like in the introductory example.

If we want the mapped cash flow to have the same volatility as the original cash flow, then

$$x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho\sigma_1\sigma_2 = \sigma^2. \tag{2.5}$$

If we combine (2.5) with any of the previous conditions, we find a unique value of (x_1, x_2).

Example 2.1.
Let's consider the same cash flow as the previous example: present value of \$1 and maturity T =1 year 55 days. We'll consider the vertices T_1 =12 months and T_2 =18 months.
Now, suppose the interest rate volatilities are 75 bp for the 12-month and 90 bp for the 18-month rate and that the correlation $\rho = 0.75$.
Let's find a cash flow mapping that preserves:

1. the volatility and the present value;
2. the volatility and duration;
3. the volatility and PV01.

Solution
We start by interpolating the volatility of the 1 year 55 day interest rate from the volatilities given above:

$$\sigma = \sigma_1 \times \frac{T_2 - T}{T_2 - T_1} + \sigma_2 \times \left(1 - \frac{T_2 - T}{T_2 - T_1}\right),$$

$$0.79583\% = 0.75\% \times \frac{540 - 415}{180} + 0.90\% \times \left(1 - \frac{540 - 415}{180}\right).$$

1. To satisfy equation (2.1), we write $x_1 = x$ and $x_2 = 1 - x$. We replace x_1 and x_2 by their new expressions in (2.5) i.e.

   $$0.63335 = 0.5625x^2 + 0.81(1 - x)^2 + 1.0125x(1 - x).$$

   The expression becomes

   $$0.36x^2 - 0.6075x + 0.17665 = 0.$$

   x could be either 1.314089 or 0.373410.
   We choose the second solution $x = 0.373410$ in order to have both mapped CFs positive.
   This means we have \$373,410 mapped to the 12-month vertex and \$626,590 mapped to the 18-month vertex, in present value terms.

2. Based on the combination of equations (2.2) and (2.5), we have to solve for x_1 and x_2 in

   $$360x_1 + 540x_2 = 415\,(x_1 + x_2) \Rightarrow 11x_1 = 25x_2,$$

   $$0.63335 = 0.5625x_1^2 + 0.81x_2^2 + 1.0125x_1x_2.$$

We find $x_1 = 0.737383$ and $x_2 = 0.324448$.
Hence, we map \$737,383 to the 12-month vertex and \$324,448 to the 18-month vertex, in present value terms.
However, the present value of the mapped cash flow is no longer \$1 million, but \$1,061,831 because obviously, we did not force the present value invariant mapping condition (equation (2.1)).

3. Based on a combination of equations (2.5) and (2.3), we need to solve for x_1 and x_2 in

$$360x_1 + 540x_2 = 415,$$

$$0.63335 = 0.5625x_1^2 + 0.81(x_2)^2 + 1.0125x_1x_2$$

A solution is $x_1 = 0.903004$ and $x_2 = 0.166515$.
We map \$903,004 to the 12-month vertex and \$166,515 to the 18-month vertex.
However, the present value of the mapped cash flow is no longer \$1 million, but \$1,069,519.

2.4 Complex Cash-Flow Map

It is ideal to want to preserve all the characteristics of a cash flow while mapping it, for maximum accuracy and logic in our calculations.
That is why there is a way to perform the complex cash flow mapping that preserves present value, duration, PV01 and volatility all at once.

In the previous section, that was impossible to perform because we only considered mapping our cash flow to two adjacent vertices. This means only two properties of the cash flow could remain invariant. However, this does not have to be the case. Neither do the vertices have to be adjacent nor do they have to be only two. In fact, when dealing with a big portfolio, we choose more than two vertices to map our cash flows.
In this section we'll show an example on how a cash flow can be mapped to three vertices while conserving the invariant characteristic of the desired properties.
First, we'll summarize the conditions for each property:
Present value invariance

$$\sum_{i=1}^{n} x_i = 1 \tag{2.6}$$

Duration invariance

$$\sum_{i=1}^{n} x_i T_i = T \sum_{i=1}^{n} x_i \tag{2.7}$$

PV01 invariance (continuous compounding)

$$\sum_{i=1}^{n} x_i T_i = T \tag{2.8}$$

Volatility invariance

$$X'VX = \sigma^2 \tag{2.9}$$

where V is the covariance matrix of the interest rates at the vertices in the mapping set, X is the vector present value of mapped cash flows and σ is the volatility of the interest rate maturity T.

Example 2.1.
Let's consider a cash flow with maturity $T = 1y65d = 425d$ and present value \$1 million to be mapped to the 6-month (180 days), 12-month (360 days) and 18-month (540 days) vertices.
The volatilities and correlations of the continuously compounded interest rates at the vertices in the mapping set are given in table 2.1.
The mapping should keep present value, PV01 and volatility invariant.

Correlations	6 month	12 month	18 month	Volatilities (bp)
6 month	1	0.8	0.7	60
12 month	0.8	1	0.75	75
18 month	0.7	0.75	1	90

Table 2.2: Volatilities and correlations of the mapping set

Solution
First, we set up the constraints ((2.6)) and ((2.7)) with $T_1 = 180$, $T_2 = 360$, $T_3 = 540$ and $T = 425$. We know that if these are satisfied then so is ((2.8)).
In Excel Solver, we use the objective $\left[X'VX - \sigma^2\right]$ with the covariance matrix

$$V = \begin{pmatrix} 0.36 & 0.36 & 0.378 \\ 0.36 & 0.5625 & 0.50625 \\ 0.378 & 0.50625 & 0.81 \end{pmatrix}$$

where σ is interpolated using the same method as the previous example.
This gives

$$(x_1, x_2, x_3) = (-0.227054, 1.092997, 0.134057)\,.$$

The result is \$227,054 mapped to the 6-month vertex, \$1,092,997 mapped to the 12-month vertex and \$134,057 mapped to the 18-month vertex, in present value terms.
This mapping satisfies all the invariance conditions specified above.

Chapter 3

Mapping an options portfolio

An options portfolio is quite different from a portfolio containing other derivatives and financial instruments. The value of the portfolio depends on the price of each option.
The price is either obtained by *marking to market* through current market price observations or by *marking to model*; where we obtain the price as a numerical solution to a stochastic differential equation.
The purpose is to map the options portfolio to a set of **risk factors** and see how its value behaves when subject to a small change in any of the related risk factors. We ignore the risk of the factor itself allowing the traders to take precautionary hedging measures to mitigate such changes.

We use this for the identification of risk exposures and their classification as well a faster approximation of the Value at Risk relative to the entire portfolio.
The procedure works through *Taylor expansion*[1].

3.1 Taylor Expansions

A Taylor expansion may be used as a **local** approximation to the change in the value of an options portfolio as a function of small changes in its risk factors.
The major risk factors of an options portfolio are the **prices of the underlying contracts** and **implied volatilities**. Minor risk factors could be interest rates or dividends.

When we think about options, the first thing that comes to mind to measure risk is the options' Greeks. When all the options in the portfolio have the same underlying, we use the *net position* Greeks in the Taylor expansion, and all the risk factor returns are included in the expansion. When the portfolio contains options of different underlyings, we use the *net value* Greeks and in that case, only the price risk factor return is included in the expansion.

The disadvantage of the Taylor expansion using the Greeks is its large computational time compared to the fast changes which could occur in the market and affect the option value instantly. It can take several minutes to calculate the net portfolio Greeks of a large and complex options portfolio because it requires complete revaluation of all the options in the portfolio for different values of the underlying risk factors. *It is quite possible that by the time the value of the Greek has been computed the underlying price will have changed, so the computed value will no longer be useful.*

[1] The n^{th} order Taylor approximation of $f(x)$ about x_0 is:
$f(x) \approx f(x_0) + (x - x_0) f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + ... + \frac{(x-x_0)^n}{n!} f^n(x_0)$

3.2 Value Delta and Value Gamma

Just like we mentioned in the previous section, we use *value Greeks* to map an options portfolio because they are additive over several underlyings.
In this section, we will explain how the value delta and value gamma of a portfolio are calculated.

First, we define the *point value* (*pv*) of an option: it is the value of the underlying contract per point. It's used to calculate the profit and loss from trading the option:

$$P\&L = \Delta g * pv \tag{3.1}$$

where Δg is the change in the option price.
An example by Alexander C.(vol.3, 2008) will clarify it better. Suppose we have exchange traded options on the FTSE 100 index with a point value of £10. If the option is bought at £60 and sold at £65 then $\Delta g = £5$ and the profit is $£10 \times £5 = £50$. We can also use the point value to convert the underlying price of the FTSE 100 index (which has no units of measurement) into a notional equivalent. The 'point' for FTSE 100 options and futures is a point on the FTSE 100 index as well. So, if the index stands at 7,000, then an option or futures on the same index is on a notional value of $£10 \times 7,000 = £70,000$.

3.2.1 Value Delta

This leads to the calculation of the *value delta*, also called *dollar delta* because it is in value terms. It's calculated by multiplying the position delta by the price of the underlying (which will be denoted by S) and the point value.
It is important to remember that the position delta is the ordinary delta (i.e. partial derivative with respect to S) multiplied by the number of units of the underlying (N) that the option contracts to trade.
Hence,

$$\delta^{\$} = \delta^{P} \times S \times pv = \delta \times N \times S \times pv \tag{3.2}$$

The value delta divided by 100 denotes the sensitivity of the nominal value of the position to a 1% change in the underlying price.
It's the **map** of the option using its delta: it is the value of a position in the underlying that has the same profit (or loss) as the options portfolio when S changes by 1%.
That's why some people like to call (3.2) the *delta equivalent*.
When we wish to move on to map the entire options portfolio on different underlyings, we'll just add up all the value deltas because they are additive, while position deltas are not. This notion will become clearer in the following example.

Example 3.1.
Suppose we have six options on two UK stocks X and Y. The details are given in table (3.1):

Stock	X	Y
Option type	Short call	Short put
N	2	4
Share price (pence)	50	75
Delta	+0.5	-0.6
Point value (£ per penny)	10	10

Table 3.1: Stocks X and Y details

We wish to calculate the value delta of this portfolio using the details from table 3.1.

- The value delta of the short calls on stock X is:

$$-0.5 \times 2 \times 50 \times £10 = -£500.$$

  This means that if the price of share X rises by 1%, we would lose approximately £5 on this position.

- The value delta of the short puts on stock Y is:

$$0.6 \times 4 \times 75 \times £10 = £1800,$$

  meaning that if the price of share Y rises by 1% we gain approximately £18 on this position.

Finally, the value delta of the portfolio is $-£500 + £1800 = £1300$. If the prices of **both** shares rise by 1% then we would gain approximately £13 on the total position.

3.2.2 Value Gamma

The value gamma has a different definition and is not as intuitive as the definition of the value delta:

$$\gamma^{\$} = \gamma \times N \times S^2 \times pv, \tag{3.3}$$

i.e. the value gamma is the position gamma multiplied by the **square** of the underlying price converted into value terms using the point value.

Value gammas are also additive across options on different underlyings, just like value deltas.

3.3 Delta-Gamma approximation: Single Underlying

This section explains how to map a portfolio of options on a single underlying to its *price* risk factor. The formula for the **delta-gamma approximation** is a quadratic risk factor mapping that captures the non-linearity of the payoff profile for options portfolios.
When the single underlying price is denoted as S, the pay-off of the portfolio is written as

$$\Delta P(S) \approx \delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2, \tag{3.4}$$

where δ^P and γ^P are the net position delta and gamma of the portfolio.

The equation (3.4) shows how to approximate the portfolio price changes while using the current value of the underlying as presented in figure 3.1 (cf. Alexander C.(vol.3, 2008)).

Using equation (3.4) and the point value of the underlying, we reach an expression of the P&L of the options portfolio:

$$P\&L \approx \left[\delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2\right] \times pv, \tag{3.5}$$

Another way to write the expression of the P&L is using the value Greeks and the return on the underlying $R = \frac{\Delta S}{S}$.

$$P\&L \approx \delta^{\$} R + \frac{1}{2}\gamma^{\$} R^2 \tag{3.6}$$

We will now consider an example to illustrate the delta-gamma approximation.

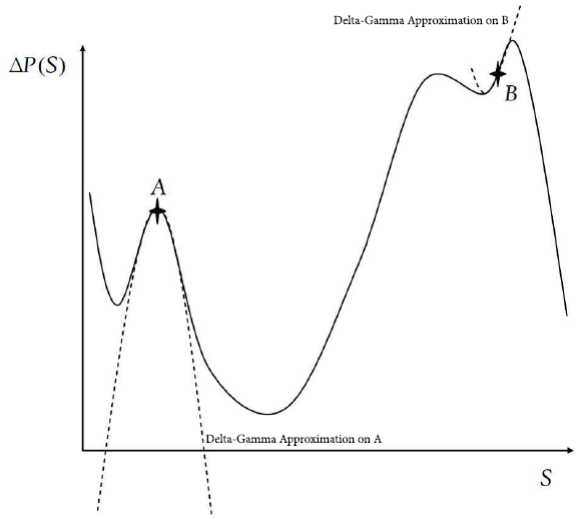

Figure 3.1: Delta-Gamma approximation

Example 3.1.
Consider a portfolio of options on the S&P500 index having a delta of -0.55 and a gamma of 0.013 when the index is at 1200.
We wish to find the value delta and value gamma of the portfolio in order to approximate the change in portfolio value if the S&P500 index increases or decreases by 1%.
The point value of futures and options on the S&P500 index is \$250. The value delta and gamma are, therefore,

$$\delta^{\$} = -0.55 \times 1200 \times \$250 = -\$165,000,$$

$$\gamma^{\$} = 0.013 \times 1200^2 \times \$250 = \$4,680,000.$$

The value delta shows that if the S&P500 index increased by 1%, we would lose approximately \$1,650 on the portfolio and if the index decreased by 1% we would gain approx. \$1,650.

The value delta is only a linear approximation. The quadratic approximation, which is better, shows that:

- If the index increases by 1%: $R = +0.01 \Rightarrow P\&L = -\$165,000 \times 0.01 + \frac{1}{2} \times \$4,680,000 \times 0.01^2$

$$= -\$1,650 + \$234 = -\$1,416,$$

- If the index decreases by 1%: $R = -0.01 \Rightarrow P\&L = \$165,000 \times 0.01 + \frac{1}{2} \times \$4,680,000 \times 0.01^2$

$$= \$1,650 + \$234 = \$1,884.$$

Using the quadratic approximation shows, in this case, that if the S&P500 index increased by 1% we would only lose \$1,416 (vs. \$1,650) and if the index decreased by 1% we would gain \$1,884 (vs. \$1,650 as well).

Clearly, having a positive gamma on an options portfolio is a good thing, because it increases the gains and lowers the severity of the losses, and that, in turn, lowers the price risk compared to a portfolio with a negative or zero gamma.

Figure 3.3 shows different graphs: the dotted line shows the delta-gamma approximation, the grey line shows the call option and the straight line shows the delta approximation.
Since call options have a positive gamma, the figure denotes the effect of a positive gamma on the P&L of a portfolio. With a positive gamma, the upward price moves are enhanced and the downward price moves are diminished.

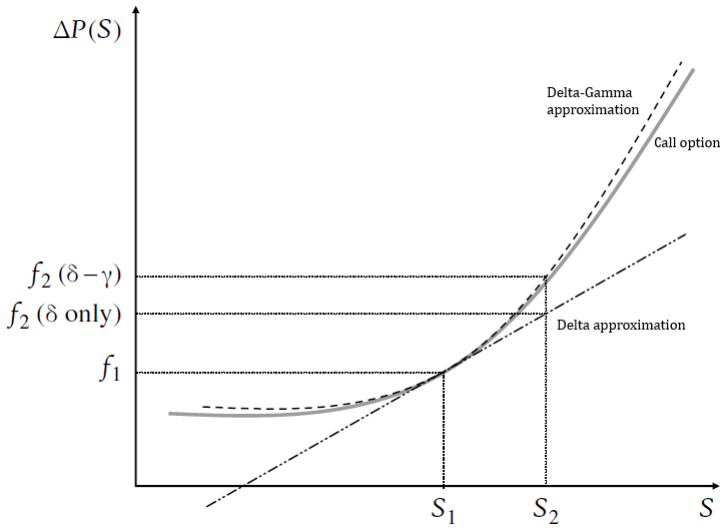

Figure 3.2: The effect of a positive Gamma

For a portfolio with a negative gamma, this is not the case. A similar figure with a negative delta and a negative gamma (from going short instead of long this option) will show that the upside gains are diminished while the downside losses are augmented, compared with the delta-only approximation.

3.4 Price Beta mapping

In the past section, we based ourselves on a portfolio containing an option on a single underlying in order to perform the mapping. However, in reality, we will get a portfolio of options on different securities and the calculation will be much more complex.
Now to make our calculations simpler and less time consuming, instead of using each individual security price as a risk factor, we choose an index to use as a risk factor (we can choose more).
Consider a portfolio containing two options, each on a different security, and the two securities have prices S_1 and S_2. Then the portfolio has a delta-gamma representation

$$\Delta P \approx \delta_1^P \Delta S_1 + \frac{1}{2}\gamma_1^P (\Delta S_1)^2 + \delta_2^P \Delta S_2 + \frac{1}{2}\gamma_2^P (\Delta S_2)^2 \tag{3.7}$$

where the deltas and the gammas are the position greeks with respect to **each** security price; hence the time consumption problem.
We need to convert this to a representation of the form of equation (3.4) where S is the price of an index different than the underlying securities.
The conversion of greeks with respect to the security price into greeks with respect to the index price is equivalent to a *change of scale*. We assume that the *percentage return* on security i is related to the *percentage return* on the index by the β coefficient of each security in the following manner:

$$\frac{\Delta S_i}{S_i} = \beta_i \frac{\Delta S}{S} \tag{3.8}$$

where β is estimated using the following model:

$$R_i = \beta_i R + \varepsilon_i \tag{3.9}$$

From the Ordinary Least Squares regression, we estimate beta like this:

$$\beta_i = \frac{Cov\,(R_i, R)}{V(R)} \tag{3.10}$$

Substituting equation (3.8) for (3.7) gives

$$\Delta P \approx \left[\delta_1^P \beta_1 \left(\frac{S_1}{S}\right) + \delta_2^P \beta_2 \left(\frac{S_2}{S}\right)\right] \Delta S + \frac{1}{2}\left[\gamma_1^P \beta_1^2 \left(\frac{S_1}{S}\right)^2 + \gamma_2^P \beta_2^2 \left(\frac{S_2}{S}\right)^2\right] (\Delta S)^2 \tag{3.11}$$

A way to generalize equation (3.11) to n securities instead of two is first by generalizing the *net position delta* and *net position gamma* as

$$\delta^P = \sum_{i=1}^{n} \delta_i^P \beta_i \left(\frac{S_i}{S}\right), \tag{3.12}$$

$$\gamma^P = \sum_{i=1}^{n} \gamma_i^P \beta_i^2 \left(\frac{S_i}{S}\right)^2, \tag{3.13}$$

and then substituting these by their new expressions and obtaining:

$$\Delta P \approx \delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2. \tag{3.14}$$

Usually, the point value of the securities options is the same as the point value of the index options. The relationships (3.12) and (3.13) imply that the value delta and value gamma of the portfolio are also written as

$$\delta^{\$} = \sum_{i=1}^{n} \delta_i^{\$} \beta_i, \tag{3.15}$$

$$\gamma^{\$} = \sum_{i=1}^{n} \gamma_i^{\$} \beta_i^2. \tag{3.16}$$

Finally, we replace by our new values in the initial equation (3.6) for P&L:

$$P\&L \approx \delta^{\$} R + \frac{1}{2}\gamma^{\$} R^2, \tag{3.17}$$

where R is now the return on the index, not on the individual security.

Example 3.1.
We shall consider a portfolio with two options on stocks A and B.
The details about the options are given in table (3.1)

Stock/Index	A	B	S&P100
Price($)	55	25	
Point value (&)	250	250	250
Beta β	1.24	1.76	
Number of long calls	30	40	
Delta δ	0.5	0.25	
Gamma γ	0.2	0.1	

Table 3.2: Option details for the delta-gamma approximation to a single underlying

Solution
First we start off by calculating the value delta and value gamma of each option from equations (3.2) and (3.3)

$$\delta_A^{\$} = 0.5 \times 30 \times \$250 \times \$55 = \$206,250$$

$$\delta_B^{\$} = 0.25 \times 40 \times \$250 \times \$25 = \$62,500$$

The portfolio value delta will be given by

$$\delta^{\$} = \$206,250 \times 1.24 + \$62,500 \times 1.76 = \$365,750$$

The value gamma of each option is

$$\gamma_A^{\$} = 0.2 \times 55 \times 30^2 \times \$250 = \$2,475,000$$

$$\gamma_B^{\$} = 0.1 \times 25 \times 40^2 \times \$250 = \$1,000,000$$

Hence, portfolio value gamma is given by

$$\gamma^{\$} = \$2,475,000 \times 1.24^2 + \$1,000,000 \times 1.76^2 = \$6,903,160$$

Finally, if the index increases by 1% (R=1%), we apply the delta-gamma approximation using equation (3.17) and obtain

$$P\&L = \$365,750 \times 0.01 + \frac{1}{2} \times \$6,903,160 \times 0.01^2 = \$4,003$$

While if the index *decreases* by 1% (R=-1%)

$$P\&L = \$365,750 \times (-0.01) + \frac{1}{2} \times \$6,903,160 \times (-0.01)^2 = \$ - 3,312$$

3.5 Delta-Gamma approximation on several underlyings

In this section we consider a portfolio of options on several underlying assets (a mix between options on stocks, options on bonds, and other kinds of financial instruments).
We will derive a quadratic mapping for the change in portfolio value. Let the underlying assets have prices $S_1, \cdots, S_n$ and let

$$\Delta S = (\Delta S_1, \cdots, \Delta S_n)'$$

be the vector of changes in underlying prices. The net position deltas and gammas of the portfolio will be considered as the ordinary partial derivatives of the portfolio price P with respect to the underlying prices according to Alexander C.(vol.3, 2008).

$$\delta_P = \begin{pmatrix} P_{S_1} \\ \vdots \\ P_{S_n} \end{pmatrix}, \Gamma_P = \begin{pmatrix} P_{S_1 S_1} & \cdots & P_{S_1 S_n} \\ \vdots & \ddots & \vdots \\ P_{S_n S_1} & \cdots & P_{S_n S_n} \end{pmatrix} \tag{3.18}$$

where subscripts denote partial differentiation.

The *cross-gamma* effect dictates that the gamma matrix is not assumed to be diagonal; the delta with respect to the ith asset may change when the kth asset price changes, even when the ith asset price does not change.

A multivariate second order Taylor approximation to the change in portfolio value may be written

$$\Delta P \approx \delta_P' \Delta S + \frac{1}{2} \Delta S' \Gamma_P \Delta S. \tag{3.19}$$

However, this cannot be extended to the traditional expression for the P&L of the portfolio, because different underlyings have different point values for their options and futures. So, let pv_i denote the point value for options on the ith asset.
This means

$$\delta_i^{\$} = P_{S_i} \times S_i \times pv_i \tag{3.20}$$

and

$$\gamma_{ij}^{\$} = P_{S_iS_j} \times S_i \times S_j \times \sqrt{pv_i pv_j} \tag{3.21}$$

We then define the *value delta vector* and the *value gamma matrix* as

$$\delta_{\$} = \left(\delta_1^{\$}, \cdots, \delta_n^{\$}\right)', \Gamma_{\$} = \left(\gamma_{ij}^{\$}\right), \tag{3.22}$$

The definition (3.21) extends the definition of value gamma to value *cross gammas* with respect to two underlying assets with (possibly) different point values.

Finally, we can write the *multivariate delta-gamma approximation* to the portfolio P&L as

$$P\&L \approx \delta_{\$}' R + \frac{1}{2} R' \Gamma_{\$} R, \tag{3.23}$$

where $R = (R_1, \cdots, R_n)'$ is the *vector* of the underlying asset returns.

Example 3.1.
Consider the example of a portfolio containing bond and stock options, for which the details will be displayed in table (3.3)
Solution

Asset	Bond A options	Stock B options
Position delta (δ)	10	20
Position gamma (γ)	2.5	0.5
Point value (\$)	1,000	250
Delta equivalent ($\delta^{\$}$)	1,000,000	5,000,000
Cross-gamma (γ_{AB})	-0.15	
Returns R	-1%	1%

Table 3.3: Data illustrating example (3.1)

First, we start by using the expression to calculate $\delta^{\$}$ to infer S_1 and S_2 to use in (3.20) and (3.21). We have

$$S_1 = \frac{\$1,000,000}{10 \times \$1,000} = \$100$$

and

$$S_2 = \frac{\$5,000,000}{20 \times \$250} = \$1,000$$

We can now calculate individual gammas to infer the gamma matrix

$$\Gamma_{\$} = \begin{pmatrix} 2.5 \times 100^2 \times 1000 & -0.15 \times 100 \times 1000 \times \sqrt{1000 \times 250} \\ -0.15 \times 100 \times 1000 \times \sqrt{1000 \times 250} & 0.5 \times 1000^2 \times 250 \end{pmatrix} = \begin{pmatrix} 25 & -7.5 \\ -7.5 & 125 \end{pmatrix} \times \$1m$$

Then we use this matrix and the delta vector (the delta equivalents) in equation (3.23) to obtain the portfolio's P&L in millions of dollars:

$$P\&L \approx \begin{pmatrix} 1 & 5 \end{pmatrix} \begin{pmatrix} -0.01 \\ +0.01 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -0.01 & 0.01 \end{pmatrix} \begin{pmatrix} 25 & -7.5 \\ -7.5 & 125 \end{pmatrix} \begin{pmatrix} -0.01 \\ +0.01 \end{pmatrix} = 0.04825.$$

This means that if bond prices fall by 1% and equity prices increase by 1%, the portfolio will make a gain of approximately \$48,250.

3.6 Including Time and Interest Rates Sensitivities

Up until now, we have considered the major risk factor affecting our portfolio; price risk. We found the value of a portfolio if the underlying price moves, but have not associated this price movement with any time period[2].
However, price changes do not happen **instantly**, they take place over time. With time, the value of an options portfolio (as well as the value of anything else) will change, even when the underlying price or volatility remains the same.

The definition behind the Taylor expansion dictates that the more it's developed, the more accuracy it will provide. In our case, it means that the more risk factors we include in our Taylor expansion, the more accurate our portfolio value will be.
In this section, we will introduce a way to capture the effect of time on an options portfolio as well as interest rates changes.

First, we include the portfolio's time sensitivity i.e. the theta of the portfolio in our original delta-gamma mapping (3.4). This provides us with the *delta-gamma-theta approximation* to the change in portfolio price

$$\Delta P \approx \theta^P \Delta t + \delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2 \tag{3.24}$$

where δ^P, γ^P and θ^P denote the portfolio's net position delta, gamma and theta.

Then to include the portfolio's interest rate sensitivity, we use the following equation: delta-gamma-theta-rho approximation

$$\Delta P \approx \theta^P \Delta t + \delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2 + \pi'_P \Delta r, \tag{3.25}$$

where π_P is the portfolio's net position rho[3] vector and r is the vector of discount rates with maturities equal to those of the options.

Since, in reality, we have several underlying assets, we generalize equation (3.25) to obtain the P&L of the portfolio

$$P\&L \approx \theta^{\$} \Delta t + \delta'_{\$} R + \frac{1}{2} R' \Gamma_{\$} R + \pi'_{\$} \Delta R, \tag{3.26}$$

where the value delta and gamma are defined in the previous section, the value theta and value rho vector are

$$\theta^{\$} = \sum_{i=1}^{n} \theta_i^P \times pv_i, \pi_{\$} = \sum_{i=1}^{n} \pi_i^P \times pv_i$$

Example 3.1.
Consider a portfolio of standard European options defined in table 3.1 as well as the risk free rates and the dividend yields corresponding to the same maturities.
We assume the market price of the underlying asset is 100 and the purpose is to find a delta-gamma-theta-rho approximation for the portfolio's P&L.

Solution
Table 3.5 presents the values of the Greek letters calculated based on the Black-Scholes-Merton formula. Net position Greeks are calculated by summing the product of the position and the option sensitivity over all options.

[2]We also ignored the need to discount the P&L to today's terms if it is measured over a risk horizon of months instead of days or weeks.

[3]We use π instead of ρ because ρ is used to denote correlation between risk factors

Option name	A	B	C	D
Option type	Put	Call	Call	Put
Strike	95.0	105.00	100.0	85.0
Market price	0.5363	1.1897	3.4899	0.2203
Maturity	**30**	**60**	**90**	**180**
Risk free rate	4.0%	3.8%	3.7%	3.6%
Dividend yield	2.0%	2.2%	2.5%	2.3%

Table 3.4: Data illustrating delta-gamma-theta-rho approximation example

Option Name	A	B	C	D		
Delta	-0.1703	0.2746	0.5275	-0.0483	Position Delta	-0.6024
Gamma	0.0443	0.0459	0.0474	0.0097	Position Gamma	0.0537
Theta	-0.0232	-0.0213	-0.0199	-0.0027	Position Theta	-0.0269
Rho	-0.0144	0.0432	0.1215	-0.0249		
Position Rho	-0.0433	-0.1727	0.2429	-0.0249		

Table 3.5: Option price sensitivities

We apply equation (3.25) in order to obtain

$$\Delta P \approx -0.0269\Delta t - 0.6024\Delta S + 0.0537(\Delta S)^2 + \begin{pmatrix} -0.0433; & -0.1727; & 0.2429; & -0.0249 \end{pmatrix} \begin{pmatrix} \Delta r_{30} \\ \Delta r_{60} \\ \Delta r_{90} \\ \Delta r_{180} \end{pmatrix}.$$

Occasionally, traders are given theta and rho limits, but it is more common to replace traditional sensitivity limits by VaR limits in major banks.
Interest rates and time are minor risk factors compared to price and implied volatility sensitivities in most options portfolios.

Chapter 4

Mapping implied volatility

Until now, in our Taylor expansion, we have considered price risk to the first and second order as well as time risk and interest rate risk. However, we haven't considered the risk of the implied volatility that comes with price risk.
In this section, the option price will be viewed as a function of two risk factors: a single underlying price S and its volatility σ. Changes in volatility are assumed to be deterministic in our calculations, because if they weren't, the option's *vega* would not be clearly defined.

We will try to extend our Taylor expansion to include vega risk[1].

4.1 Vega Risk in Options Portfolios

The same way we included time and interest rate risk in our Taylor expansion in the previous section, we denote the *delta-gamma-vega* approximation by the following

$$\Delta g \approx \delta \Delta S + \frac{1}{2}\gamma(\Delta S)^2 + v\Delta\sigma \tag{4.1}$$

If all the options in the portfolio are on the same underlying, we can extend the approximation (4.1) to a *delta-gamma-vega-theta-rho* approximation which includes all of the sensitivities mentioned previously

$$\Delta P \approx \theta^P \Delta t + \delta^P \Delta S + \frac{1}{2}\gamma^P(\Delta S)^2 + \pi'_P \Delta r + v'_P \Delta\sigma, \tag{4.2}$$

where σ represents the vector of implied volatilities; one for each option.
If we apply delta-gamma-vega-theta-rho approximation to example (3.1), we obtain the following representation of the portfolio

$$\Delta P \approx -0.0269\Delta t - 0.6024\Delta S + 0.0537(\Delta S)^2 + \begin{pmatrix}-0.0433; & -0.1727; & 0.2429; & -0.0249\end{pmatrix} \begin{pmatrix}\Delta r_{30}\\ \Delta r_{60}\\ \Delta r_{90}\\ \Delta r_{180}\end{pmatrix}$$

$$+ \begin{pmatrix}0.0726; & 0.1349; & 0.1994 & 0.0703\end{pmatrix} \begin{pmatrix}\Delta\sigma_{95,30}\\ \Delta\sigma_{105,60}\\ \Delta\sigma_{100,90}\\ \Delta\sigma_{85,180}\end{pmatrix}.$$

[1] Volatility as a risk factor

Should we wish to generalize the approximation so it could include options on several underlying assets, we would need the portfolio's value greeks, including the value vega, and for that we need the net position vega.
How do we calculate the net position vega and the value vega? We face an issue: each option vega represents the sensitivity of the option value to *its own implied volatility*, which differs for every option according to the strike and maturity of each one. It is simply not feasible to have one volatility risk factor for every option. This is why we have to find a way to reduce the number of volatility risk factors.

4.2 Vega Bucketing

A solution for the problem mentioned above is to classify all the volatility risk factors according to buckets.
We already established the delta-gamma-vega-theta-rho approximation for a portfolio of options on a single underlying

$$\Delta P \approx \theta^P \Delta t + \delta^P \Delta S + \frac{1}{2}\gamma^P (\Delta S)^2 + \pi_P' \Delta r + v_P' \Delta \sigma, \tag{4.3}$$

where σ is the vector of volatility risk factors and the vector v_P is the net position vega.
More generally, the approximation applied on a portfolio of options on several underlying assets is

$$\Delta P \approx \theta^{\$} \Delta t + \delta_{\$}' R + \frac{1}{2} R' \Gamma_{\$} R + \pi_{\$}' \Delta r + v_{\$}' \Delta \sigma, \tag{4.4}$$

where $v_{\$}$ is the *net value vega*, also called the *dollar vega*.
But how are the position vega in (4.3) and the value vega in (4.4) defined?
This is when we resort to an approach called *vega bucketing*. It means that the options on the same underlying with similar maturities are grouped together into a single maturity **bucket**. The volatility risk factor corresponding to each bucket is usually given as the ATM[2] implied volatility of the **average** maturity in the bucket, which is intuitive and logical.

This implies that the sensitivity to this ATM volatility, i.e. the net position vega for that specific bucket is calculated as the sum of the position vegas of all options in that bucket: $v^P = \sum_{i=1}^n v_i^P$.
Hence, the value vega will be $v^{\$} = v^P \times pv = \sum_{i=1}^n v_i^P \times pv$ where pv is the point value of the underlying.

For example, all GBP/USD options with maturity up to 2 months could be put into a bucket together, with the 1-month ATM volatility on GBP/USD as the reference volatility for this bucket, and so on.

Even though this approach provides the trader with a clear perspective on his volatility exposure[3], Alexander C. (vol. 3, 2008) brings to our attention a constraint we haven't considered while vega bucketing; the assumption that the implied volatilities of all options in the same bucket are perfectly correlated with the risk factor for that bucket. Realistically, this is not always the case.
Also, vega bucketing becomes quite unmanageable for a large portfolio where there are many different underlying assets, and several buckets for each underlying.

[2] At-The-Money
[3] Especially for traders who always deal in options on the same underlying

4.3 Volatility Beta Mapping

We reason the same way as the price beta mapping method explained in section (3.4) in order to come out with a more practical result in risk assessment.
The volatility beta mapping strategy chooses a small set of reference volatility risk factors for each price risk factor in the portfolio. In order to keep our analysis simple, we will assume that only one reference volatility risk for each underlying asset (this infers one maturity bucket per underlying).
To calculate net position and value vegas corresponding to each bucket, we start by calculating the volatility beta with respect to the reference volatility risk factor. However, beta is calculated with the help of some historical data on implied volatilities and the OLS regression[4] of the variation in the option's implied volatility on the absolute changes[5] in the risk factor volatility.
After having estimated the betas, the net position vega is given by

$$v^P \approx \sum_{i=1}^{n} v_i \times \beta_i^v \times N_i, \tag{4.5}$$

$$v^{\$} = \sum_{i=1}^{n} v^P \times pv. \tag{4.6}$$

where N_i denotes the number of units of the underlying that the i^{th} option contracts to buy or sell, β_i^v is the volatility beta of the i^{th} option and pv is the point value of the options.

[4]Or forecast a beta using the GARCH time series model which estimates volatilities

[5]We use the absolute changes instead of changes in logarithm of the volatilities, since it is the absolute change in the implied volatility that appears in the delta-gamma-vega approximation.

Part II

Capital Requirements under Basel Regulations

Chapter 5

Banking Regulations History and Main Changes until Basel III

5.1 Banking Regulations History until Basel II

The main purpose of bank regulations is to ensure that a bank keeps enough capital for the risks it takes.
1988 is the year which marked the start of serious banking regulations (cf. Sayah, M. (2017)). Before that year, there used to be a minimum capital to total asset ratio which differed from one country to another. This left some banks loosely regulated and exposed to a lot of risk.

5.1.1 The 1988 Accord

In 1974, the Basel Committee was formed by representatives from several countries, under the supervision of the Bank for International Settlements: BIS. They started working on a set of rules to be implemented by banks and, in 1988, they came out with what is known as the Basel Accord (or Basel I).
The governments were worried about systemic risk[1]. The Accord introduced the Cooke ratio which took into consideration credit risk exposures on and off the balance sheet based on risk weighted assets (RWA). The Accord also introduced the rule that forced banks to hold at least 8% of RWA as a capital requirement.

5.1.2 The 1996 Amendment

Later came the 1996 Amendment (also known as BIS 98). It introduced a standardized approach for calculating market risk capital charge. It also introduced an internal models based approach which could be used by the more sophisticated banks. However, it did not take into account the correlations between different instruments[2] Banks were required to perform back testing for the one-day 99% VaR over the previous 250 days.
The credit risk capital charge included exposures to the same items in the trading and banking book like the 1988 Accord except (a) debt and equity traded securities and (b) positions in commodities and foreign exchange.
This means the total capital required for credit risk and market risk under the 1996 Amendment is

$$Total\ Capital = 0.08 \times (credit\ risk\ RWA + market\ risk\ RWA) \tag{5.1}$$

[1]The risk that a failure by a large bank will lead to failures by other large banks and a collapse of the financial system
[2]The formula for calculating the market risk capital charge using the Internal Models approach is $max\left(VaR_{t-1}, m_c \times VaR_{avg}\right) + SRC$ where m_c is a multiplicative factor, SRC is a *specific risk charge*, VaR_{t-1} is the previous day's value at risk and VaR_{avg} is the average value at risk over the past 60 days.

5.1.3 Basel II

In 1999, a new set of rules was proposed, revised and finally published in 2004. It is the one now known as Basel II. It's applied by all banks of Europe and only the large banks of the US.
It is mainly composed of three pillars:

- Pillar 1: *Minimum Capital Requirements*
  - Market risk capital requirements are the same as in the 1996 Amendment.
  - Credit risk capital requirements are calculated using a new way which reflects the credit risk of counterparties.
  - There is a new capital charge for operational risk.

  The general requirement in Basel I that banks hold a total capital equal to 8% of RWA remains unchanged. When the capital requirement for a particular risk is calculated directly rather than in a way involving RWA, it is multiplied by 12.5 to convert it into a RWA-equivalent.
  As a result, it is always the case that

$$Total\ capital = 0.08 \times [credit\ risk\ RWA + market\ risk\ RWA + operational\ risk\ RWA] \tag{5.2}$$

- Pillar 2: *Supervisory Review Process*
  Supervisors are required to:
  - ensure that a bank has a process in place for ensuring that capital levels are maintained,
  - intervene early when problems arise and cover risks not covered by Pillar 1 (e.g. concentration risks),
  - encourage banks to use better risk management techniques.
- Pillar 3: *Market Discipline*
  Banks are required to reveal more information about risks they are taking and how they are allocating capital for them. This pressures banks into being more conservative investors when the shareholders know more about the deals.

5.2 Basel II.5 and Basel III

5.2.1 Basel II.5

The severe financial crisis of 2007-2009 was a wake up call for the Basel committee to change the regulatory framework. Three main changes were implemented under Basel II.5: a new market risk capital charge, incremental risk charge (IRC) and a new measure of risk taking into account correlation between instruments.

Market Risk Capital Requirements

Only the internal models approach has been changed for the calculation of market risk capital requirements. The standardized method remains the same.
Under the internal models approach (IMA), banks now have to calculate two VaRs instead of one. One is the usual VaR (based on the previous one to four years of market movements). The other is the **stressed VaR**, calculated from a stressed period of 250 days[3].

[3]The formula for the market risk capital charge becomes $max(VaR_{t-1}, m_c \times VaR_{avg}) + max(sVaR_{t-1}, m_s \times sVaR_{avg})$ where VaR_{t-1} and $sVaR_{t-1}$ are the VaR and the stressed VaR calculated on the previous day with a 99% confidence level and a 10-day time horizon, and VaR_{avg} and $sVaR_{avg}$ are the average of VaR and stressed VaR calculated over the previous 60 days, also with a 99% confidence level and a 10-day horizon. m_s and m_c are multiplicative factors determined by bank supervisors and are floored at 3.

Incremental Risk Charge

Banks tended whenever possible to hold credit-dependent instruments in the trading book because the trading-book calculation usually gave rise to a much lower capital charge than the banking-book calculation[4]. The Basel Committee took this into consideration by introducing the incremental default risk charge (IRDC) in 2005 that took into consideration instruments in the trading book that were sensitive to default risk. In the credit crisis of 2008, they realized that most of the losses were not only a result of defaults, but of several other factors such as

- changes in credit ratings,
- widening of credit spreads,
- loss of liquidity.

That is why the incremental risk charge (IRC) was introduced. It is given by the one-year 99% VaR for losses from credit-sensitive products in the trading book taking both credit rating changes and defaults into account. The objective is to set capital equal to the maximum of that obtained using trading book calculations and that obtained using banking book calculations.

Comprehensive Risk Measure (CRM)

The comprehensive risk measure takes into account instruments that are sensitive to the **correlation** between the default risks of different assets[5] The CRM is a single capital charge replacing the incremental risk charge and the specific risk charge for instruments dependent on credit correlation.

External Credit Assessment	**AAA to AA-**	**A+ to A-**	**BBB+ to BBB-**	**BB+ to BB-**	**Below BB- or Unrated**
Securitisations	1.6%	4%	8%	28%	Deduction
Resecuritisations	3.2%	8%	28%	52%	Deduction

Table 5.1: Standardized Capital Charge for Correlation-Dependent Instruments

Table 5.1 shows a standardized approach summary for calculating the CRM. It is not surprising that capital charges are higher for resecuritisations than for securitisations. A deduction means that the principal amount is subtracted from capital, which is equivalent to a 100% capital charge.

5.2.2 Basel III

In alignment with its continuous improvement of the rules, the Basel Committee introduced brand new regulatory controls. The important key elements and differences from Basel II can be understood as follows:

- capital definition and requirements,
- capital conservation buffers,
- countercyclical buffers,
- leverage ratio,
- liquidity risk,

[4]If a bank created ABSs from loans in the banking book, and then bought all the resultant tranches for its trading book, regulatory capital requirements would be lower even though the bank's risks would be unchanged.

[5]such as asset backed securities (ABSs) and collateralized debt obligations (CDOs).

- counterparty credit risk.

They have been summarized and explained by John Hull in his book on risk management (cf. Hull, J. 2018) as specified in the following paragraphs.
The Fundamental Review of the Trading Book (FRTB) has also been created to lay out rules for the market risk capital charge calculation. It will be discussed in detail in chapter (7).

Capital Definition and Requirements

Under the Basel III framework, the bank's total capital is divided into tiers: Tier 1 equity capital, Additional Tier 1 capital, and Tier 2 capital. Each of these parts has its own definition and capital requirement, all summarized in table (5.2).

Tier	**Definition**	**Capital requirement**
Tier 1 equity capital	Share capital and retained earnings	At least 4.5% of RWA
Additional Tier 1 capital	Non cumulative preferred stocks	
Total Tier 1 capital	Tier 1 equity + Additional Tier 1	At least 6% of RWA
Tier 2 capital	Subordinated debt	
Total capital	Tier 1 + Tier 2	At least 8% of RWA

Table 5.2: Capital definition and requirements

The Basel III rules are obviously more demanding than before because of an increase in percentages and a tighter definition of what qualifies as equity capital for regulatory purposes.
The Basel Committee also calls for more capital for 'systemically important' banks[6].

Capital Conservation Buffer

In addition to the minimum capital requirements defined in section (5.2.2), banks are required to set aside an amount equal to **2.5% of the risk weighted assets**. In a non-crisis period, it is easier for a bank to raise capital. That is why this new buffer must be present during normal times in order to cover losses should a crisis occur.

Countercyclical Buffer

We add the countercyclical buffer to the capital conservation buffer and the minimum capital requirements. However, it's not a fixed percentage that should be present all the time. It's set by the local authorities and based on the cyclicality of bank earnings. It ranges **between 0% and 2.5%** of total risk weighted assets. Some countries set this buffer to be higher than the one recommended by the Basel Committee for major banks presenting high systemic risk.

Leverage Ratio

A main factor which led to the subprime crisis is leverage. Too much leverage caused banks to default and in return create a domino effect which led to everyone defaulting and the crisis to reach its peak. That is why the Basel Committee specified a minimum leverage ratio equal to 3%. This ratio is the ratio of capital to total exposure, including all on-balance sheet assets and all exposures[7].

[6]The Committee provides a way to assess whether a bank is Globally Systemically Important (G-SIB) or not, and it is found here: https://www.bis.org/bcbs/publ/d296.pdf

[7]The exposure measure is the sum of (a) on-balance sheet exposures, (b) derivatives exposures, (c) securities financing transaction exposures and (d) off-balance sheet items. Derivatives exposures are calculated as "replacement cost plus add-on".

Liquidity risk

Following the subprime crisis, it has come to the attention of regulators that it is not enough to have sufficient capital to sustain a bank during a crisis. Many of the problems encountered by financial institutions were instead a result of liquidity risks they have taken.
Liquidity risks arise because banks have a tendency to finance long-term needs with short-term funding. That is why Basel III has introduced requirements involving two liquidity ratios that are designed to ensure that banks can survive liquidity pressures:

- Liquidity Coverage Ratio (LCR)
- Net Stable Funding Ratio (NSFR)

The LCR emphasizes a bank's ability to survive a 30-day period of liquidity disruption. The formula for the LCR is given by

$$LCR = \frac{High\ Quality\ Liquid\ Assets}{Net\ Cash\ Outflows\ in\ a\ 30-Day\ Period} \tag{5.3}$$

where the high quality liquid assets (HQLA) are defined by Basel III. This ratio is required to be higher than 100% so that the bank's liquid assets are enough to survive these pressures.

The NSFR focuses on liquidity management over a period of one year:

$$NSFR = \frac{Amount\ of\ Stable\ Funding}{Required\ Amount\ of\ Stable\ Funding} \tag{5.4}$$

where the numerator is calculated by multiplying each category of funding[8] by an available stable funding (ASF) factor defined by Basel III. The denominator is calculated from the items requiring funding, which in turn, have their own required stable funding (RSF) factors also defined by Basel III. The NSFR is also required to be larger than 100%.

Counterparty Credit Risk

On the OTC market, one of the biggest risks encountered is Counterparty Credit Risk. It can be split into two kinds of risks: the default of the counterparty and the decrease in its credit quality.
For computing default counterparty credit risk, the Standardized Counterparty Credit Risk approach (SA-CCR) is followed.
To capture the risk of a decrease in credit quality, the Credit Valuation Adjustment (CVA) capital charge is calculated.
Both approaches are explained thoroughly in chapters 6 and 7.

[8] Capital, wholesale, deposits, retail deposits, etc.

Chapter 6

Applied Capital Requirements for Counterparty Risk

According to Basel, counterparty risk is the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows.
Counterparty risk includes two main components: Counterparty Credit Risk (CCR) and Credit Valuation Adjustment (CVA).

6.1 Counterparty Credit Risk (CCR)

CCR arises from two major classes of products: OTC derivatives[1] and securities financing transactions[2]. CCR arising from OTC derivatives is more significant due to the size of their market and its diversity.
It's actually different from lending risk (or credit risk) in the sense that it is bilateral because both parties are exposed to it. The other difference is that when it comes to lending risk, the notional amount at risk, at any time during the lending period, is usually known with a certain degree of certainty. In most cases, with CCR, the value of the contract in the future is uncertain.
A complete understanding of CCR is complemented by knowledge in market risk, credit risk, operational risk, and liquidity risk.
The formula for the CCR capital charge is given by

$$CCR_{RWA} = EAD \times Risk\ Weight_{counterparty} \tag{6.1}$$

where:

- EAD designates the Exposure At Default, and is given by

  $$EAD = alpha \times (RC + PFE)$$

  RC stands for Replacement Cost and PFE stands for Potential Future Exposure.
  $alpha$ is set at 1.4. These two parameters are calculated differently for **margined** and **unmargined** netting sets. **Only unmargined transactions will be considered in this paper**.

- $Risk\ Weight_{counterparty}$ is also specified by Basel according to the counterparty's credit rating and type.

[1] Such as interest rate swaps, FX forwards, credit default swaps
[2] These include repos, reverse repos, securities borrowing and lending

Unmargined Transactions

For unmargined transactions, if a counterparty were to default and were closed out of its transactions immediately, the replacement cost (RC) is the parameter representing the losses that would occur. It is calculated at the netting set level. As for the potential future exposure (PFE), it represents a potential conservative increase in exposure over a one-year time horizon from the present date. It is calculated per asset class within netting sets and then aggregated.

Replacement Cost (RC)
Mathematically, the replacement cost for unmargined transactions will be written in the following manner:

$$RC = \max\{V - C; 0\} \tag{6.2}$$

where V is the value of the derivative transactions in the netting set and C is the haircut value of net collateral held. When we assume there is no collateral, the replacement cost becomes

$$RC = \max\{MtM; 0\}$$

where MtM stands for Mark-to-Market.
Banks are allowed by the SA-CCR to hold excess collateral (even for unmargined transactions) for further protection and reduction of the PFE, but that does not affect the replacement cost.
Instruments with a one-way margining agreement in favor of the counterparty (when the bank posts collateral but does not collect any) must be treated as unmargined transactions.

Potential Future Exposure (PFE)
The PFE is the product of an aggregate add-on component and a multiplier which takes into account the presence of excess collateral:

$$PFE = multiplier \times AddOn^{aggregate} \tag{6.3}$$

The multiplier is activated when the current value of the derivative transactions is negative. Mathematically:

$$multiplier = \min\left\{1; Floor + (1 - Floor) \times \exp\left(\frac{V - C}{2 \times (1 - Floor) \times AddOn^{aggregate}}\right)\right\} \tag{6.4}$$

where $Floor = 5\%$ and $AddOn^{aggregate} = \sum_a AddOn^{(a)}$; the add-on factor is the simple sum of each asset class add-on.
The following step-by-step approach is followed to compute the add-on factor per transaction:

- Define the primary risk factor;
- Find the corresponding asset class among these:
  - Interest Rate (IR),
  - Foreign Exchange (FX),
  - Equity,
  - Credit,
  - Commodity;
- Compute the adjusted notional amount $d_i^{(a)}$ (the duration is taken into consideration in IR and credit asset classes);
- Find the maturity factor M_i defined as the latest date when the contract may still be active;
- Multiply the adjusted notional by a parameter called *supervisory delta* δ_i;

- Multiply the latter by a given supervisory factor reflecting volatility;
- Aggregate by hedging sets and asset-class levels.

Aside from the maturity M_i, three date parameters appear in the SA-CCR:

- S_i: For Interest Rate and Credit derivatives, S_i references the start date of the contract.
- E_i: For Interest Rate and Credit derivatives, E_i references the end date of the contract.
- T_i: For options in all asset classes, T_i references the latest contractual exercise date.

Adjusted Notional $d_i^{(a)}$

- Interest Rate (IR) and Credit derivatives

$$d_i^{(IR)} = trade\ notional \times SD_i$$

where $SD_i = \frac{\exp(-0.05 \cdot S_i) - \exp(-0.05 \cdot E_i)}{0.05}$ is the supervisory duration, in which S_i and E_i are respectively the start and end of the time period referenced by the derivative floored by 10 business days.

- Foreign Exchange (FX)
The adjusted notional is the notional of the foreign currency leg converted to the domestic currency. That applies to both legs if they are both denominated in foreign currency.

- Equity and Commodity
The adjusted notional is defined as the price of one unit of the stock or commodity multiplied by the number of units.

Mostly, there is no ambiguity concerning the trade notional amount. It is defined clearly and fixed until maturity. However, when there is some confusion, the Basel committee set rules to help with defining it. They are stated in The Standardised Approach for Measuring Counterparty Credit Risk Exposures (March 2014)[3] under paragraph 158.

Supervisory Delta
The supervisory delta adjustments are parameters which reflect the direction of a transaction and its non-linearity and are applied to the adjusted notional amounts. They are specified in tables (6.1) and (6.2) below.

δ_i	Long in the primary risk factor	Short in the primary risk factor
Instruments that are not options or CDO tranches	+1	-1

δ_i	Bought	Sold
Call Options	$+\phi\left(\frac{\ln(P_i/K_i)+0.5\cdot\sigma_i^2\cdot T_i}{\sigma_i\cdot\sqrt{T_i}}\right)$	$-\phi\left(\frac{\ln(P_i/K_i)+0.5\cdot\sigma_i^2\cdot T_i}{\sigma_i\cdot\sqrt{T_i}}\right)$
Put Options	$-\phi\left(-\frac{\ln(P_i/K_i)+0.5\cdot\sigma_i^2\cdot T_i}{\sigma_i\cdot\sqrt{T_i}}\right)$	$+\phi\left(-\frac{\ln(P_i/K_i)+0.5\cdot\sigma_i^2\cdot T_i}{\sigma_i\cdot\sqrt{T_i}}\right)$

Table 6.1: Supervisory delta adjustments for instruments that are not CDO tranches

[3] https://www.bis.org/publ/bcbs279.pdf

δ_i	Purchased (long protection)	Sold (short protection)
CDO tranches	$+\frac{15}{(1+14*A_i)(1+14*D_i)}$	$-\frac{15}{(1+14*A_i)(1+14*D_i)}$

Table 6.2: Supervisory delta adjustments for CDO tranches

In table (6.1), K_i designates the strike price and σ_i is a supervisory volatility specified in the following paragraph along with the supervisory factor.
In table (6.2), A_i and D_i designate respectively the attachment and detachment point of the CDO tranche.

Supervisory Factor

The supervisory factor $SF_i^{(a)}$ is a factor specific to each asset class and is used to convert the effective notional into effective EPE (Expected Positive Exposure).
The supervisory factors are prescribed and presented in the following table along with prescribed supervisory volatilities.
In the table, we will also find prescribed *supervisory correlations* $\rho_i^{(a)}$ only applied to the equity, credit and commodity asset classes, but not interest rate and foreign exchange classes.

Asset Class	**Subclass**	**Supervisory Factor**	**Correlation**	**Supervisory option volatility**
Interest rate		0.50%	N/A	50%
Foreign exchange		4.00%	N/A	15%
Credit, Single Name	AAA	0.38%	50%	100%
	AA	0.38%	50%	100%
	A	0.42%	50%	100%
	BBB	0.54%	50%	100%
	BB	1.06%	50%	100%
	B	1.60%	50%	100%
	CCC	6.00%	50%	100%
Credit, Index	IG	0.38%	80%	80%
	SG	1.06%	80%	80%
Equity, Single Name		32%	50%	120%
Equity, Index		20%	80%	75%
Commodity	Electricity	40%	40%	150%
	Oil/Gas	18%	40%	70%
	Metals	18%	40%	70%
	Agricultural	18%	40%	70%
	Other	18%	40%	70%

Table 6.3: Summary table of supervisory parameters

Time Risk Horizons
For unmargined transactions, the minimum time risk horizon for the SA-CCR is given by:

$$MF_i^{(unmargined)} = \sqrt{\frac{\min\left(M_i; 1\ year\right)}{1\ year}} \tag{6.5}$$

Hedging sets
Basel III rules define what hedging sets are in order to aggregate the add on factors later on. They are summarized in the following table:

Derivatives	**Hedging set**
Interest rate	A hedging set per individual currency
FX	A hedging set per currency pair
Credit	Single hedging set
Equity	Single hedging set
Commodity	A hedging set per category: energy, metals, agricultural, other

Table 6.4: Hedging sets definitions

Add-on factor
As mentioned before, each asset class has a tailored step-by-step approach to compute the effective notional and, therefore, the add-on factor.

- Calculate the effective notional per bucket k:

$$D_{jk}^{(IR)} = \sum_{i \in Ccy_j, MB_k} \delta_i \cdot d_i^{(IR)} \cdot MF_i^{(type)}$$

- Aggregate across maturity buckets to obtain the effective notional per hedging set j:

$$EffectiveNotional_j^{(IR)} = \left[\left(D_{j1}^{(IR)}\right)^2 + \left(D_{j2}^{(IR)}\right)^2 + \left(D_{j3}^{(IR)}\right)^2 + 1.4 \cdot D_{j1}^{(IR)} \cdot D_{j2}^{(IR)} + 1.4 \cdot D_{j2}^{(IR)} \cdot D_{j3}^{(IR)} + 0.6 \cdot D_{j1}^{(IR)} \cdot D_{j3}^{(IR)}\right]^{\frac{1}{2}}$$

- Calculate the add-on factor at the hedging set level:

$$AddOn_j^{(IR)} = SF_j^{(IR)} \cdot EffectiveNotional_j^{(IR)}$$

- Aggregate across hedging sets to obtain final add-on:

$$AddOn^{(IR)} = \sum_j AddOn_j^{(IR)}$$

The add-on formulas per hedging set for IR and FX are the same

- Compute the effective notional per hedging set HS_j:

$$EffectiveNotional_j^{(FX)} = \sum_{i \in HS_j} \delta_i \cdot d_i^{(FX)} \cdot MF_i^{(type)}$$

- Calculate the add-on per hedging set HS_j:

$$AddOn_{HS_j}^{(FX)} = SF_j^{(FX)} \cdot |EffectiveNotional_j^{(FX)}|$$

- Aggregate add-on factors across hedging sets

Credit derivatives are treated a bit differently. The add-on is calculated at the entity level at first, and then aggregated while taking into consideration systematic and idiosyncratic risks through a prescribed correlation factor.
At the entity level:

- Effective notional per entity k:

$$EffectiveNotional_k^{(Credit)} = \sum_{i \in Entity_k} \delta_i \cdot d_i^{(Credit)} \cdot MF_i^{(type)}$$

- Add-on per entity k:

$$AddOn(Entity_k) = SF_k^{(Credit)} \cdot EffectiveNotional_k^{(Credit)}$$

Mathematically, the aforementioned aggregation is performed like so:

$$AddOn^{(Credit)} = \left[\left(\sum_k \rho_k^{(Credit)} \cdot AddOn(Entity_k) \right)^2 + \sum_k \left(\left(1 - (\rho_k^{(Credit)})^2 \right) \right) \cdot (AddOn(Entity_k))^2 \right]^{\frac{1}{2}}$$

The calculation process of the add-on for equity derivatives is very similar to the process for credit derivatives:

- Effective notional per entity k:

$$EffectiveNotional_k^{(Equity)} = \sum_{i \in Entity_k} \delta_i \cdot d_i^{(Equity)} \cdot MF_i^{(type)}$$

- Add-on factor per entity k:

$$AddOn(Entity_k) = SF_k^{(Equity)} \cdot EffectiveNotional_k^{(Equity)}$$

- Aggregated add-on:

$$AddOn^{(Equity)} = \left[\left(\sum_k \rho_k^{(Equity)} \cdot AddOn(Entity_k) \right)^2 + \sum_k \left(\left(1 - (\rho_k^{(Equity)})^2 \right) \right) \cdot (AddOn(Entity_k))^2 \right]^{\frac{1}{2}}$$

The following step-by-step approach is followed for the computation of the commodity add-on factor:

- Effective Notional per commodity type k:

$$EffectiveNotional_k^{(Com)} = \sum_{i \in Type_k^j} \delta_i \cdot d_i^{(Com)} \cdot MF_i^{(type)}$$

- Add-on factor per hedging set j and commodity type k:

$$AddOn(Type_k^j) = SF_{Type_k^j}^{(Com)} \cdot EffectiveNotional_k^{(Com)}$$

- Add-on factor per hedging set HS_j:

$$AddOn_{HS_j}^{(Com)} = \left[\left(\rho_j^{(Com)} \cdot \sum_k AddOn(Type_k^j) \right)^2 + \left(1 - (\rho_k^{(Equity)})^2 \right) \cdot \sum_k \left(AddOn(Type_k^j) \right)^2 \right]^{\frac{1}{2}}$$

- Overall add-on factor for the commodity asset class:

$$AddOn^{(Com)} = \sum_j AddOn_{HS_j}^{(Com)}$$

6.2 Credit Valuation Adjustment (CVA)

While Basel II standards covered the risk of a counterparty default, they did not cover losses associated with the deterioration in the credit worthiness of a counterparty. During the financial crisis, such losses were greater than those arising from simple defaults.
Basel III introduces several approaches for calculating the CVA capital charge: a basic approach (BA-CVA), a standardized approach (SA-CVA) and an internal approach (IMM-CVA).

- The BA-CVA has been presented for the use of most banks (when Basel III was first introduced) as it is simple to apply.
- Throughout the years and following several reforms, the SA-CVA was introduced. It closely resembles the sensitivity-based approach used for the calculation of market risk capital requirements.
- Banks wanting to follow the IMM-CVA have to obtain a special approval from authorities to do so, just like the market risk capital charge calculation.

In this paper we will be following the basic approach for the CVA capital charge calculation. In the 2017 Basel III reforms[4], two basic approaches were presented: one where hedges are recognized, and another reduced approach where hedges are not recognized.

Reduced BA-CVA

The CVA capital requirement under the reduced method is computed as follows:

$$K_{reduced} = \sqrt{\left(\rho \cdot \sum_c SCVA_c\right)^2 + (1-\rho)^2 \cdot \sum_c SCVA_c^2} \tag{6.6}$$

where $\rho = 50\%$ designates the supervisory correlation parameter and:

$$SCVA_c = \frac{1}{\alpha} \cdot RW_c \cdot \sum_{NS} M_{NS} \cdot EAD_{NS} \cdot DF_{NS} \tag{6.7}$$

designates the stand-alone CVA capital per counterparty c. In equation (6.7), the parameters are defined as follows:

- RW_c: counterparty risk weight (prescribed according to credit quality in a table below)
- M_{NS}: effective maturity of the netting set, calculating following the Basel II framework[5]
- EAD_{NS}: exposure at default of the netting set NS calculated the same way as in the SA-CCR
- $DF_{NS} = \frac{1-e^{-0.05 \cdot M_{NS}}}{0.05 \cdot M_{NS}}$: supervisory discount factor
- $\alpha = 1.4$

[4]Basel III: Finalising post-crisis reforms
https://www.bis.org/bcbs/publ/d424.pdf
[5]In paragraphs 320 to 323, without the application of the five year cap (par. 320)
https://www.bis.org/publ/bcbs107.pdf

The prescribed risk weights RW_c are given in the table below:

Sector of counterparty	Credit quality of counterparty	
	IG	HY and NR
Sovereigns including central banks, multilateral development banks	0.5%	3.0%
Local government, government-backed non-financials, education and public administration	1.0%	4.0%
Financials including government-backed financials	5.0%	12.0%
Basic materials, energy, industrials, agriculture, manufacturing, mining and quarrying	3.0%	7.0%
Consumer goods and services, transportation and storage, administrative and support service activities	3.0%	8.5%
Technology, telecommunications	2.0%	5.5%
Health care, utilities, professional and technical activities	1.5%	5.0%
Other sector	5.0%	12.0%

Figure 6.1: CVA Reduced Risk Weights

Full BA-CVA

When hedges are recognized in the computation of the CVA charge, the full version of the BA-CVA is followed.
First, Basel III defines eligible hedges as one of the following:

- Single-name CDS,
- Single-name contingent CDS[6],
- Index CDS.

The instruments listed above must refer directly to the counterparty or to its sector and region. They could also refer to an entity legally related to it.
The CVA capital charge using the full approach is computed using the following step-by-step approach:

- Calculate the hedging misalignment parameter per counterparty HMA_c:

$$HMA_c = \sum_{h \in c} (1 - r_{hc}^2) \cdot \left(RW_h \cdot M_h^{SN} \cdot B_h^{SN} \cdot DF_d^{SN} \right)^2 \tag{6.8}$$

  where

  - r_{hc} is the supervisory correlation between the credit spread of counterparty c and single-name hedge h of c,
  - M_h^{SN} is the remaining maturity of the single-name hedge h,
  - B_h^{SN} is the notional of the single-name hedge h[7],
  - $DF_h^{SN} = \frac{1-e^{-0.05 \cdot M_h^{SN}}}{0.05 \cdot M_h^{SN}}$ is the supervisory discount factor,
  - RW_h is the supervisory risk weight defined in figure (6.2).

- Calculate IH, the parameter recognizing the reduction in CVA risk as a result of the use of index hedges, as follows:

$$IH = \sum_i RW_i \cdot M_i^{ind} \cdot B_i^{ind} \cdot DF_i^{ind} \tag{6.9}$$

  where:

[6]It is a CDS where an additional triggering event is required, such as a noteworthy change in an overall measure of the economy.
[7]If it is a contingent CDS, the notional is determined by the current market value of the instrument

- M_i^{ind} is the remaining maturity of the index hedge i,
- B_i^{ind} is the notional of the index hedge i,
- $DF_i^{ind} = \frac{1-e^{-0.05 \cdot M_i^{ind}}}{0.05 \cdot M_i^{ind}}$ is the supervisory discount factor,
- RW_i is the supervisory risk weight of the index hedge i; and is defined as 70% of the risk weight prescribed in figure (6.2) and or 70% of the average of risk weights when it comes to multiple sectors.

- Compute SNH_c, the parameter recognizing the reduction in CVA risk as a result of the use of single-name hedges of credit spread risk, as follows:

$$SNH_c = \sum_{h \in c} r_{hc} \cdot RW_h \cdot M_h^{SN} \cdot B_h^{SN} \cdot DF_h^{SN} \tag{6.10}$$

where:

- r_{hc} is the prescribed correlation between the credit spread of the single-name hedge h of counterparty c and the credit spread of c (possible values for r_{hc} are prescribed in a table below),
- M_d^{SN} is the remaining maturity of hedge h,
- B_h^{SN} is the notional of the hedge h,
- $DF_d^{SN} - \frac{1-e^{-0.05 \cdot M_h^{SN}}}{0.05 \cdot M_h^{SN}}$ is the supervisory discount factor,
- RW_h is the supervisory risk weight of the single-name hedge h; possible risk weights are prescribed in figure (6.2).

- Compute the stand-alone CVA capital SVA_c as described in the reduced BA-CVA
- Infer the part of capital recognizing eligible hedges as follows:

$$K_{hedged} = \sqrt{\left(\rho \cdot \sum_c (SCVA_c - SNH_c) - IH\right)^2 + (1-\rho^2) \cdot \sum_c (SCVA_c - SNH_c)^2 + \sum_c HMA_c} \tag{6.11}$$

- Calculate $K_{reduced}$ as in the reduced BA-CVA,
- Conclude the full CVA capital charge as follows:

$$K_{full} = \beta \cdot K_{reduced} + (1-\beta) \cdot K_{hedged} \tag{6.12}$$

where $K_{reduced}$ is computed using the reduced BA-CVA mentioned previously and $\beta = 0.25$.

The possible values for the supervisory correlation parameter r_{hc} are laid out as follows:

Single-name hedge h of counterparty c	Value of r_{hc}
references counterparty c directly	100%
has legal relation with counterparty c	80%
shares sector and region with counterparty c	50%

Figure 6.2: CVA Prescribed Correlations r_{hc}

Chapter 7

Applied Capital Requirements for Market Risk

In January 2016, the Basel Committee on Banking Supervision (BCBS) published what is now known as the Fundamental Review of the Trading Book[1]. These standards address the material undercapitalisation of trading book exposures during the financial crisis of 2007-2008.
The Basel committee has introduced a strong boundary between the banking book and the trading book[2]; it is notable then that trading book positions are subject to mark to market and so attract higher market risk capital charges.
Under the standardized approach, the market risk capital charge is equal to the sum of the three following risk charges:

- sensitivities-based method risk charge,
- default risk charge,
- residual risk add-on.

Each of these components will be explained in detail throughout the following sections.

7.1 Standardized Approach: Sensitivities-Based Method

7.1.1 Full Sensitivities-Based Method (SA-SbM)

SA-SbM: Main definitions, structure and correlation risk

Main Definitions
The main concepts of the standardized approach are covered by the following definitions:

- *Risk class:*
  - General interest rate risk (GIRR)
  - Credit spread risk (securitisations in a non-correlation trading portfolio)
  - Credit spread risk (securitisations in a correlation trading portfolio)
  - Equity risk
  - Commodity risk
  - Foreign exchange risk
- *Risk factor:* variables within a pricing function decomposed from trading book instruments

[1] formerly known as the Standards for Minimum Capital Requirements for Market Risk and found here: https://www.bis.org/publ/bcbs265.pdf

[2] Positions marked as being held to maturity are defined for regulatory purposes to be in the banking book while positions that are tradable are defined to be in the trading book.

- *Risk position:* the main input entering the risk charge computation. For delta and vega risks, it's the sensitivity to a certain risk factor. For curvature risk, it's the worst loss of two stress scenarios. Delta, vega, and curvature risks will be defined in the following paragraph.
- *Risk charge:* the amount of capital that a bank should hold as a consequence of the risks it takes. It's computed as the aggregation of risk positions at the bucket level and then at across buckets within a risk class: $\sum_{risk\ class}(\sum_{bucket} risk\ positions)$.
- *Bucket:* a set of risk positions grouped together by common characteristics.

Structure of the approach

The risk charge under the sensitivities-based method is the sum of the three following risk measures:

- *Delta:* a risk measure based on sensitivities of a bank's trading book to regulatory delta risk factors.
- *Vega:* a risk measure also based on sensitivities to regulatory vega risk factors; used the same way as the delta risk measure as an input in the aggregation formula for the capital requirement.
- *Curvature:* a risk measure which captures incremental risk not captured by delta risk of price changes **in the value of an option**. The eventual risk position is determined as the **worst loss of two stress scenarios** involving an upward shock and a downward shock to a given risk factor.

Correlation risk

The Basel committee realized that the risk that correlations may increase or decrease in periods of financial stress and induce much bigger losses. That's why **three** risk charge figures must be calculated for each risk class based on three different scenarios on specified values for the correlation parameters ρ_{kl}[3] and γ_{bc}[4].

Instruments subject to delta, vega and curvature risk

- Instruments with optionality[5] and instruments with embedded options are subject to vega and curvature risks.
- Instruments whose cash flows can be written as a **linear function** of underlying notional are without optionality. That's why they are not subject to vega risk nor curvature risk.

Methodology

Delta and vega risks are captured using the same aggregation formula through the following approach:

- Find a net sensitivity s_k across instruments to each risk factor k
- Find the weighted sensitivity using a defined risk weight RW_k:

$$WS_k = RW_k s_k$$

- Find the risk position for Delta and Vega within a bucket b

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{k \neq l} \rho_{kl} WS_k WS_l}$$

where the quantity within the square root is floored at zero.

[3] correlation between risk factors within a bucket

[4] correlation across buckets within a risk class

[5] such as calls, puts, caps, floors, swaptions, barrier options, exotic options, etc.

Variable	Meaning	
i	instrument subject to curvature risk	
x_k	current level of risk factor k	
$V_i(x_k^{(RW^{curvature})+})$, $V_i(x_k^{(RW^{curvature})-})$	price of instrument i after upward and downward shock	
	FX and Equity risk classes	**GIRR, CSR, Commodity risk classes**
$RW_k^{curvature}$	Relative shocks equal to delta risk weights	Parallel shift of all vertices for each curve based on the highest delta risk weight per risk class
s_{ik}	Delta sensitivity of instrument i per risk factor	Sum of delta sensitivities to all tenors of the relevant curve of i per risk factor k

Table 7.1: Equation (7.1) variables

- Find the Delta and Vega risk charges:

$$Delta\ (resp.\ Vega) = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c}$$

where $S_b = \sum_k WS_k$ in bucket b. However, if $\sum_k K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c < 0$, then $S_b = \max\left[\min\left(\sum_k WS_k, K_b\right), -K_b\right]$ in bucket b.

As mentioned before, in order to compute the curvature risk charge, we must perform a set of stress scenarios on given risk factors. Here is a step-by-step approach to compute the curvature risk charge:

- For each risk factor k, two stress scenarios[6] are performed; they will generate two values for possible losses across the instruments. The worst possible loss between the two is detected and the delta risk position is deduced to obtain the curvature risk position for k.
- The curvature risk charge CVR_k for the risk factor k is then computed:

$$CVR_k = -\min \begin{bmatrix} \sum_i \left\{ V_i\left(x_k^{(RW^{(curvature)+})}\right) - V_i(x_k) - RW_k^{curvature} \cdot s_{ik} \right\} \\ \sum_i \left\{ V_i\left(x_k^{(RW^{(curvature)-})}\right) - V_i(x_k) + RW_k^{curvature} \cdot s_{ik} \right\} \end{bmatrix} \tag{7.1}$$

  The variables for this equation are explained in table (7.1.1).

- Negative curvature risk exposures are ignored unless they hedge a positive curvature risk exposure.
- The curvature risk exposure is aggregated within buckets using the ρ_{kl} correlation in the following way:

$$K_b = \sqrt{\max\left(0, \sum_k \max(CVR_k, 0)^2 + \sum_k \sum_{k \neq l} \rho_{kl} CVR_k CVR_l \psi\left(CVR_k, CVR_l\right)\right)} \tag{7.2}$$

where $\psi(x,y) = \begin{cases} 0 & if\ x < 0\ and\ y < 0 \\ 1 & if\ else \end{cases}$

[6]For example, for a given currency, all vertices must be shifted upward and downward.

- The curvature risk positions are then aggregated across buckets within each risk class using the γ_{bc} correlation:

$$Curvature\ risk = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)} \tag{7.3}$$

  where $S_b = \sum_k CVR_k$ in bucket b and ψ has already been specified.

- If, however, S_b and S_c cause $\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)$ to be negative, the curvature risk charge will be calculated using an alternative definition for S_b; $S_b = \max\left[\min\left(\sum_k CVR_k, K_b\right), -K_b\right]$ in bucket b.

7.1.2 Simplified Alternative to the Standardized Approach

In June 2017, the Basel committee released a document proposing a more simplified alternative approach to the market risk capital charge computation. It could either be a reduced sensitivities-based method (R-SbM) or a recalibrated version of the Basel II standardized approach.

Reduced Sensitivity-based Method (R-SbM)

The proposed simplified approach has been created for smaller banks with usually not enough infrastructure for the SA-SbM. Banks who can follow the R-SbM procedure must have specifications set by the Basel committee[7].
It is worth noting that banks which follow the criteria mentioned above but have fairly complex risks in specific risk classes must apply the full SbM method and not the reduced one.

In the R-SbM, **vega and curvature risk charges have been dropped**, and only a delta risk charge will be computed.
The risk weight and bucket definitions have been changed as well. However, the definitions of the sensitivities remain the same.
To capture delta risk, the following steps must be followed:

- Find a net sensitivity s_k to each risk factor k.

- Compute the weighted sensitivity WS_k using prescribed risk weights RW_k and the sensitivity s_k:

$$WS_k = RW_k s_k$$

- Aggregate the weighted sensitivities using correlations ρ_{kl} across risk factors:

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_k \sum_{k \neq l} \rho_{kl} WS_k WS_l}$$

  where the quantity within the square root function is floored at zero.

- Aggregate risk positions across buckets within risk class using correlations γ_{bc}:

$$Delta = \sqrt{\left| \sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \right|}$$

  where $S_b = \sum_k WS_k$ for all risk factors in bucket b.

[7] https://www.bis.org/bcbs/publ/d436.pdf

Recalibrated Basel II Standardized Approach

The Basel Committee also suggested the use of the Basel II capital requirements but recalibrated them using new scaling factors in order not to have history repeat itself.
The four risk classes considered are interest rate risk, equity risk, foreign exchange risk, and commodity risk and the capital requirement is calculated as in the following formula:

$$Capital\ requirement = CR_{IRR} \times SF_{IRR} + CR_{EQ} \times SF_{EQ} + CR_{FX} \times SF_{FX} + CR_{COMM} \times SF_{COMM} \tag{7.4}$$

where CR stands for Capital Requirement[8] and SF stands for Scaling Factor.
The scaling factors are specified as:

- SF_{IRR} = Scaling factor of [1.50-2.00];
- SF_{EQ} = Scaling factor of [3.00-3.50];
- SF_{COMM} = Scaling factor of [1.50-2.50];
- SF_{FX} = Scaling factor of [1.25-1.50].

7.2 The Default Risk Charge (DRC)

In order to cover jump-to-default risk, we must compute the default risk charge by following this step-by-step approach:

- Compute the jump-to-default risk of each instrument separately[9];
- Derive the "net jump-to-default" (net JTD) risk positions;
- Allocate net JTD risk positions to buckets and weigh them by prescribed risk weights.

7.3 Residual Risk Add-on (RRAO)

RRAO covers what is not covered by the risk charges mentioned above. It's calculated separately and added to other components of the capital requirement.
It's computed by adding together gross notional amounts of the instruments bearing residual risk, and multiplying that sum by a 1.0% risk weight for instruments with an exotic underlying and a 0.1% risk weight for instruments bearing other residual risks.

Instruments containing exotic underlyings are classified trading book instruments with an underlying exposure not covered by the delta, vega, or curvature risk treatment and the default risk charge.
Instruments bearing **other** residual risks include CTP instruments and instruments subject to vega or curvature charges but with payoffs that cannot be written as a finite linear combination of vanilla options with a single underlying equity[10] price.

Other residual risk types include but are not limited to: gap risk, correlation risk, and behavioral risk. These are futher defined in the Basel III document specifying market risk capital requirements[11].

[8] The Basel II Capital Requirements are specified in Basel II, Section VI (C), each one under its corresponding subsection.
https://www.bis.org/publ/bcbs128.pdf

[9] It's a function of notional amount and market value of the instrument and prescibed LGD

[10] or commodity, exchange rate, bond price, CDS price, or interest rate swap

[11] https://www.bis.org/bcbs/publ/d352.htm

Part III

Capital Charge Calculation and Stress Test

Chapter 8

Chosen portfolios

To assess the sensitivity of the capital charges described in chapters (6) and (7), the first step is constructing two portfolios of derivatives and calculating their capital charges.
The first portfolio is a back-to-back portfolio (also known as a sales portfolio in banks) in which each position taken is backed by another opposing position with another counterpart. It contains the following derivatives:

- A long and short call with a USDTRY underlying and a USD 1 Mn notional and a maturity of half a year each,
- A long and short put with a USDEUR underlying and a USD 50 Mn notional and a maturity of 1.25 years each,
- Two swaps with opposing positions on USD with a USD 20 Mn notional and a maturity of 5 years each,
- Two cross-currency swaps with opposing positions on USDTRY with a USD 10 Mn notional (or the equivalent in the Turkish currency which is TRY 53,254,000 while considering an FX rate of about 0.19) and a maturity of 5 years each.

The second portfolio chosen is a trading portfolio. It contains the following derivatives:

- A long call with a USDEUR underlying and a EUR 1 Mn notional and a 0.87 year maturity,
- A short put with a USDTRY underlying and a USD 1 Mn notional and a 1.25 year maturity,
- A receiving-fixed swap in USD with a USD 20 Mn notional and a 6 year maturity,
- A EUR-USD cross-currency swap (receiving fixed in EUR) with a USD 20 Mn notional and a 5.3 year maturity.

The valuation of these instruments is performed on May 13th, 2019. Since the goal is to stress test based on sensitivities and **not** redo the entire capital charge calculation, the valuation from May 13th is the only one relied upon.

Chapter 9

Capital Charges per Portfolio under Basel III

9.1 Counterparty Risk Capital Charge

As mentioned before, Basel III divides counterparty risk into two sub-categories: Counterparty Credit Risk (which we account for using the SA-CCR approach) and CVA, which we account for using the reduced BA-CVA approach. Another more advanced approach, the full BA-CVA, is available for use by most banks but the reduced version is applied here because the full version adds a CVA capital accounting for hedges between instruments per counterpart which is not applied here.
In the SA-CCR, every instrument is assigned a risk class. Risk classes in our portfolio are either FX or IR. The SA-CCR focuses on the risk class per instrument instead of the counterpart itself.
However, the CVA procedure focuses on the nature of the counterpart and its rating as well as the instrument risk class.

9.1.1 Counterparty Credit Risk: SA-CCR

To compute the SA-CCR according to the procedure mentioned in chapter (6.1), we set the following assumptions:

- There is no posted collateral for any of the derivatives;
- The options' volatilities are set at 15% according to Basel III regulations.

Based on the procedure described in section (6.1), we obtain the following capital charges per portfolio.

Portfolio	**CCR Capital (USD)**
Back-to-back	7,208,948
Trading	2,139,632

Table 9.1: Initial CCR Capital Charge per Portfolio

The capital charge for the trading portfolio is significantly lower because we have half as much derivatives as in the first portfolio, and more than triple the amount in notional in the first portfolio than in the second one. However, the numbers represent respectively 4.45% and 4.77% of total notional. So in proportion, the capital charges are alike per portfolio.

9.1.2 Credit Valuation Adjustment: BA-CVA

The following assumptions are taken into consideration when computing the CVA capital charge:

- The first, third, fifth and seventh positions in the back-to-back portfolio are held with central banks with a CVA risk weight of 0.5% and their opposing positions are held with clients with a CVA risk weight of 0.3% (they could be in the basic materials field or consumer goods and services);
- In the trading portfolio, the USDEUR option and the swap on USD are held with central banks with a CVA risk weight of 0.5% and the two other positions (USDTRY option and EUR-USD cross-currency swap) are held with counterparts having a 3.0% CVA risk weight.

The obtained capital charges for both portfolios are given below:

Portfolio	**CCR Capital (USD)**
Back-to-back	217,042
Trading	127,945

Table 9.2: Initial CVA Capital Charge per Portfolio

Both the CCR and the CVA charges are affected by fluctuations in the market through the replacement cost, which changes directly according to the mark-to-market per instrument.

9.2 Market Risk Capital Charge

As stated in chapter (7), the market risk capital charge under Basel III is the sum of the default risk charge (DRC) and the three metrics delta, vega and curvature. There is also a charge for residual risk add-on (RRAO) that accounts for residual risks present in complex derivatives with exotic underlyings, but such derivatives do not exist in the portfolios studied.
The risk classes assigned to the instruments are the same as the CCR.
For the computation of the default risk charge (DRC), each counterpart is assumed to be unrated and thus having a jump-to-default risk weight of 15%.
Using the procedure described in chapter (7), we obtain the following capital requirements for market risk per portfolio:

Portfolio	**Delta + Vega + Curvature (USD)**	**DRC (USD)**	**Total (USD)**
Back-to-back	3,035,422	12,223,768	15,259,190
Trading	688,486	140,544	829,030

Table 9.3: Initial Market Risk Capital Charge per Portfolio

The delta, vega and curvature metrics rely on the initial sensitivity of the instruments to changes in the market. As seen in the procedure detailed in Appendix F on page 84, only the default risk charge is affected by a change in P&L.

Chapter 10

Applied Shocks

The instruments chosen in this study are sensitive to many risk factors in the market, whether they were FX rates, interest rates, or even implied volatilities.

Option Risk Factors
The options' sensitivity is based on the options mapping technique described in part (I). A delta-gamma-vega-rho map is performed, meaning that shocks are applied to the spot (FX rate), the volatility (implied option volatility) and the interest rate.
The table below shows the risk factors of each option:

Portfolio	Option	Remaining maturity	Spot	Volatility	IR
BtB	1	1M	USDTRY	1M ATM implied volatility	TRY 1M
	2	1M	USDTRY	1M ATM implied volatility	TRY 1M
	3	4.5M	USDEUR	4.5M ATM implied volatility	EUR 4.5M
	4	4.5M	USDEUR	4.5M ATM implied volatility	EUR 4.5M
Trading	1	1M	USDEUR	1M ATM implied volatility	EUR 1M
	2	6M	USDTRY	6M ATM implied volatility	TRY 6M

Table 10.1: Option Risk Factors

Swap Risk Factors
Swaps are sensitive to fluctuations in interest rates. Since the purpose here is to check for a change in market value and not the calculation of the VaR or any other metric, the cash flow mapping technique described in part I will not be applied here. The change in market value of each swap will be simply the product of its DV01 and the shock applied.
The table below presents the risk factors to which are exposed the swaps in the two portfolios examined.

Portfolio	Swap	Swap type	Fixed risk factor	Floating risk factor	IR swap risk factor
BtB	1	IR			USD 3M
	2	IR			USD 3M
	3	CCS	USD 3M	TRY 3M	
	4	CCS	USD 3M	TRY 3M	
Trading	1	IR			USD 3M
	2	CCS	EUR 3M	USD 3M	

Table 10.2: Swap Risk Factors

Chosen shocks
The stress test will be performed over three scenarios: mild, severe and severely adverse.
The shocks for the severe scenario are chosen based on the 10-day Expected Shortfall (at a 97.5% probability) of the 5-year historical changes in each risk factor.
The shocks for the mild and severely adverse scenarios are chosen based on expert judgment.
The table below presents the chosen shocks per risk factor:

Risk factor type	Risk factor	Mild shock	Severe shock	Severely adverse shock
FX	USDTRY	6%	12.73%	30%
	USDEUR	2%	4.51%	10%
IR	USD 3M	-0.25%	0.38%	0.625%
	TRY 1M	1.5%	2.80%	5%
	TRY 3M	1.7%	2.70%	5%
	TRY 6M	1.7%	2.51%	5%
	EUR 1M	-0.25%	0.008%	0.25%
	EUR 3M	-0.25%	0.0058%	0.25%
	EUR 4.5M	-0.25%	0.0064%	0.25%
Volatility	USDTRY 1M	5%	15.31%	30%
	USDTRY 6M	5%	8.27%	20%
	USDEUR 1M	2%	3.56%	10%
	USDEUR 4.5M	0.5%	1.81%	5%

Table 10.3: Shocks for the three scenarios per risk factor

The stress level of the severe scenario has been chosen based on five-year historical data showing mostly positive movement in interest rates. However, the most recent variations in American and European markets have a tendency to be negative.

What could cause such fluctuations and place us in a mild, severe or severely adverse scenario?

Under the spotlight today are the tensions caused by the US-China trade war.
If the tensions persist but do not take an extreme turn, as is happening right now, investors tend to move their investments from the economy to the swap markets (but keeping them in the American markets). That would result in lower interest rates and place us in a mild scenario. The Fed could cut rates as a reaction to Trump's trade dispute with China, leading to a global economic recession (cf. Lee, Y.N. & Roy Choudhury, S. (2019)).
However, if tensions ease between the two economies, there would be more activity in the economy which causes higher interest rates, but in a positive way. This has already been observed throughout the years 2005-2007 right before the financial crisis of 2008 when interest rates increased and there was a lot of circulation in the economy.
One unwanted scenario is where tensions rise between the US and China and take an extreme turn (China could impose more severe conditions on the US for example). This will cause investors to withdraw their money completely from the American markets and place them in European markets (such as in Switzerland, for example) thus causing an unexpected increase in rates in the US.

Chapter 11

Effect on the Capital Charge and the Capital Adequacy Ratio

After having applied the shocks mentioned in chapter 10, the new capital charges have been computed and put in the summary tables below:

	Baseline	Mild	Severe	Severely Adverse
Market Risk	3,035,422	3,035,422	3,035,422	3,035,422
DRC	12,223,768	12,209,358	12,188,371	12,148,109
Total	15,259,190	15,244,779	15,223,792	15,183,531
CCR	7,208,948	7,207,140	7,411,910	8,166,700
CVA	217,042	220,701	225,538	238,309

Figure 11.1: Back-to-Back portfolio new capital charges in three scenarios

	Baseline	Mild	Severe	Severely Adverse
Market Risk	688,486	688,486	688,486	688,486
DRC	140,544	143,109	146,328	153,372
Total	829,030	831,595	834,814	841,858
CCR	2,139,632	2,178,895	2,247,851	2,387,180
CVA	127,945	128,119	129,200	131,182

Figure 11.2: Trading portfolio new capital charges in three scenarios

The changes of the capital charge in response to the shocks are visualized in the following charts:

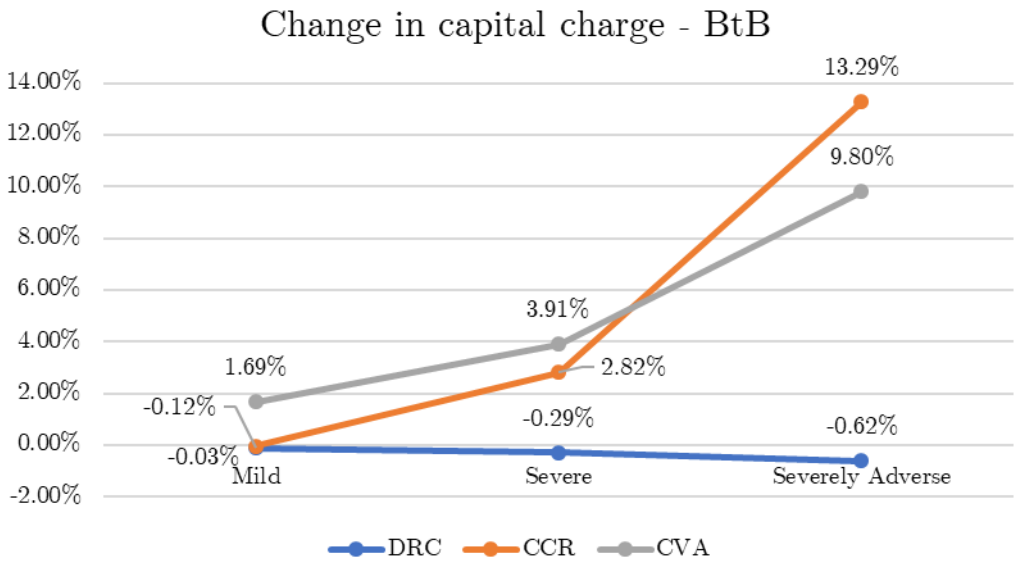

Figure 11.3: Back to back portfolio changes in capital charge - all scenarios

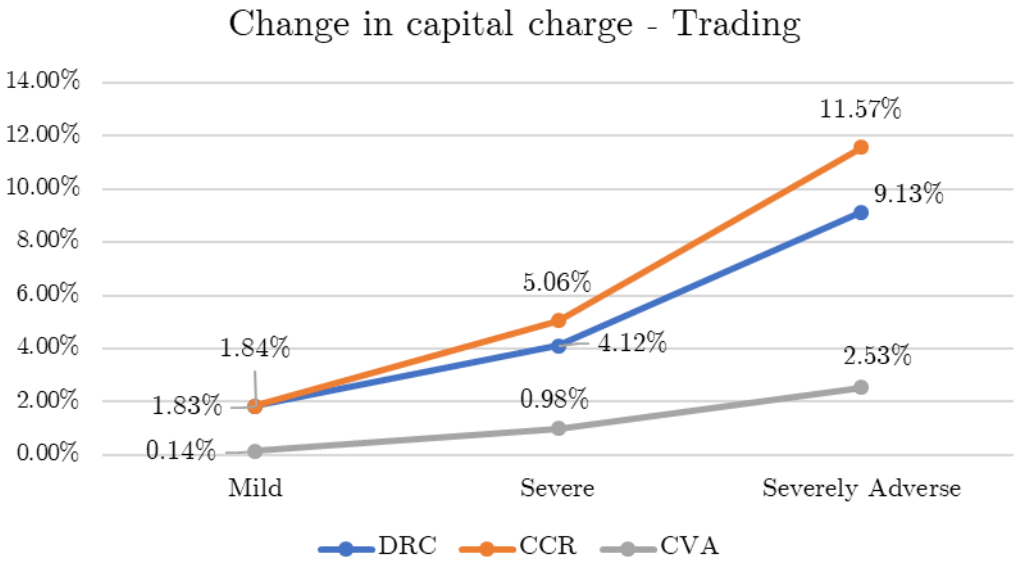

Figure 11.4: Trading portfolio changes in capital charge - all scenarios

It is intuitive that the capital charges all increase with fluctuations in the market, as is happening with the CCR in both portfolios and both capital charges in the trading portfolio.
However, the default risk charge slightly decreases in the back-to-back portfolio. That is explained by the fact that the DRC acknowledges positive long positions and negative short positions (see Appendix F on page 84). In the back-to-back portfolio, the shocks caused a negative change in market value for the long positions and a positive change in market value for short positions, thus decreasing the JTD per position. This causes the overall DRC for the back-to-back-portfolio to decrease slightly.
In the trading portfolio, there is only one long position which increased in market value, thus increasing the overall DRC per portfolio with every shock.

A closer look at the severely adverse scenario shows the following changes to the required capital:

Changes in required capital in the severely adverse scenario

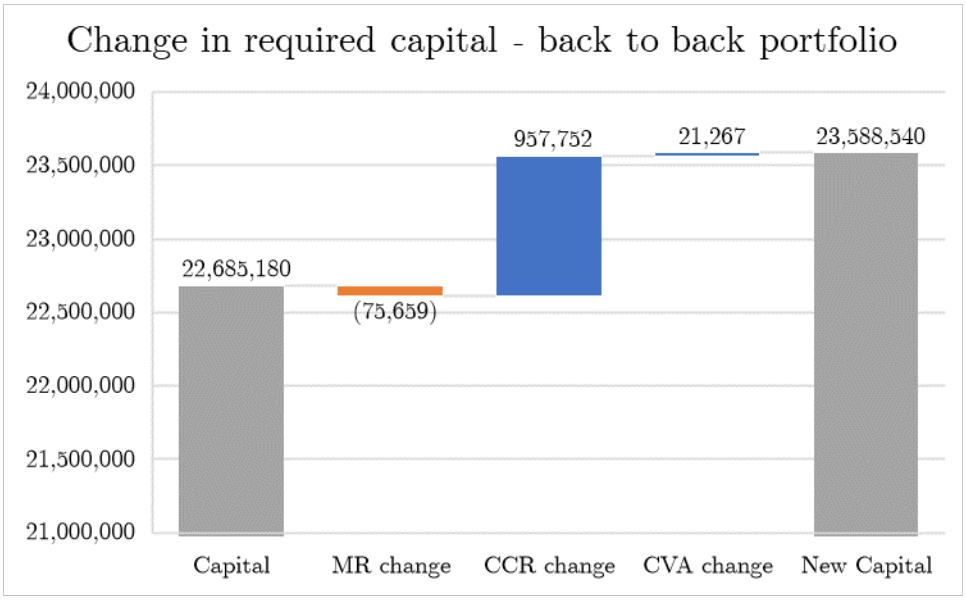

Figure 11.5: Change in required capital in the severely adverse scenario - Back to back portfolio

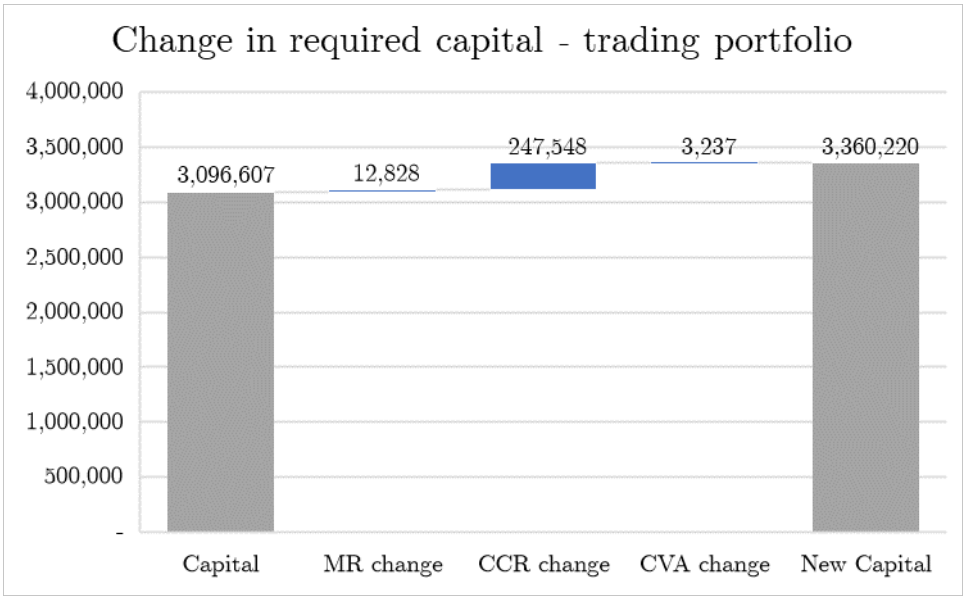

Figure 11.6: Change in required capital in the severely adverse scenario - Trading portfolio

Changes in the capital adequacy ratio in the severely adverse scenario

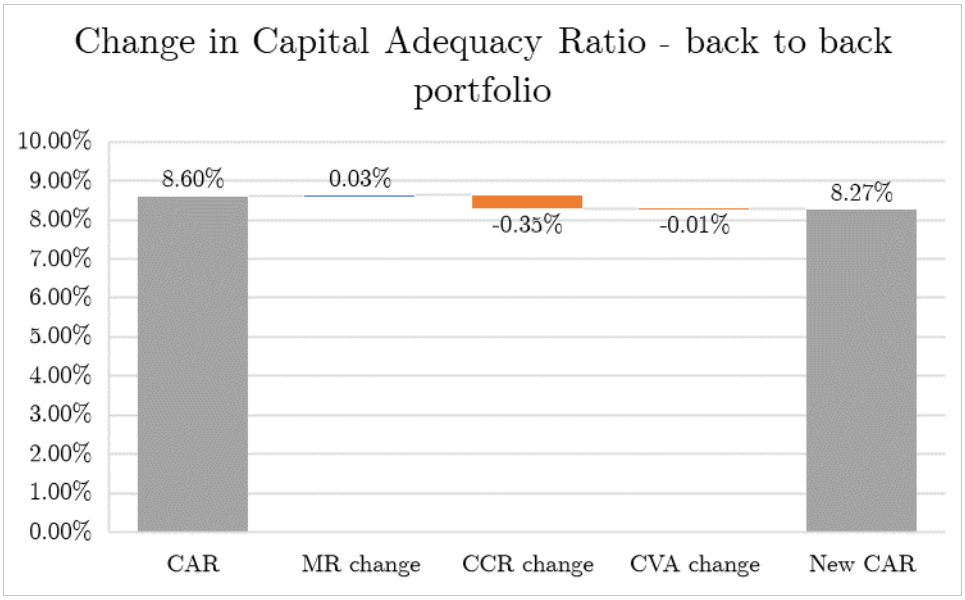

Figure 11.7: Change in the capital adequacy ratio in the severely adverse scenario - Back to back portfolio

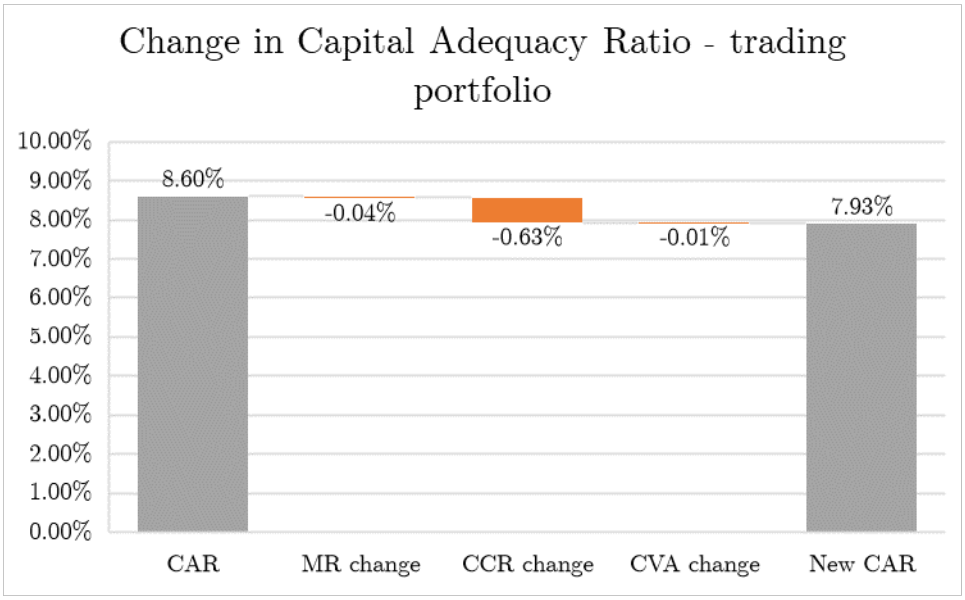

Figure 11.8: Change in the capital adequacy ratio in the severely adverse scenario - Trading portfolio

It is obvious the shocks in the severely adverse scenario disturb the capital adequacy ratio by increasing required capital under Basel III rules.
The CAR goes down from 8.6% to 8.27% in the back-to-back portfolio and down to 7.93% in the trading portfolio.

Chapter 12

Conclusion

The drop in the CAR for the trading portfolio is more severe than the drop in the back-to-back portfolio, which is a result consistent with intuition. Each position in the sales portfolio by another one that is the exact opposite, thus avoiding (although not completely) the risk that comes with it. Even though the new reforms to Basel rules are highly conservative and include counterparty risk in a lot of components (even market risk through the default risk charge), it is still sensitive to fluctuations in the market, even in the case of a back-to-back portfolio.
This highlights the importance of **accounting for counterparty risk** while avoiding a crisis.

In real life, financial institutions and banks hold sales and trading portfolios that are much larger; they could contain hundreds or even thousands of instruments much more complex and exotic than the ones examined in this study. This means that the reevaluation of the portfolio takes up time and resources, not to mention the recalculation of the required capital every time and at every scenario. The availability of **sensitivities** of instruments to fluctuations in the market (such as the Greeks and the DV01 metric) opens the door for the use of the cash flow mapping technique as well as the options map through Taylor approximations. These methods allow us to compute a change in market value at every desired scenario almost instantly without having to recalculate the required capital, thus saving time and effort.

The following flowchart shows how to reach the initial capital adequacy ratio:

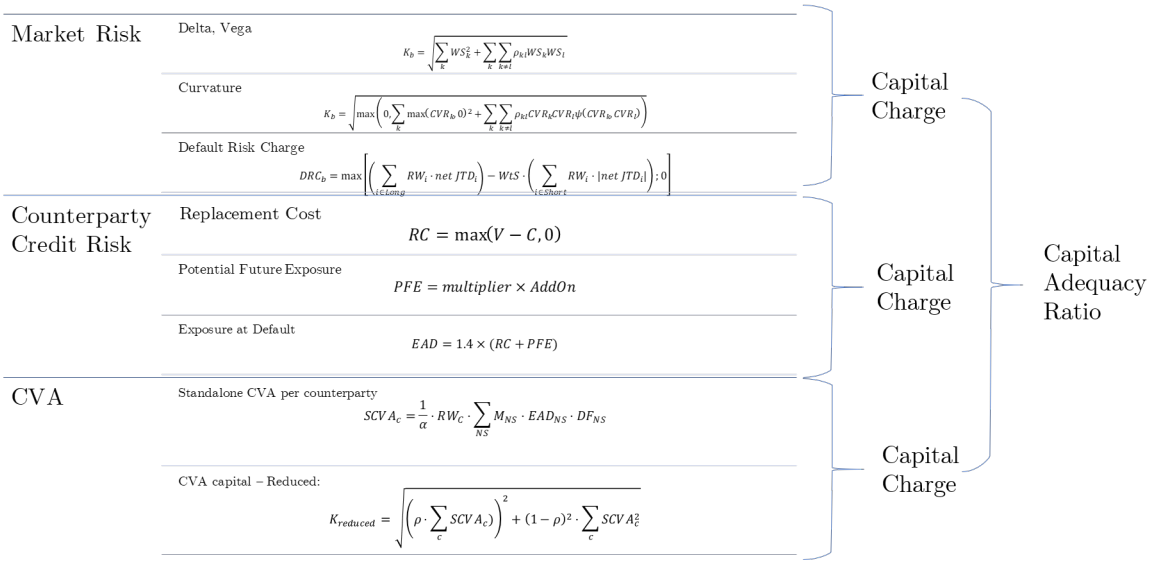

Figure 12.1: Reaching the initial capital adequacy ratio

In the following flowchart, the parameters highlighted in red are the ones affected by the change in market value after the shock:

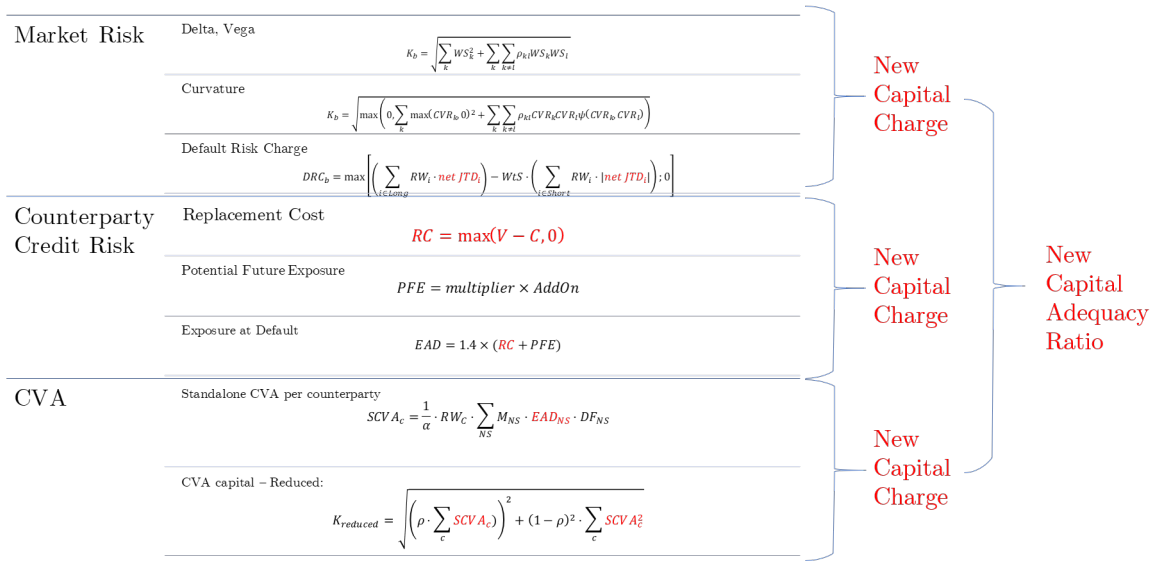

Figure 12.2: Reaching the shocked capital adequacy ratio

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Appendices

Appendix A: Full SbM - Risk factor definitions

	Delta	
	Risk factor	**Notes**
GIRR	Risk free yield curve per currency	Constructed using trading book instruments with lowest credit risk or market-implied offered swap curves such as LIBOR[1]
	Flat curve of market-implied inflation rates	Risk factor only relevant when a cash flow is functionally dependent on a measure of inflation Considered in addition to the sensitivity to interest rates
	Currency basis risk factor	Basis of each currency over USD Basis of each currency over EUR Considered in addition to the sensitivity to interest rates
Credit Spread Risk (CSR) - non securitisation	Relevant issuer credit spread curves	Along vertices: 0.5, 1, 3, 5, 10 years
CSR securitisation: non-CTP	Tranche Credit spread curves	 Along vertices: 0.5, 1, 3, 5, 10 years
CSR securitisation: CTP	Relevant underlying credit spread curves	Along vertices: 0.5, 1, 3, 5, 10 years
Equity	All equity spot prices and all equity repo rates	
Commodity	All commodity spot prices depending on contract grade and legal terms and time to maturity	Along vertices: 0, 0.25, 0.5, 1, 2, 3, 5, 10, 15, 20, 30 years
Foreign Exchange (FX)	All the exchange rates between the currency in which an instrument is denominated and the reporting currency	

Table 12.1: Delta risk factors

[1]If data is insufficient, the yield curve is derived from the most appropriate sovereign bond curve.

	Vega	
GIRR	**Risk factor** Implied volatilities of options that reference GIRR-sensitive underlyings, defined under two dimensions:	**Notes** Maturity of the option Residual of the underlying of the option at the expiry date of the option
Credit Spread Risk (CSR) - non securitisation	Implied volatilities of options that reference the credit issuer names as underlyings	Implied volatility of the option as mapped to maturity vertices specified in delta risk factor
CSR securitisation: non-CTP	Implied volatilities of options that reference non-CTP credit spreads as underlyings	Implied volatility of the option as mapped to maturity vertices specified in delta risk factor
CSR securitisation: CTP	Implied volatilities of options that reference CTP credit spreads as underlyings	Implied volatility of the option as mapped to maturity vertices specified in delta risk factor
Equity	Implied volatilities of options that reference equity spot rates as underlyings	No vega risk capital charge for equity repo rates
Commodity	Implied volatilities of options that reference commodity spot prices as underlyings	Defined along the implied volatility of the option as mapped to vertices: 0.5, 1, 3, 5, 10 years
Foreign Exchange (FX)	Implied volatilities of options that reference exchange rates between currency pairs	Implied volatility of the option as mapped to one or several of vertices: 0.5, 1, 3, 5, 10 years

Table 12.2: Vega risk factors

	Curvature	
	Risk factor	**Notes**
GIRR	Risk-free yield curve per currency	All vertices are to be shifted in parallel There is no curvature risk charge for inflation and cross currency basis risks
Credit Spread Risk (CSR) - non securitisation	Relevant issuer credit spread curves	All vertices are to be shifted in parallel
CSR securitisation: non-CTP	Relevant tranche credit spread curves	All vertices are to be shifted in parallel
CSR securitisation: CTP	Relevant underlying credit spread curves	All vertices are to be shifted in parallel
Equity	All equity spot prices	No curvature risk charge for equity repo rates
Commodity	Constructed curve per commodity spot price	All vertices are to be shifted in parallel
Foreign Exchange (FX)	All the exchange rates between the currency in which an instrument is denominated and the reporting currency	

Table 12.3: Curvature risk factors

Appendix B: Full SbM - Sensitivities definitions

Risk class	Delta Sensitivity
GIRR	$s_{k,r_t} = \frac{V_i(r_t+0.0001,cs_t)-V_i(r_t,cs_t)}{0.0001}$
CSR non-securitisation	$s_{k,cs_t} = \frac{V_i(r_t,cs_t+0.0001)-V_i(r_t,cs_t)}{0.0001}$
CSR securitisation	Same as Delta CSR non-securitisation
Equity spot	$s_k = \frac{V_i(1.01EQ_k)-V_i(EQ_k)}{0.01}$
Equity repos	$s_k = \frac{V_i(RTS_k+0.0001)-V_i(RTS_k)}{0.0001}$
Commodity	$s_k = \frac{V_i(1.01CTY_k)-V_i(CTY_k)}{0.01}$
FX	$s_k = \frac{V_i(1.01FX_k)-V_i(FX_k)}{0.01}$

Table 12.4: SbM Delta Sensitivities Formulas

Option level	Portfolio level
Product of the option's vega and its implied volatility	Simple sum of option-level vega sensitivities to each risk factor

Table 12.5: SbM Vega Sensitivities Formulas

Appendix C: Full SbM - Delta Risk Weights and Correlations

Delta GIRR

Buckets

Each individual currency exposure to GIRR is represented by a bucket.

Vertex (year)	Risk Weight
0.25	2.4%
0.5	2.4%
1	2.25%
2	1.88%
3	1.73%
5	1.5%
10	1.5%
15	1.5%
20	1.5%
30	1.5%

Table 12.6: Delta GIRR Risk Weights

Risk Weights

Risk weights are specified in the table above.

For the inflation and cross currency basis risk factors, a risk weight of 2.25% is set. Risk weights for selected currencies by the Basel committee[2] are divided by the square root of 2 at the discretion of the bank.

Correlations

The correlations for Delta GIRR are prescribed in table (12.7) below.

Same bucket	Same vertex	Same curve	100%
		Different curve	99.90%
	Different vertex	Same curve	$\max\left[e^{\left(-\theta\frac{\lvert T_k-T_l\rvert}{\min(T_k;T_l)}\right)};40\%\right]^3$
		Different curve	Above parameter $\times$ 99.90%

Table 12.7: Delta GIRR Correlations

[2]EUR, USD, GBP, AUD, JPY, SEK, CAD, and the domestic currency

where T_k is the vertex related to WS_k, and θ is set at 3%.
In addition to the correlations specified in table (12.7), we must note the following prescribed correlations:

- ρ_{kl} between WS_k to the inflation curve and a sensitivity WS_l to a vertex of the relevant yield curve is 40%;
- There is no correlation between a sensitivity WS_k to a cross currency basis curve and a sensitivity WS_l to a given vertex of the relevant yield curve or another cross currency basis curve;
- To aggregate between different currencies, $\gamma_{bc} = 50\%$.

[3] T_k is the vertex related to WS_k, and θ is set at 3%.

Delta CSR Non-Securitisations

Buckets

The prescribed buckets for delta CSR (non securitisations) are laid out in the following manner:

82. Sensitivities or risk exposures should first be assigned to a bucket according to the following table:

Bucket number	Credit quality	Sector
1	Investment grade (IG)	Sovereigns including central banks, multilateral development banks
2		Local government, government-backed non-financials, education, public administration
3		Financials including government-backed financials
4		Basic materials, energy, industrials, agriculture, manufacturing, mining and quarrying
5		Consumer goods and services, transportation and storage, administrative and support service activities
6		Technology, telecommunications
7		Health care, utilities, professional and technical activities
8		Covered bonds[27]
9	High yield (HY) & non-rated (NR)	Sovereigns including central banks, multilateral development banks
10		Local government, government-backed non-financials, education, public administration
11		Financials including government-backed financials
12		Basic materials, energy, industrials, agriculture, manufacturing, mining and quarrying
13		Consumer goods and services, transportation and storage, administrative and support service activities
14		Technology, telecommunications
15		Health care, utilities, professional and technical activities
16	Other sector[28]	

Figure 12.3: Delta CSR non-securitisation buckets

Risk Weights

The prescribed risk weights for delta CSR (non-securitisations) are laid out in the following manner:

Correlations

Within the same bucket, the correlation between two sensitivities WS_k and WS_l is expressed as:

$$\rho_{kl} = \rho_{kl}^{(name)} \cdot \rho_{kl}^{(tenor)} \cdot \rho_{kl}^{(basis)} \tag{12.1}$$

where:

- $\rho_{kl}^{(name)} = \begin{cases} 1 & if\ sensitivities\ k\ and\ l\ have\ identical\ names \\ 35\% & if\ else \end{cases}$
- $\rho_{kl}^{(tenor)} = \begin{cases} 1 & if\ sensitivities\ k\ and\ l\ have\ identical\ vertices \\ 65\% & if\ else \end{cases}$
- $\rho_{kl}^{(basis)} = \begin{cases} 1 & if\ sensitivities\ have\ the\ same\ curve \\ 99.90\% & if\ else \end{cases}$

This definition of ρ_{kl} should not be applied in the curvature risk context, and should not be applied to the 'other sector' bucket.

In fact, the capital requirement for the 'other sector' bucket is calculated separately per risk class and added to the overall risk class capital, simply by aggregating the absolute values of net weighted sensitivities, like so:

$$K_{b(other\ bucket)} = \sum_k |WS_k| \tag{12.2}$$

Bucket	Risk Weight
1	0.5%
2	1.0%
3	5.0%
4	3.0%
5	3.0%
6	2.0%
7	1.5%
8	4.0%
9	3.0%
10	4.0%
11	12.0%
12	7.0%
13	8.5%
14	5.5%
15	5.0%
16	12.0%

Table 12.8: Delta CSR non securitisation Risk Weights

Across buckets, the correlation parameter γ_{bc} is calculated as follows:

$$\gamma_{bc} = \gamma_{bc}^{(rating)} \cdot \gamma_{bc}^{(sector)} \tag{12.3}$$

where:

- $\gamma_{bc}^{(rating)} = \begin{cases} 1 & if\ buckets\ b\ and\ c\ have\ the\ same\ rating\ category^4 \\ 50\% & if\ else \end{cases}$
- $\gamma_{bc}^{(sector)} = 1$ if buckets b and c have the same sector. If else, the following numbers:

Bucket	1 / 9	2 / 10	3 / 11	4 / 12	5 / 13	6 / 14	7 / 15	8
1 / 9		75%	10%	20%	25%	20%	15%	10%
2 / 10			5%	15%	20%	15%	10%	10%
3 / 11				5%	15%	20%	5%	20%
4 / 12					20%	25%	5%	5%
5 / 13						25%	5%	15%
6 / 14							5%	20%
7 / 15								5%
8								

Figure 12.4: Delta CSR non securitisation correlation across buckets

[4] Either IG or HY/NR

Delta CSR Securitisations (CTP)

Buckets
The same bucket structure as the one for the CSR non-securitisation framework is applied.

Risk Weights
Risk weights are the same for all vertices[5] and are presented in table (12.9).

Bucket	Risk weight
1	4.0%
2	4.0%
3	8.0%
4	5.0%
5	4.0%
6	3.0%
7	2.0%
8	6.0%
9	13.0%
10	13.0%
11	16.0%
12	10.0%
13	12.0%
14	12.0%
15	12.0%
16	13.0%

Table 12.9: Delta CSR Securitisation (CTP) Risk Weights

Correlations
The correlation parameters ρ_{kl} are the same as the ones in the Delta CSR non-securitisations case. However, one parameter changes:

$$\rho_{kl}^{(basis)} = \begin{cases} 1 & if\ sensitivities\ have\ the\ same\ curve \\ 99.00\% & if\ else \end{cases}$$

[5] 0.5 yr, 1 yr, 3 yr, 5 yr, 10 yr

Delta CSR Securitisations (non-CTP)

Buckets

Sensitivities or risk exposures must be assigned to buckets according to the following table:

93. Sensitivities or risk exposures must first be assigned to a bucket according to the following table:

Bucket number	Credit quality	Sector
1	Senior Investment grade (IG)	RMBS – Prime
2		RMBS – Mid-Prime
3		RMBS – Sub-Prime
4		CMBS
5		ABS – Student loans
6		ABS – Credit cards
7		ABS – Auto
8		CLO non-correlation trading portfolio
9	Non-Senior Investment grade (IG)	RMBS – Prime
10		RMBS – Mid-Prime
11		RMBS – Sub-Prime
12		CMBS
13		ABS – Student loans
14		ABS – Credit cards
15		ABS – Auto
16		CLO non-correlation trading portfolio
17	High yield (HY) & non-rated (NR)	RMBS – Prime
18		RMBS – Mid-Prime
19		RMBS – Sub-Prime
20		CMBS
21		ABS – Student loans
22		ABS – Credit cards
23		ABS – Auto
24		CLO non-correlation trading portfolio
25	Other sector[29]	

Figure 12.5: Delta Securitisation (non-CTP) buckets

Risk Weights
The buckets 1 to 8 refer to Senior Investment grade instruments. The risk weights for these buckets are laid out in the following table:

Bucket	Risk weight
1	0.9%
2	1.5%
3	2.0%
4	2.0%
5	0.8%
6	1.2%
7	1.2%
8	1.4%

Table 12.10: Delta CSR securitisations non-CTP risk weights - buckets 1 to 8

Risk weights for buckets 9 to 16 (Non-Senior Investment grade) are obtained by respectively scaling up each Senior Investment grade risk weight by 1.25.
Risk weights for buckets 17 to 24 (High yield and non-rated) are computed by scaling up risk weights of buckets 1 to 8 by 1.75.
The risk weight for bucket 25 is equal to 3.5%.

Correlations
The correlation parameter ρ_{kl} between two sensitivities WS_k and WS_l is computed the following manner:

$$\rho_{kl} = \rho_{kl}^{(tranche)} \cdot \rho_{kl}^{(tenor)} \cdot \rho_{kl}^{(basis)} \tag{12.4}$$

where:

- $\rho_{kl}^{(tranche)} = \begin{cases} 1 & if\ k\ and\ l\ indicate\ the\ same\ securitisation\ tranche\ in\ the\ same\ bucket^{6} \\ 40\% & if\ else \end{cases}$
- $\rho_{kl}^{(tenor)} = \begin{cases} 1 & if\ sensitivities\ k\ and\ l\ have\ the\ same\ vertices \\ 80\% & if\ else \end{cases}$
- $\rho_{kl}^{(basis)} = \begin{cases} 1 & if\ sensitivities\ are\ related\ to\ the\ same\ curves \\ 99.90\% & if\ else \end{cases}$

However, the correlations for the 'other sector' bucket (in this case, bucket number 25) are not computed as mentioned above. The capital requirement for this bucket is computed by simply aggregating the absolute values of the corresponding net weighted sensitivities:

$$K_{b(other\ bucket)} = \sum_k |WS_k|$$

[6] With more than 80% overlap in notional terms

Equity Risk

Buckets

Sensitivities are assigned to buckets according to paragraph 102 of the Basel III document on capital requirements for market risk[7].

102. Sensitivities must first be assigned to a bucket as defined in the following table:

Bucket number	Market cap	Economy	Sector
1	Large	Emerging market economy	Consumer goods and services, transportation and storage, administrative and support service activities, healthcare, utilities
2			Telecommunications, industrials
3			Basic materials, energy, agriculture, manufacturing, mining and quarrying
4			Financials including government-backed financials, real estate activities, technology
5		Advanced economy	Consumer goods and services, transportation and storage, administrative and support service activities, healthcare, utilities
6			Telecommunications, industrials
7			Basic materials, energy, agriculture, manufacturing, mining and quarrying
8			Financials including government-backed financials, real estate activities, technology
9	Small	Emerging market economy	All sectors described under bucket numbers 1, 2, 3 and 4
10		Advanced economy	All sectors described under bucket numbers 5, 6, 7 and 8
11	Other sector[30]		

Figure 12.6: Delta buckets and risk weights for equity risk

Some definitions accompany this table:

- Market capitalization is the sum of market caps of the same legal entity or group of legal entities across all stock markets globally.
- According to Basel III, the market cap is qualified as large when it is equal to or greater than USD 2 billion, and as small when it is less than USD 2 billion.
- Countries with an advanced economy are Canada, the United States, Mexico, the euro area, the non-euro area western European countries[8], Japan, Oceania[9], Singapore and Hong Kong SAR.

[7] https://www.bis.org/bcbs/publ/d352.pdf
[8] The United Kingdom, Norway, Sweden, Denmark and Switzerland
[9] Australia and New Zealand

Risk Weights

The risk weights to the sensitivities to Equity spot price and Equity repo rate are set out in the following manner:

Bucket	Risk weight: Equity spot price	Risk weight: Equity repo rate
1	55%	0.55%
2	60%	0.60%
3	45%	0.45%
4	55%	0.55%
5	30%	0.30%
6	35%	0.35%
7	40%	0.40%
8	50%	0.50%
9	70%	0.70%
10	50%	0.50%
11	70%	0.70%

Table 12.11: Equity risk: delta risk weights

Correlations

When in the same bucket, correlations between two sensitivities to an Equity spot price (or an Equity repo rate) are defined in the table below:

Large market cap	Emerging market economy	15%
	Advanced economy	25%
Small market cap	Emerging market economy	7.50%
	Advanced economy	12.50%

Table 12.12: Equity risk correlations

Within the same bucket, the correlation ρ_{kl} between two sensitivities WS_k to an equity spot price and WS_l to an equity repo rate (related to the same Equity issuer name) is set at 99.90%.
When it comes to different Equity issuer names, the correlations defined in the table above are multiplied by 99.90%.
As mentioned in different risk classes, the 'other sector' bucket is treated separately and its capital charge is calculated as the sum of absolute values of its weighted sensitivities, like so:

$$K_{b(other\ bucket)} = \sum_k |WS_k|$$

If buckets b and c fall within bucket numbers 1 to 10, the correlation parameter γ_{bc} across buckets is set at 15%.

Commodity Risk

Buckets and risk weights

For commodity risk, the buckets and the risk weights are prescribed in the same table:

Bucket	Commodity bucket	Examples of commodities allocated to each commodity bucket (non-exhaustive)	Risk weight (percentage points)
1	Energy - Solid combustibles	coal, charcoal, wood pellets, nuclear fuel (such as uranium)	30%
2	Energy - Liquid combustibles	crude oil (such as Light-sweet, heavy, WTI and Brent); biofuels (such as bioethanol and biodiesel); petrochemicals (such as propane, ethane, gasoline, methanol and butane); refined fuels (such as jet fuel, kerosene, gasoil, fuel oil, naptha, heating oil and diesel)	35%
3	Energy - Electricity and carbon trading	electricity (such as spot, day-ahead, peak and off-peak); carbon emissions trading (such as certified emissions reductions, in-delivery month EUA, RGGI CO2 allowance and renewable energy certificates)	60%
4	Freight	dry-bulk route (such as capesize, panamex, handysize and supramax); liquid-bulk/gas shipping route (such as suezmax, aframax and very large crude carriers)	80%
5	Metals – non-precious	base metal (such as aluminium, copper, lead, nickel, tin and zinc); steel raw materials (such as steel billet, steel wire, steel coil, steel scrap and steel rebar, iron ore, tungsten, vanadium, titanium and tantalum); minor metals (such as cobalt, manganese, molybdenum)	40%
6	Gaseous combustibles	natural gas; liquefied natural gas	45%
7	Precious metals (including gold)	gold; silver; platinum; palladium	20%
8	Grains & oilseed	corn; wheat; soybean (such as soybean seed, soybean oil and soybean meal); oats; palm oil; canola; barley; rapeseed (such as rapeseed seed, rapeseed oil, and rapeseed meal); red bean, sorghum; coconut oil; olive oil; peanut oil; sunflower oil; rice	35%
9	Livestock & dairy	cattle (such as live and feeder); hog; poultry; lamb; fish; shrimp; dairy (such as milk, whey, eggs, butter and cheese)	25%
10	Softs and other agriculturals	cocoa; coffee (such as arabica and robusta); tea; citrus and orange juice; potatoes; sugar; cotton; wool; lumber and pulp; rubber	35%
11	Other commodity	industrial minerals (such as potash, fertilizer and phosphate rocks), rare earths; terephthalic acid; flat glass	50%

Figure 12.7: Commodity Risk - Buckets and Risk Weights

Correlations
Within the same bucket, ρ_{kl} is calculated as follows:

$$\rho_{kl} = \rho_{kl}^{(cty)} \cdot \rho_{kl}^{(tenor)} \cdot \rho_{kl}^{(basis)} \tag{12.5}$$

where:

- $\rho_{kl}^{(cty)} = \begin{cases} 1 & if\ commodities\ of\ k\ and\ l\ are\ identical \\ intra-bucket\ correlation & if\ else \end{cases}$
- $\rho_{kl}^{(tenor)} = \begin{cases} 1 & if\ k\ and\ l\ have\ identical\ vertices \\ 99.90\% & if\ else \end{cases}$
- $\rho_{kl}^{(basis)} = \begin{cases} 1 & if\ contract\ grade\ and\ delivery\ location\ are\ identical \\ 99.90\% & if\ else \end{cases}$

and the intra-bucket correlaion referred to in the above list is shown in the following table:

Bucket	Commodity bucket	Correlation (ρ_{kl})
1	Energy - Solid combustibles	55%
2	Energy - Liquid combustibles	95%
3	Energy - Electricity and carbon trading	40%
4	Freight	80%
5	Metals – non-precious	60%
6	Gaseous combustibles	65%
7	Precious metals (including gold)	55%
8	Grains & oilseed	45%
9	Livestock & dairy	15%
10	Softs and other agriculturals	40%
11	Other commodity	15%

Table 12.13: Intra-bucket Correlations

For sensitivities across buckets, if both buckets fall within bucket numbers 1 to 10, $\gamma_{bc} = 20\%$. If either bucket is bucket number 11, $\gamma_{bc} = 0\%$.

Foreign Exchange Risk

Risk weights
A unique risk weight of 30% applies to all FX risk exposures.

Correlations
A uniform correlation of $\gamma_{bc} = 60\%$ applies to all FX risk exposure pairs.

Appendix D: Full SbM - Vega Risk Weights and Correlations

Buckets

Vega buckets are the same as the delta buckets specified in Appendix C.

Risk Weights

Vega risk factors take into account liquidity risk, through the assignment of different liquidity horizons to each risk class ($LH_{risk\ class}$) and incorporating them in the risk weight formula below:

$$RW_k = \min\left[RW_\sigma \cdot \frac{\sqrt{LH_{risk\ class}}}{\sqrt{10}}; 100\%\right] \tag{12.6}$$

where:

- $RW_\sigma = 55\%$,
- $LH_{risk\ class}$ is prescribed in table (12.14).

Risk class	$LH_{risk\ class}$
GIRR	60
CSR non-securitisations	120
CSR securitisations (CTP)	120
CSR securitisations (non-CTP)	120
Equity (large cap)	20
Equity (small cap)	60
Commodity	120
FX	40

Table 12.14: Prescribed liquidity horizons for vega risk weight calculations

Correlations

The ρ_{kl} correlation parameter

GIRR Risk Class
The correlation parameter ρ_{kl} is calculated as follows:

$$\rho_{kl} = \min\left[\rho_{kl}^{(option\ maturity)} \cdot \rho_{kl}^{(underlying\ maturity)}; 1\right] \tag{12.7}$$

where:

- $\rho_{kl}^{(option\ maturity)} = e^{-\alpha \frac{|T_k - T_l|}{\min[T_k; T_l]}}$ where:
  - T_k is the maturity of the option from which the vega sensitivity is derived (see table (12.5));
  - α is set at 1%.
- $\rho_{kl}^{(underlying\ maturity)} = e^{-\alpha \frac{|T_k^U - T_l^U|}{\min\left(T_k^U; T_l^U\right)}}$ where
  - T_k^U is the maturity of the underlying option from which the vega sensitivity is derived;
  - α is set at 1%.

Other Risk Factors
The correlation parameter ρ_{kl} is calculated as follows:

$$\rho_{kl} = \min\left[\rho_{kl}^{(DELTA)} \cdot \rho_{kl}^{(option\ maturity)}; 1\right] \tag{12.8}$$

where:

- $\rho_{kl}^{(DELTA)}$ is the correlation between the delta risk factors corresponding to vega risk factors k and l;
- $\rho_{kl}^{(option\ maturity)}$ is the same as the one from equation (12.7).

The γ_{bc} correlation parameter
Whether the risk class was GIRR or non-GIRR, the γ_{bc} correlation parameter is the same as specified for delta correlations in the previous section.

Appendix E: Full SbM - Curvature Risk Weights and Correlations

Buckets

The delta bucket classification is replicated for curvature risk.
The first level of aggregation between curvature risk positions remains the bucket within each risk class.

Risk Weights

The curvature risk weights depend on the risk class we are dealing with:

FX and Equity curvature risk factors: Relative shifts ('shocks') equal to the **delta** risk weights;

GIRR, CSR and Commodity curvature risk factors: Parallel shift of all vertices for each curve based on the highest prescribed delta risk weight for each risk class.

Correlations

To calculate ρ_{kl} and γ_{bc}, we simply square the delta correlation parameters. For example,

$$\rho^{GIRR}_{(CVR_{EUR},CVR_{USD})} = 50\%^2 = 25\%$$

Appendix F: Default Risk Charge

The step-by-step approach to calculate the default risk charge is specified in (7.2). In this appendix, we will specify the way to calculate JTD amounts used in the computation of the DRC.

Non-Securitisations

Gross JTD

The gross JTD risk position is computed exposure by exposure[10].
It is obtained as a function of the LGD, the notional amount (or face value) and the cumulative P&L already realized on the position (cumulative mark-to-market loss or gain):

$$JTD_{long} = \max\left(LGD \times notional + P\&L, 0\right) \tag{12.9}$$

$$JTD_{short} = \min\left(LGD \times notional + P\&L, 0\right) \tag{12.10}$$

Instrument	LGD
Equity	100%
Non-senior debt	
Senior debt	75%
Covered bonds	25%

Table 12.15: Prescribed LGD values for non-securitisations

Net JTD

When it comes to the same obligor, long and short exposures can be offset when the short exposure has the same or lower seniority relative to the long exposure[11]. Exposures with different maturities can be offset under some conditions:

- If they have maturities longer than the capital horizon of one year, then they can be fully offset.
- If there is a mix of long and short exposures to the same obligor with a maturity less than the capital horizon of one year, they must be weighted by the ratio of the exposure's maturity to the capital horizon[12].
- The scaling can be applied to both long and short offsetting exposures having a maturity under one year.

[10]For example, if a bank is long a bond on Apple, and short another bond on Apple, it must compute two JTD exposures

[11]For example, a short exposure in an equity may offset a long exposure in a bond. However, the inverse is not applicable.

[12]For example, with a one-year capital horizon, a three-month short exposure should be weighted so that its benefit against long exposures with a maturity longer than one year would be reduced to one quarter of the exposure size.

Default Risk Charge for non-securitisations

Default Risk Weights

Net JTD amounts are then weighted according to credit rating quality, without taking into consideration counterparty type. The risk weights are given in the following table:

Credit quality category	Default risk weight
AAA	0.5%
AA	2%
A	3%
BBB	6%
BB	15%
B	30%
CCC	50%
Unrated	15%
Default	100%

Table 12.16: Default risk weights by credit quality categories for non-securitisations

Buckets for net JTD

Then, each weighted net JTD is allocated to one of three buckets:

- Corporates;
- Sovereigns;
- Local governments/municipalities.

Hedge Benefit Ratio

The next step would be to compute a **hedge benefit ratio**, in order to recognize hedging relationship between long and short positions, using the following procedure:

- Calculate the simple sum of net long and net short JTD amounts across credit quality categories: $\sum net\ JTD_{long}$ and $\sum net\ JTD_{short}$
- Compute the hedge benefit ratio (also called Weighted to Short ratio - WtS):

$$WtS = \frac{\sum net\ JTD_{long}}{\sum net\ JTD_{long} + \sum |net\ JTD_{short}|} \tag{12.11}$$

Overall default risk charge per bucket

$$DRC_b = \max\left[\left(\sum_{i \in Long} RW_i \cdot net\ JTD_i\right) - WtS \cdot \left(\sum_{i \in Short} RW_i \cdot |net\ JTD_i|\right); 0\right] \tag{12.12}$$

where DRC_b stands for 'default risk charge in bucket b' and i refers to an instrument in that bucket. Finally, the total capital charge for default risk non-securitisations is the simple sum of bucket-level capital charges.

Securitisations: non-correlation trading portfolio

Gross JTD

The same approach is followed as for non-securitisations, except that no LGD ratio is applied to the exposure. It is taken into consideration in the securitisation risk weights.
In order to avoid double counting of the LGD, we consider the JTD for securitisations to be simply the market value of the corresponding exposure[13].

Net JTD

Offsetting securitisation exposures is limited and can only be done under certain conditions:

- No offsetting can be done between securitisation exposures with different underlying, even if attachment and detachment points are the same;
- No offsetting can be done between exposures arising from different tranches within the same portfolio.

Otherwise, securitisations that are identical but with different maturities can be offset with the same restrictions as described for non-securitisation exposures.

Default Risk Charge for Securitisations (non-CTP)

To calculate the default risk charge, the same approach as for non-securitisations is applied. However, the exposures are sorted by tranche instead of credit quality.
The risk weights applied are specified in a the Basel III Revisions for Securitisations document[14], and they match treatment procedures for the banking book. In this procedure, the maturity component is set to zero (a maturity of one year is assumed) and the determination of risk weights is applied at the underlying pool level.
The buckets for non-CTP securitisations are defined as follows:

- A unique bucket is assigned to corporates
- The rest of the buckets are defined along the two following dimensions:
  - **Asset class:** ABCP, Auto Loans/Leases, RMBS, Credit Cards, CMBS, Collateralised Loan Obligations, CDO-squared, Small and Medium Enterprises, Student Loans, Other retail, Other wholesale.
  - **Region:** Asia, Europe, North America, All other.

The default risk charge within buckets is determined as in the non-securitisations procedure. No hedging is recognized between buckets.

[13] Tranche positions, for example, have a JTD equal to their market value
[14] https://www.bis.org/bcbs/publ/d303.pdf

Securitisations: Correlation Trading Portfolio

Gross JTD

For CTP securitisations, the same approach as before is applied. For non-securitisations in the CTP, the JTD is defined as their market value.
N^{th}-to-default products are treated as tranched products with attachment and detachment points defined as:

(a) attachment point=(N-1)/Total Names

(b) detachment point=N/Total Names

Net JTD

Offsetting is permitted for exposures that only differ in maturity. Detailed instructions on how to offset exposures by family or by tranche are specified in the Basel III Minimum Capital Requirements for Market Risk[15], in paragraph number 169.

Default Risk Charge for securitisations (CTP)

The default risk charge per bucket is determined by:

$$DRC_b = \left(\sum_{i \in Long} RW_i \cdot net\ JTD_i \right) - WtS_{ctp} \cdot \left(\sum_{i \in Short} RW_i \cdot |net\ JTD_i| \right) \tag{12.13}$$

where WtS designates the hedge benefit ratio as defined previously in (12.11).
The risk weights used are based on the banking book risk weights found in a separate Basel Committee publication[16]. Since migration risk in the trading book will already be captured by the credit spread risk charge, we assume a maturity of one year the banking book securitisation framework.
The bucket-level default risk charges are aggregated in the following manner:

$$DRC_{CTP} = \max \left[\sum_b \left(\max \left[DRC_b, 0 \right] + 0.5 \times \min \left[DRC_b, 0 \right] \right), 0 \right] \tag{12.14}$$

[15] https://www.bis.org/bcbs/publ/d457.pdf
[16] https://www.bis.org/bcbs/publ/d303.pdf

Appendix G: Reduced SbM

Buckets, Risk Weights and Correlations

Delta GIRR

Buckets
Each individual currency exposure to GIRR is assigned to a bucket.

Risk weights
The risk weight is set at 5% for all maturities.
A risk weight of 3% is set for the inflation risk factor and the cross currency basis risk factors.

Correlations
The correlation parameter is set as:

- 20% between a weighted sensitivity to an increase in the short end of the yield curve and a sensitivity to an increase in the long end of the yield curve,
- 40% between a weighted sensitivity to the inflation curve and a sensitivity to the yield curve (either short or long end),
- 0% between a weighted sensitivity to a cross-currency basis curve and a weighted sensitivity to any other GIRR risk factor.

The γ_{bc} parameter is set at 50% between all buckets.

Delta CSR non-securitisations

Buckets
The buckets for Delta CSR non-securitisation exposures are defined as follows:

Bucket number	Credit quality	Sector
1	Investment grade (IG)	Sovereigns including central banks, multilateral development banks Local government, government-backed non-financials, education, public administration
2		Financials including government-backed financials
3		Other sector
4	High yield (HY) & non-rated (NR)	Sovereigns including central banks, multilateral development banks Local government, government-backed non-financials, education, public administration
5		Financials including government-backed financials
6		Other sector

Figure 12.8: Delta CSR non-securitisation buckets for the R-SbM

Risk Weights
Risk weights for the buckets defined in the previous paragraph are defined as follows:

Bucket number	Risk weight
1	1.0% for Sovereigns including central banks, multilateral development banks 5.0% for Local government, government-backed non-financials, education, public administration
2	10.0%
3	15.0%
4	5.0% for Sovereigns including central banks, multilateral development banks 10.0% for Local government, government-backed non-financials, education, public administration
5	25.0%
6	30.0%

Figure 12.9: Delta CSR non-securitisation risk weights for the R-SbM

Correlations

The correlation ρ_{kl} beween two sensitivities within the same bucket is equal to 1 when the sensitivities have the same name and 35% otherwise.

Between buckets, γ_{bc} is defined as follows:

Bucket	1	2	3	4	5	6
1		10%	0%	50%	5%	0%
2			0%	5%	50%	0%
3				0%	0%	50%
4					10%	0%
5						0%
6						

Figure 12.10: Delta CSR non-securitisation correlations for the R-SbM

Delta CSR securitisations

Buckets

There are 12 buckets for CSR securitisation exposures, and they are defined as follows:

Bucket number	Credit quality	Sector
1	Senior Investment grade (IG)	RMBS
2		Non-mortgage retail securitisations
3		CMBS
4		Other
5	Non-Senior Investment grade (IG)	RMBS
6		Non-mortgage retail securitisations
7		CMBS
8		Other
9	High yield (HY) & non-rated (NR)	RMBS
10		Non-mortgage retail securitisations
11		CMBS
12		Other

Figure 12.11: Delta CSR securitisation buckets for the R-SbM

Risk Weights

The risk weights assigned to the buckets in the previous paragraph are defined as follows:

Bucket number	Risk weight
1	2.0%
2	5.0%
3	5.0%
4	10.0%
5	5.0%
6	6.25%
7	6.25%
8	12.5%
9	3.5%
10	8.75%
11	8.75%
12	17.5%

Figure 12.12: Delta CSR securitisation risk weights for the R-SbM

Correlations

The ρ_{kl} parameter is equal to 1 when sensitivities k and l belong to the same bucket and belong to the same securitisation tranche, and 40% otherwise.
γ_{bc} is set at 0%.

Equity Risk

Buckets

There are 6 buckets to which we assign sensitivities as follows:

Bucket number	Market cap	Economy	Sector
1	Large	Emerging market economy	Financials including government-backed financials, real estate activities, technology
2			Other sector
3		Advanced economy	Financials including government-backed financials, real estate activities, technology
4			Other sector
5	Small	Emerging market economy	All sectors
6		Advanced economy	All sectors

Figure 12.13: Equity risk buckets for the R-SbM

where market cap, large market cap, small market cap, and advanced economies are defined as in section (12).

Risk Weights

Risk weights for the buckets mentioned above are laid out as follows:

Bucket number	Risk weight for Equity spot price
1	60%
2	60%
3	50%
4	40%
5	70%
6	60%

Figure 12.14: Equity risk weights for the R-SbM

Correlations
Within each bucket, ρ_{kl} is defined as follows:

Bucket number	Correlation (ρ_{kl})
1	15%
2	15%
3	25%
4	25%
5	7.5%
6	12.5%

Figure 12.15: Equity risk correlations for the R-SbM

Across buckets, the γ_{bc} correlation parameter is set at 15%.

Commodity Risk

Buckets, Risk Weights and Correlations
The following table presents the definitions of each bucket as well as the corresponding risk weights and correlations:

Bucket	Commodity category	Risk Weight	Correlation
1	Solid combustibles (eg coal, charcoal, wood, nuclear fuel)	40%	55%
2	Liquid combustibles (eg crude oil, heating oil, gasoline, diesel, aviation fuel, bioethanol)	45%	95%
3	Electricity and carbon trading	70%	40%
4	Freight	90%	80%
5	Non-precious metals	50%	60%
6	Gaseous combustibles (eg natural gas, methane, city gas)	55%	65%
7	Precious metals (including gold)	30%	55%
8	Grains and oilseed	45%	45%
9	Livestock and dairy	35%	15%
10	Softs and other agriculturals	45%	40%
11	Other commodity	60%	15%

Table 12.17: Commodity risk buckets, risk weights and correlations for the R-SbM

Across buckets, γ_{bc} is defined as follows:

- 20% if b and c fall within buckets 1-10,
- 0% if any bucket b or c is bucket 11.

Foreign Exchange Risk

Risk Weights A unique risk weight of 15% is applied to all FX net sensitivities.
However, banks may apply a risk weight of 32% to specified currency pairs by the Basel Committee[17]

Correlations
Across buckets, $\gamma_{bc} = 60\%$.

[17]USD/EUR, USD/JPY, USD/GBP, USD/AUD, USD/CAD, USD/CHF, USD/MXN, USD/CNY, USD/NZD, USD/RUB, USD/HKD, USD/SGD, USD/TRY, USD/KRW, USD/SEK, USD/ZAR, USD/INR, USD/NOK, USD/BRL, EUR/JPY, EUR/GBP, EUR/CHF and JPY/AUD.