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Titre du mémoire: Implementation of Machine Learning techniques to estimate the impact of a change in the future management of asset allocation on the Solvency II Own Funds.		
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Graduation Thesis

Implementation of Machine Learning techniques to estimate the impact of a change in the future management of asset allocation on the Solvency II Own Funds.

Mathilde Sabathié

April 2021

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Résumé et mots clés

Un apport du Machine Learning pour estimer les fonds propres SII.

Sensibilité à une modification de l'allocation d'actif.

Introduction :

Notre point de départ est le modèle actif-passif stochastique d'un assureur.

Ce modèle, nommé ALIM, est lourd : il nécessite une heure pour calculer le ratio de solvabilité $\frac{Fonds\ Propres\ SII}{SCR}$ dans un an, pour une allocation d'actif donnée.

Nous avons testé quatre algorithmes de Machine Learning et avons mesuré leurs efficacités à retrouver les fonds propres SII calculés par le modèle ALIM. Nous avons sélectionné celui qui était le plus proche en terme de résultats.

L'algorithme sélectionné réplique le modèle ALIM à l'aide du Machine Learning afin d'obtenir un résultat rapide (cinq secondes) ce qui représente un gain de temps important.

Nous avons recommencé la procédure pour différentes allocations d'actifs.

Nous avons alors pu déterminer les allocations d'actifs efficientes par rapport au critère de rentabilité de l'assureur (l'accroissement des fonds propres) et à son critère de risque (l'accroissement du SCR).

L'algorithme de Machine Learning n'a pas pour but de remplacer complètement le modèle ALIM mais de présélectionner les allocations cibles qui seront ensuite testées dans le modèle ALIM. Tout cela dans le but d'augmenter l'efficience du processus de sélection.

Définition de l'allocation stratégique d'actifs:

Une allocation d'actifs désigne la façon dont les actifs d'une compagnie sont investis et répartis parmi les différentes classes d'actifs.

L'allocation stratégique d'actifs détermine une allocation cible supposée optimale suivant un critère de rentabilité et un critère de risque et tient compte de la structure de l'actif et du passif.
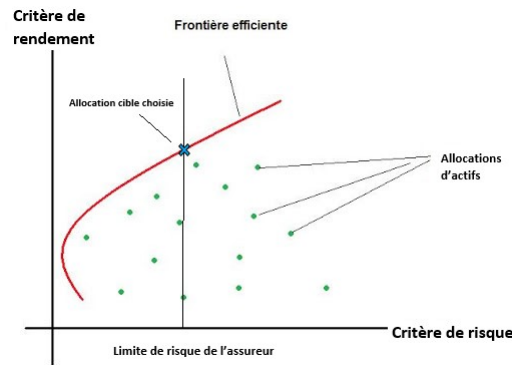
Figure 1: Frontière efficiente d'allocations d'actifs

Au sein de l'ensemble des allocations efficientes (c'est-à-dire dont il n'est pas possible d'améliorer un critère sans dégrader l'autre) l'assureur choisit une allocation cible, généralement celle qui maximise le critère de rentabilité en ne dépassant pas son appétence au risque.

Le modèle de projection actif / passif ALIM:

ALIM est un outil de projection actif-passif des contrats d'assurance-vie. Il a pour vocation principale de projeter les cash-flows futurs des produits vie selon une approche stochastique ainsi que les flux d'impôts.

Ces projections sont faites au niveau des principaux portefeuilles euro et UC de la compagnie d'assurance.

Une allocation cible est entrée en input dans le modèle. ALIM estime les fonds propres dans un an.

Pour une allocation d'actifs donnée, ALIM projette sur 60 ans, selon 1000 scénarios financiers les éléments du bilan (placements à l'actif, provisions au passif) et par différence on obtient les fonds propres SII.

ALIM estime le SCR suite à un changement d'allocation cible de la manière suivante :

1. Recalcule des fonds propres après choc de chaque facteur de risque. Calcul de la différence avant et après changement d'allocation cible.

2. Agrégation des risques avec une matrice de corrélation.

3. Estimation de l'impact de la fiscalité et des participations bénéficiaires.

Le Machine Learning:

Le Machine Learning utilise des approches mathématiques et statistiques pour donner aux ordinateurs la capacité d' apprendre à partir de données, c'est-à-dire d'améliorer leurs performances à résoudre des tâches sans être explicitement programmés pour chacune.

Ici, le modèle d'apprentissage utilise des données d'entrée et de sortie du modèle ALIM.

Nous avons choisi de nous concentrer sur les données du plus gros portefeuille d'assurance-vie de la compagnie dans le but d'obtenir de meilleurs résultats en réduisant le champ d'application.

Une fois l'algorithme de Machine Learning entrainé il peut remplacer ALIM.

Les Algorithmes de Machines Learning :

Nous avons testé quatre algorithmes de machines learning afin de sélectionner celui qui avait les résultats les plus proches du modèle ALIM. Un grid-search à été effectué afin d'optimiser les paramètres de chaque modèle puis leurs erreurs ont été comparées.

Les quatre algorithmes sont:

- Suport Vector Machine (SVM)

- Gradient Boosting Machine (GBM)

- Feed Forward Neural Network (FFNN)

- Long Short-Term Memory (LSTM)

Le Long Short-Term Memory (LSTM) à été sélectionné comme le modèle donnant les meilleurs résultats.

Le Long Short-Term Memory est un algorithme de type réseaux de neurones qui a la particularité d'avoir la capacité de mémoriser des données. C'est un algorithme qui est connu pour bien performer sur les données contenants des pas de temps et il n'est pas étonnant qu'il ai obtenu la meilleure performance sur nos données.

Le LSTM fait partie des algorithmes de réseau de neurones récurrents.

Comparaison des fonds propres déterminés par ALIM et par le Machine Learning:

Changement d'allocation cible %				Fonds propres			
Produits dérivés	Actions	Investissements Alternatifs	Immobilier	ALIM	Machine Learning	erreur absolu	erreur %
0%	0%	0%	0%	5 268 562 513	5 266 663 526	1 898 987	-0,04%
0%	0%	+0,5%	0%	5 235 223 005	5 204 984 187	30 238 819	-0,58%
0%	0%	0%	+1%	5 238 179 800	5 226 166 666	12 013 135	-0,23%
0%	+0,5%	0%	0%	5 249 361 096	5 244 707 632	4 653 465	-0,09%

Le tableau suivant compare les fonds propres SII calculés par le modèle ALIM et ceux calculés par le modèle LSTM de Machine Learning.

On peut noter que le modèle à tendance à surestimer légèrement les fonds propres.

On constate que les meilleurs résultats, on été obtenus lorsque l'allocation stratégique d'actifs ne différait pas de celle de l'année précédente.

L'algorithme peut raisonnablement être utilisé pour des allocations d'actif pas trop éloignées de l'allocation de départ. Nous ne recommandons pas de l'utiliser pour des allocations très différentes.

Application à la sélection de l'allocation stratégique d'actif:

Nous voulons déterminer une allocation cible qui correspond aux objectifs et aux attentes des investisseurs. Nous partons de l'allocation existante et nous la faisons varier avec un pas de 0,5%.

La première étape consiste à sélectionner un sous-ensemble optimal au sens où il permet d'avoir le meilleur ratio de solvabilité pour une donnée de critère de risque.
La seconde étape consiste à fixer un niveau maximal de risque acceptable.

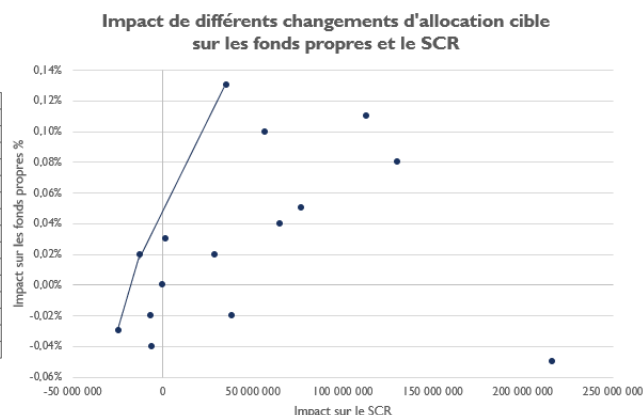L'allocation cible ainsi obtenue donnera la meilleure rentabilité par rapport au risque fixé.
Rappelons que l'assureur a choisi l'accroissement des fonds propres comme critère de rentabilité et l'accroissement du SCR comme critère de risque.

Le Machine Learning apporte un gain de temps qui permet de projeter le rendement/risque pour beaucoup plus d'allocation d'actifs. Il permet de tester une allocation cible en 5 secondes au lieu d'une heure et 500 allocations cibles en 1 heure au lieu de trois semaines.

Le Machine Learning pourra tester beaucoup plus d'allocation cible que le modèle ALIM. Il permettra de présélectionner les allocations cibles testées par ALIM et d'améliorer le processus de sélection de l'allocation cible.

Premiers résultats pour l'application à la sélection de l'allocation cible:

Changement d'allocation cible %				Impact sur le SCR	Impact sur les fonds propres %
Produits dérivés	Actions	Investissements Alternatifs	Immobilier		
0%	0%	0%	0%	0	0%
-0,5%	0%	0%	0%	-24 707 457	-0,03%
0,5%	0,5%	0,5%	1%	35 445 830	0,13%
0,5%	0,00%	0,5%	1%	113 117 811	0,11%
0,5%	0,00%	0%	0%	1 429 235	0,03%
0%	-0,5%	0%	0%	38 668 752	-0,02%
0%	0,5%	0,5%	1%	56 845 175	0,10%
0%	0,5%	0%	0%	-12 470 442	0,02%
0%	0%	0,5%	0%	216 354 825	-0,05%
0%	0%	0,5%	0%	77 216 198	0,05%
0%	0%	0,5%	1%	130 641 070	0,08%
0%	0%	0%	-0,5%	-6 579 223	-0,02%
0%	0%	0%	-1%	-6 092 774	-0,04%
0%	0%	0%	0,5%	28 878 110	0,02%
0%	0%	0%	1%	64 900 116	0,04%

Impact de différents changements d'allocation cible sur les fonds propres et le SCR

Les premiers résultats obtenus ne permettent pas encore de déterminer réellement une frontière efficiente ni d'observer de tendances pour l'instant . Il faudra encore tester des milliers d'allocations cible avant de pouvoir sélectionner celles qui performent le mieux.

Critique de l'utilisation du SCR comme métrique de risque pour la sélection de l' allocation cible:

Le choix du SCR, ou la variation du SCR, comme métrique de risque dans l'optimisation des fonds propres SII ne va pas de soi.

Les textes de Solvabilité II précise que le SCR est un montant de capital réglementaire calculé à partir de l'effet de critères de risque sur les fonds propres.

Certains risques ne sont pas pris en compte (risque souverains, risque d'inflation …).

Le calibrage des risques à été effectué après de nombreux arbitrages politique et conduit à privilégier certains actifs au détriment d'autres. L'EIOPA prévoit d'ailleurs de corriger prochainement certains chocs de la formule standard pour moins minimiser le risque de taux et mieux traiter les actions (augmentation de 10 à 17% des corrections contracycliques de choc action).

Mots clés: gestion actif-passif, stratégie d'investissement, assurance vie, modèle actif-passif, solvabilité 2, fonds propres, ALIM, ALM, Stratégie d'allocation d'actifs (SAA),Machine Learning.

Summary and Keywords

The contribution of Machine Learning to estimate the Solvency II Own Funds.

Sensitivity to an asset allocation change.

Introduction :

Our starting point is the stochastic asset-liability model of an insurer.

This model, called ALIM, is time-consuming: it requires an hour to calculate the solvency ratio $\frac{Solvency\ II\ Own\ Funds}{SCR}$ in a year, for a given asset allocation.

We tested four Machine Learning algorithms and measured their efficiency to estimate the Own Funds calculated by the model ALIM. We selected the one that was the closest in terms of results.

The selected algorithm replicates the model ALIM using machine learning in order to obtain a fast result (five seconds) which allows significant time saving.

We restarted the procedure for different asset allocations.

We were then able to determine the efficient asset allocations according to the insurer's profitability criterion (increase in the Own Funds) and its risk criterion (increase in the SCR).

The aim of the Machine Learning algorithm is not to completely replace the ALIM model but to preselect the strategic asset allocations which will then be tested in the ALIM model. All this with the aim of increasing the efficiency of the selection process.

Definition of a strategic asset allocation:

An asset allocation refers to the way a company's assets are invested and distributed among the different asset classes.

A strategic asset allocation determines a supposedly optimal strategic asset allocation based on a profitability criterion and a risk criterion and takes into account the structure of assets and liabilities.
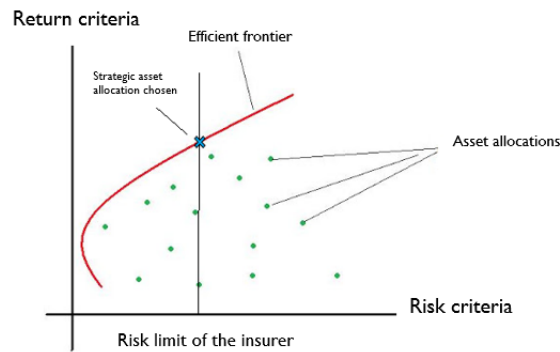
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Figure 2: Efficient frontier of asset allocations

Within the set of efficient allocations (i.e. for which it is not possible to improve one criterion without degrading the other) the insurer chooses a strategic asset allocation, generally the one that maximizes the criterion of profitability without exceeding its risk appetite.

The asset/liability projection model ALIM:

ALIM is an asset-liability projection tool for *assurance vie* contracts. Its main purpose is to project future cash flows from *assurance vie* products using a stochastic approach as well as tax flows.

These projections are made at the portfolio level. They include the main *euro* and *UC* portfolios of the insurance company.

A strategic asset allocation is entered as input into the model. ALIM estimates the Own Funds in one year.

For a given asset allocation, ALIM projects over 60 years, according to 1000 financial scenarios the balance sheet items (investments in assets, provisions in liabilities) and by difference we obtain the solvency II Own Funds.

ALIM estimates the SCR following a change of strategic asset allocation as follows:

1. Recalculates the Own Funds after shock of each risk factor. Calculation of the difference before and after strategic asset allocation change.

2. Aggregation of risks with a correlation matrix.

3. Estimation of the impact of taxation and profit sharing.

Machine Learning:

Machine learning uses mathematical and statistical approaches to give computers the ability to "learn" from data, that is, to improve their performance in solving tasks without being explicitly programmed to do so.

Here, the Machine Learning model uses input and output data from the model ALIM.

We have chosen to focus on data from the company's largest *assurance vie* portfolio in an effort to achieve better results by narrowing the scope.

Once the machine learning algorithm has been trained it can replace ALIM.

Machine Learning Algorithms:

We tested four machine learning algorithms to select the one that had the closest results to the model ALIM . A grid-search was carried out to optimize the parameters of each model and then their errors were compared.

The four algorithms are:

- Suport Vector Machine (SVM)

- Gradient Boosting Machine (GBM)

- Feed Forward Neural Network (FFNN)

- Long Short-Term Memory (LSTM)

The Long Short-Term Memory (LSTM) has been selected as the model giving the best results.

Long Short-Term Memory is a neural network algorithm that has the particularity of having the ability to memorize data. This is an algorithm that is known to perform well on data containing time steps and it not surprising that it was the one that performed the best on our data.

LSTM is part of the recurrent neural network algorithms.

Comparison of the Own Funds determined by ALIM and by Machine Learning:

| Stategic Asset Allocation change % | | | | | LSTM | | |
Derivatives	Equity	Alternative Investments	Real Estate	ALIM Output	Machine Learning Output	abs-error	%-error
0%	0%	0%	0%	5 268 562 513	5 266 663 526	1 898 987	-0,04%
0%	0%	+0,5%	0%	5 235 223 005	5 204 984 187	30 238 819	-0,58%
0%	0%	0%	+1%	5 238 179 800	5 226 166 666	12 013 135	-0,23%
0%	+0,5%	0%	0%	5 249 361 096	5 244 707 632	4 653 465	-0,09%

The following table compares the Own Funds calculated by ALIM and those calculated by the Machine Learning model.

It can be noted that the model tends to slightly overestimate the Own Funds.

We can see that the best results were obtained when the strategic asset allocation did not differ from that of the previous year.

The algorithm can reasonably be used for asset allocations not too far removed from the original allocation. We do not recommend using it for very different allocations.

Application to the selection of the strategic asset allocation:

We want to determine a strategic asset allocation that matches the objectives and expectations of investors. We start from the existing allocation and modify it with a step of 0.5 %.

The first step is to select an "optimal" subset in the sense that it provides the best solvency ratio for a given risk criterion.ue.
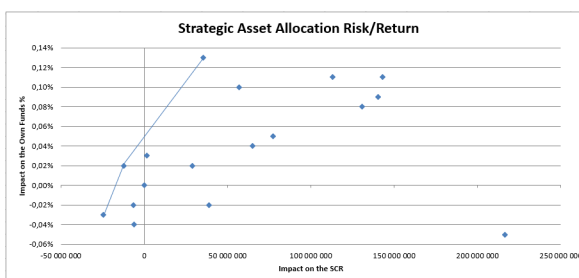The second step is to set a maximum acceptable level of risk.
It must be recalled that the insurer has chosen the increase in the Own Funds as a criterion of profitability and increase in the SCR as a criterion of risk.

Machine learning saves time that allows us to project the return / risk for much more asset allocations. It allows us to test a strategic asset allocation in 5 seconds instead of an hour and 500 target allocations in 1 hour instead of three weeks.

Machine Learning permits to test much more Strategic Asset Allocations and will make it possible to pre-select the stategic asset allocations tested by ALIM and improve the process of selecting the strategic asset allocation.

First results for the application to the selection of the strategic asset allocation:

Strategic Asset Allocation change %					
Derivatives	Equity	Alternative Investments	Real Estate	Impact on the SCR	Impact on the Own Funds
0%	0%	0%	0%	0	0,00%
-0,5%	0%	0%	0%	-24 707 457	-0,03%
0,5%	0,5%	0,5%	1%	35 445 830	0,13%
0,5%	0%	0,5%	1%	113 117 811	0,11%
0,5%	0%	0%	0%	1 429 235	0,03%
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0%	0%	0,5%	0%	77 216 198	0,05%
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0%	0%	0%	-1%	-6 092 774	-0,04%
0%	0%	0%	0,5%	28 878 110	0,02%
0%	0%	0%	1%	64 900 116	0,04%

The first results obtained do not yet allow us to determine an efficient frontier or to observe notable trends for the moment. Thousands of strategic asset allocations will still have to be tested before we can select the best performing ones.

Critique of the use of the SCR as a risk metric for the selection of the strategic asset allocation:

The choice of the SCR, or the variation of the SCR, as a risk metric in SII capital optimization is not obvious.

The Solvency II texts specify that the SCR is an amount of regulatory capital calculated from the effect of different risk criteria on the Own Funds .

Certain risks are not taken into account (sovereign risk, inflation risk ...).

The risk calibration was carried out after numerous political arbitration and leads to favoring certain assets to the detriment of others. EIOPA also plans to correct some standard formula shocks in the near future to minimize interest rate risk and better deal with equities (10 to 17% increase in counter-cyclical equity shock corrections).

Contents

Introduction

In today's low interest rate environment, insurance companies seek, among others,new ways of increasing their Own Funds. One of these ways can be by improving the selection of the company's Strategic Asset Allocation.

A Strategic Asset Allocation (SAA) determines the way a company invests and the amount of money targeted in each category of investments.
Its main purpose is to determine, for each of the company's portfolios, an optimal asset allocation that optimizes the balance between the risk taken by the insurer and the expected return, while respecting the characteristics of liabilities.

Determining a more efficient Strategic Asset Allocation according to the insurer's profitability and risk criterion would mean increasing profit while keeping the same risk. This would only be possible if the insurer is capable of testing many more strategies in order to find the best performing one.

However our insurance company encounters a calculation run time problematic when calculating impacts on the own funds associated with a change of Strategic Asset Allocation. It's stochastic asset-liability model, called ALIM, is time-consuming: it requires an hour to calculate the solvency ratio $\frac{Solvency\ II\ Own\ Funds}{SCR}$ in a year, after an asset allocation change.

For a given asset allocation, ALIM projects over 60 years, according to 1000 financial scenarios the balance sheet items (investments in assets, provisions in liabilities) and by difference we obtain the solvency II Own Funds. ALIM applies financial sensitivities to all the future liability cash flows of the company calculated in run off. The process is time-consuming due to complexity of calculation.

This is partly because a change of Strategic Asset Allocation affects also the liability cash flows of the company. They represents the amount of money that a company will need to pay during its operational activities. Liabilities are linked to assets and are affected by asset changes and therefore Strategic Asset Allocations changes.

One solution to improve efficiency of selection is approximating the ALIM model to decrease calculation time and test more Strategic Asset Allocations. This in order not to completely replace ALIM but to preselect the Strategic Asset Allocations which will then be tested in the ALIM model.

This paper describes the approximation of the Solvency II Own Fund's calculation, via Machine Learning Algorithms in response to a change of the Strategic Asset Allocation (SAA). The goal being to reduce calculation time.

Once the impacts on the Own Funds are calculated it is possible to calculate the Solvency Capital Requirement impacts of a SAA change with a method used by the insurance company that has similarities to the standard formula.

To determine the efficient asset allocations , the insurer chose the increase in the Own Funds as its profitability criterion and the increase in the SCR as its risk criterion.

We have chosen to focus on data from the company's largest *assurance vie* portfolio in an effort to achieve better results by narrowing the scope.

Machine learning is an artificial intelligence technology that allows computers to learn without being explicitly programmed for this purpose. It allows among others, to approximate complex algorithms.

We tested four Machine Learning algorithms and measured their efficiency to estimate the Own Funds calculated by the model ALIM. We selected the one that was the closest in terms of results. The selected Machine Learning algorithm had a fast calculation time of five seconds.

In this thesis the Machine Learning process and its results will be presented after defining of all the concepts used in this paper.

We will now describe the steps followed in the description of this study:

- Introduction to the insurance world and its norms of calculation. Definition of the concepts widely spread in this study.

- Presentation of Asset Liability Management and its main objectives. Focus on the Internal Asset-Liability Interaction Model (ALIM).

- Description of the calculation run time problematic and the choice of solution to deal with the issue.

- Overview of the four different Machine Learning algorithms tested and their performances.

- Presentation of the selected Machine Learning algorithm, its selection criteria,limits and results.

- Application of this method to Strategic Asset Allocation selection and critics of this process and of the risk metric chosen by the insurer.

Chapter 1

Insurance market and regulations

1.1 Presentation of the *assurance vie* market

A french *assurance vie* contract between the policyholder and the insurer is a saving plan invested in a portfolio composed of different assets. At the end of the contract, if the policyholder is alive he recovers its capital and its interest, if he is dead a beneficiary that has been designated at the time of subscription will inherit the contract.

Assurance vie is the most used tax sheltered investment vehicle for French residents. It is mainly due to its attractive taxation compared to other investment products.

The French law on taxation of insurance products changed in 2018 and there is a different tax treatment on gains depending on whether the premiums were paid before or after 27th September 2017. In *assurance vie* the tax is applicable once a withdrawal has been decided.

The tax on gains if premiums were paid before 27th September 2017 is a 35% flat rate if the withdrawal happens between years 1 to 4, 15% years 4 to 8 and 7.5% years 8 and beyond. After year 8, €4600 can be withdrawn annually free of tax or €9200 for a married couple. At this flat rate is added a social tax of 17.2%, regardless of the year of withdrawal.

The tax on gains if premiums were paid after 27th September 2017 is a flat rate of 30% made up of 12.8% income tax and 17.2% social tax. After 8 years however the income tax is only at 7.5% if the premiums paid does not exceed €150,000. If it exceeds €150,000 the income tax stays at 12.8%. Tax deductions of € 4,600 for individuals and of € 9,200 for married couples are still available, but once recorded, the earnings are treated as interest and dividends.

This reduced taxation after 8 years is one of the main financial advantages of *assurance vie*. *Assurance vie* is also advantageous when it comes to inheritance as it is subjected to different rules than normal estate. It could in theory be left to a chosen beneficiary without going through the french legal inheritance process.

Insurers invest the funds to receive an interest and gives a financial compensation to the policyholder. There are two types of funds in an *assurance vie* contract:

- **Euro funds** with regular performances, these funds invest mainly on long bonds and keep them until maturity. The capital invested is guaranteed.

-- **Unit-linked funds**, a type of fund managed by a management company whose value rises and falls depending on the shares or bonds of the funds, without guarantees.

In France the *assurance vie* market is more and more concentrated between few actors. According to *Autorité de Contrôle Prudentiel et de Résolution* (ACPR), the french supervisor of the insurance and banking industries, the 6 largest bank-insurers companies and their subsidiaries represented 61% of the premiums collected.

The current economic context has a negative impact on the profitability of the *assurance vie* sector. The current economical situation is characterized by a low interest rate environment. The European Central Bank announced that low interest rates will be maintained at least until mid 2020. This environment can decrease the profitability and attractiveness of insurance contracts but also the value of the company's assets. It can also have a negative impact on the risk profile of insurance companies.
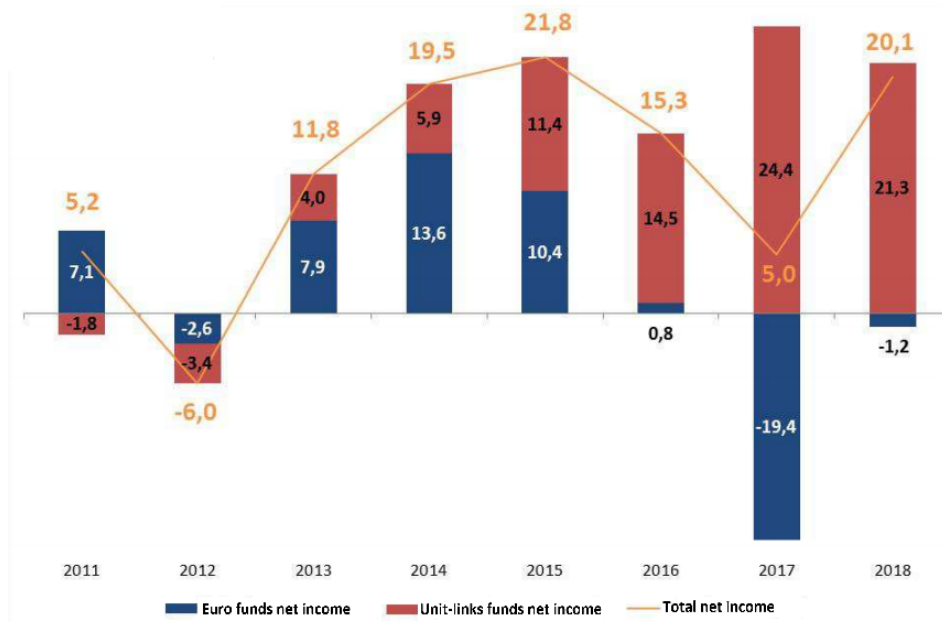

Figure 1.1: Net income by type of fund in billions of euros

According to ACPR, insurance companies net inflows was affected a lot by this economic environment in 2017 totaling only 5 billions euros but recovered sharply in 2018 with a net income of 20.1 billions euros. This is due to a 8% decrease of benefits paid and a important premium collect in Unit-linked funds.

Since 2016 insurance companies have been encouraging Unit-linked contracts that are less risky for the insurer as is does not require a minimum guaranteed interest rate. In 2018 insurers have collected 21,3 billions euros of Unit-linked net income and have had a negative income in Euro Funds of -1,2 billion.

Figure 1.1 : Net income by type of fund in billions of euros. URL: https://acpr.banque-france.fr/sites/default/files/medias/documents/as_100_2019_le_marche_francais_de_lassurance_vie_en_2018_vf.pdf

1.2 Solvency II

Solvency II is a set of rules establishing the solvency regime applicable to insurance companies in the European Union, effective since 1 January 2016.
Solvency II was put in place to impose stricter rules on the way insurance companies perform their risk calculation. It replaces the previous Solvency I directive in place since 1970.

The main objectives of this reform are:

-- An economic valuation (in market value, or ''fair value'') of assets and liabilities

-- Better protection for insurance clients

-- Harmonize insurance legislation at a European level

-- Encourage insurers to consider all types of risks (technical, financial, counterpart, operational)

-- Create a competitive European market

The Solvency II prudential rules divides into three pillars of requirements applicable to insurance companies :
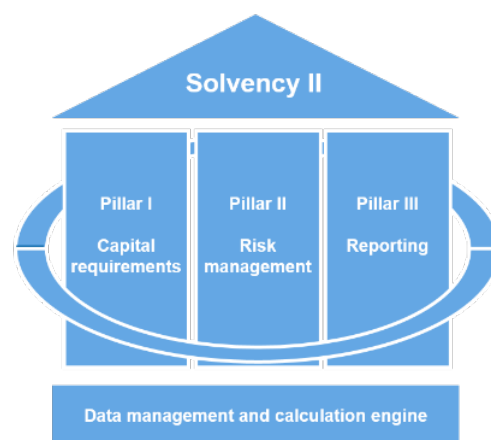

Figure 1.2: Solvency II pillars

- **Pillar I** of Solvency II combines the quantitative solvency requirements or valuation rules for assets and liabilities, as well as the capital requirements and their method of calculation.
  Capital requirements can be calculated using the Standard Formula determined by the Solvency II regulation or using a full or partial internal model. Organizations have the possibility to request different authorizations in order to adapt the quantitative requirements to their specific risk profile.

- **Pillar 2** contains the qualitative requirements for the internal governance and risk management processes. It establishes that insurance companies need to submit a report called ORSA (Own Risk and Solvency Assessment) every year to the supervisory authority.

- **Pillar 3** is about market discipline and transparency, it sets the requirements in terms of the transmission of financial information to the public and the supervisory authorities. It aims to harmonize at a European level the information published by insurance organizations as well as those given to supervisors.
  The information, both quantitative and qualitative, must be submitted annually or in some cases quarterly. In addition to European reports, specific French elements are also requested.

In France, good compliance with Solvency II standards is monitored by the *Autorité de Contrôle Prudentiel et de Résolution* (ACPR).

1.2.1 Market Value Balance Sheet (MVBS)

Under Solvency 2, financial statements are limited to one type of balance sheet. This Solvency 2 balance sheet is presented in market value as follows :
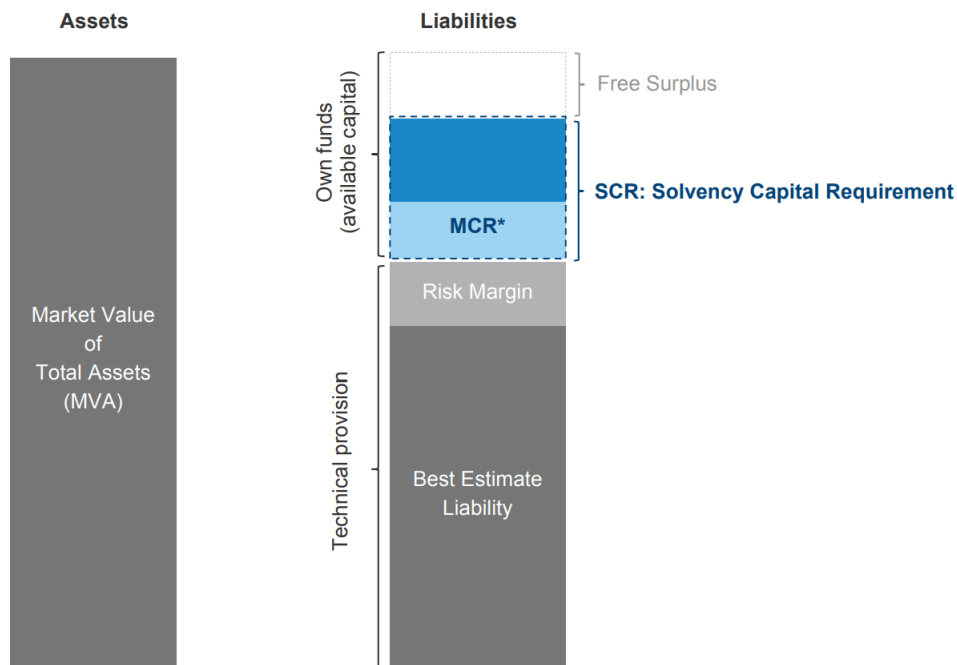
Figure 1.3: Market Value Balance Sheet

The Market Value Balance Sheet includes several elements that will be briefly described in this paragraph, some whom will be explained more precisely through this chapter:

-- **Market Value of Assets (MVA)** The current market value of all the assets of the company. The market value is the unbiased estimate of market price of assets. It is used to have the company's assets calculated in coherence with the financial markets.

-- **Technical Provision** : A capital reserved used by the insurer to meet the capital requirements of the insurance liabilities. It includes the Best Estimate Liability and the Risk Margin.

 – **Best Estimate Liability**: measures the risk of occurrence of claims and defines the fair amount of reserves necessary to cover it. It is estimated by discounting future cash flows of liabilities. The Best Estimate Liability is the average of expected present value of future cash flows weighted by their probability bases on the risk-free interest rate curve.

 – **Risk Margin** : The Risk Margin represents the cost of immobilizing the capital of the Best Estimate Liability.

-- **Own Funds:** They represent the surplus of capital available when the liabilities are deducted from the assets.

-- **Solvency Capital Requirement (SCR)**: The level of capital that the insurer must immobilize in order to avoid a situation of ruin the following year in 99.5% of cases. The risk measure used is called Value-at-Risk (VaR).
The Minimum Capital Requirement (MCR) is included in the SCR.

-- **Minimum Capital Requirement (MCR)**: The minimum level of capital that the insurer must immobilize otherwise it will be unable to honor its commitments and below which the supervisory authority intervenes systematically to restore the health of the establishment concerned.

-- **Free Surplus**: excess of capital that does not require to be immobilized and can be used freely by the insurance company.

Liabilities are composed of the risk margin and the best estimate liabilities. The sum of these two items is called technical provision. The technical provision risks are assessed either in terms of fair value for ''hedgeable'' risks on the markets or in term of Best Estimate for other risks.

SCR and MCR are calculated from the economic balance sheet of the company in question, either by a Standard Formula proposed by the European Commission in the Solvency II regulation, or by an internal model (complete or partial) developed by the company and approved by the supervisory authorities.

In the case of French companies, subject to the prudential accounting principle, the French balance sheet and the Solvency II Market Value Balance Sheet (MVBS) differ.

Indeed, french accounting caution requires capital gains and losses to be recognized only once realized; as soon as recorded. However the results appear not to be representative enough of the true value of the business.

Companies operating under Solvency II have the obligation to calculate their Solvency Ratio in order to estimate their repayment capacity. Its value must be superior to 100%.

The Solvency Ratio is calculated by dividing the amount of Own Funds by the Solvency Capital Requirement.

Solvency Ratio : $\frac{Own\ Funds}{SCR} > 100\%$

Insurance companies are required to maintain their Solvency Ratio over 100% at all times. If they fall below this level, they would need to inform the regulator and present a realistic recovery plan to bring the Solvency Ratio to 100% over the following six months.

1.2.2 Solvency Capital Requirement (SCR)

Solvency Capital Requirement (SCR), is a required capital insurance companies need to immobilize in order to insure a probability of ruin in the following year of less than 99.5%.

As an example the SCR of our insurance company was valued in 2018 at €33,486,614 according to its Solvency and Financial condition report.

Solvency II allows two different approaches when calculating the SCR; using an internal model or a standard formula. The main difference between the two methods lies in the treatment of interactions between different risks.

The Standard Formula

The Solvency II directive proposes a method for calculating the SCR based on the aggregation of certain risks which insurance companies are exposed to. This Standard Formula aggregates risks using correlation matrices.

In this approach the SCR is determined for each source of risk. The capital requirement is calculated using a stress scenario measuring the impact of a shock on the tested parameter.

The elementary risks are aggregated using correlation matrices to determine the need for capital of the major risk classes (market risk, life / non-life underwriting risk / health...). These capital requirements are themselves aggregated on the same principle, using correlation matrices.

In the Standard formula all quantifiable risks are not explicitly included. In consequence, some risks that are not explicitly included in the standard formula may be relevant to a particular company. In the same way, there may be some specific risks entities belonging to a group that are not covered in the standard formula.

The main risks included in the Standard Formula :

- **Market**: risk arising from the volatility or level of the market prices of financial instruments on the asset side of the company. Market risk exposure is measured by calculating the impact on the company, of the market's prices or rate movements. *

- **Health**: risk related to the health insurance's activity that comes from covered claims or mismanagement of the activity.

- **Default**: risk of loss caused by unexpected default of a counterpart, or deterioration of its credit standing, in the next 12 months. Insurance companies can minimize this risk by using reassurance, derivatives etc.

- **Life**: risk in *assurance vie* business comes from directly covered claims or mismanagement of the activity.

- **Non-life** : risk related to non-life insurance business, is caused from covered claims or poor management of the activity. This risk includes also the risk related to uncertainty about the behavior of the insured clients and the Catastrophe risk (CAT): risk of occurrence of a natural disaster (eg tsunami, cyclone,storm, ...) or of human origin (terrorist attack for example).

- **Intangible**: risk of a decline in the value of intangible assets in the balance sheet, such as for example the brand of the company.

- **Operational**: risk of suffering a loss due to a inaccurate or inappropriate internal process, from an error committed by an agent or a computer system, or from external events. This does not take into account reputational risk (included in the intangible risk) but includes legal risks (non-compliance with the insurance code, for example).

For each of these risks, an SCR is calculated according to a formula specified by the regulation measures, then aggregated according to the tree shown below.
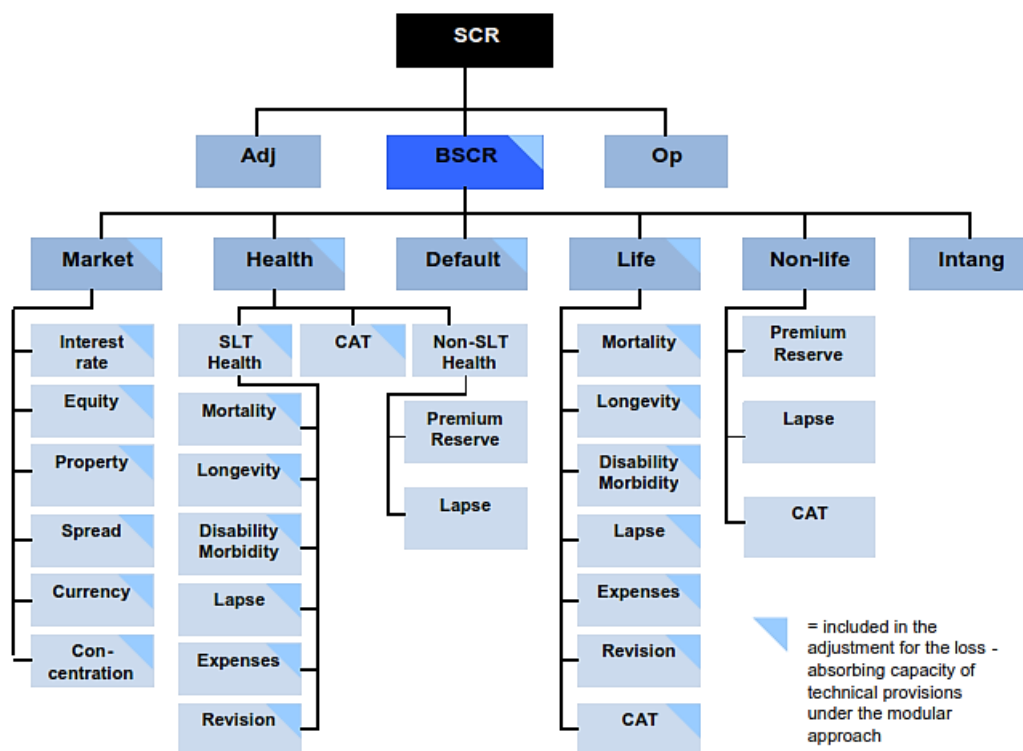
Figure 1.4: The overall structure of the standard formula

Risk Diversification is included in the general structure of the standard formula. When aggregating fund requirements using correlation matrices, the effects of risk diversification are taken into account.

The overall SCR is obtained by aggregating the results of the different modules following linear correlation techniques. The correlation coefficients retained must be representative of potential dependencies in the tail of the distribution and the stability of correlation hypotheses under stress conditions.

Some advantages of the Standard formula are the low necessities for human and financial contribution and a simplified comparison of the solvency of insurance companies but is also has disadvantages like not representing the effective risk profile of the company.

The Internal Model

An internal model is a simulation tool intended to provide projections of results and business activity. It may consist of one or more software programs exploited in a coordinated way.

The internal model approach considers the impact on the risk as a whole and not risk per risk as it is the case in the standard formula. The use of an internal model is subject to the approval of the activity control authorities.

Figure 1.4 : The overall structure of the standard formula. URL: https://service.betterregulation.com/document/191850

The approach of determining economic scenarios can be deterministic or stochastic. Depending on the nature of the economic scenarios, the internal model will be called deterministic or stochastic. A stochastic internal model, although more complex to implement, provides a probabilistic modeling of the result, which is very precise and useful for calculating the SCR.

The risk measure of the model must be calibrated to be equivalent to a VaR (Value at Risk) at the 99.5% confidence level and within one year.
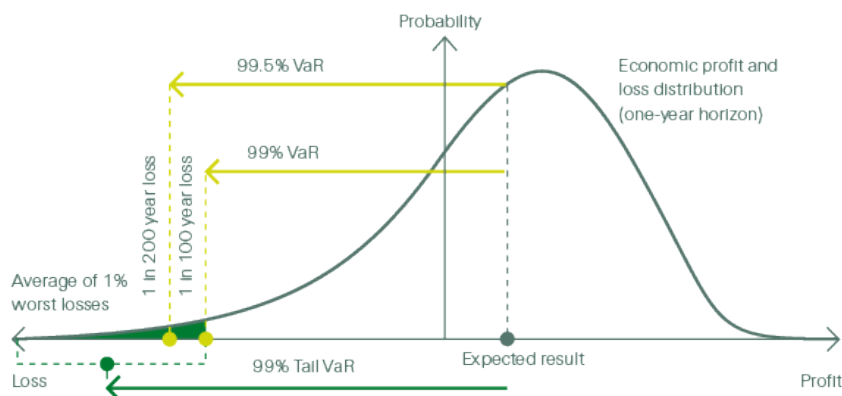

Figure 1.5: Value at Risk

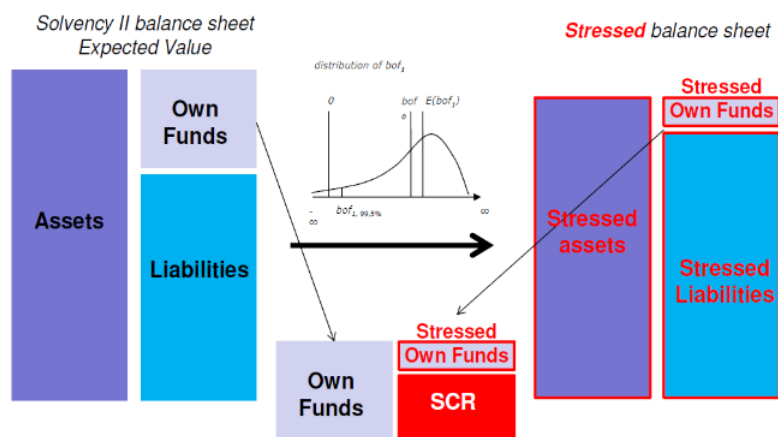The Value at Risk is calibrated by applying shocks to the Own Funds.

Figure 1.6: Solvency Capital Requirement deduction

Our insurance company uses an internal model approved by the regulator, however some entities still calculate risk with the standard formula. This internal model is based on a Value at Risk (VaR) approach using Monte Carlo simulation.

This partial internal model takes into account the interactions between the different risks and can be described as a generator of economic scenarios.

Figure 1.5: Value at Risk. URL: https://reports.swissre.com/2015/financial-report/risk-management/risk-assessment/risk-modelling-and-risk-measures.html

Figure 1.6: Solvency Capital Requirement calibration. URL: https://eiopa.europa.eu/Publications/Standards/EIOPA-14-322_Underlying_Assumptions.pdf

It allows to better take into account the integrated components of the market risk (equities, interest rates, credit, liquidity, etc.) in the hope of having a better measure of the diversification effect.

A fundamental difference between the standard formula and internal model is that the standard formula uses factor-based shock scenarios while the internal model derives the SCR by simulating each risk driver (and its corresponding economic profit and loss impact) based on its assumed distribution and interdependence with other risk drivers.

An internal model permits to define a realistic image of the risk and a precise capital need. It also help have coherent financial indicators on the company's financial situation.

However the calculation time is often time consuming. Our insurance company encounters a calculation run time due to the internal model being time consuming that will be described in section 4.1.

Chapter 2

Asset Liability Management (ALM)

2.1 ALM approach of the insurer

All the aspects of Asset Liability Management will not be described in this thesis, the focus will be made on elements that are relevant for the study carried out.

Asset Liability Management (ALM) ensures the consistency of balance sheet management. Its main goal is to define an optimal investment strategy for each of the company's portfolios while taking into account the characteristics of liabilities.

Asset Liability Management has a long term horizon of study, and is geared towards the future. It uses a realistic approach, when analyzing risks or formulating hypothesis. The risks taken into account are mostly financial.

At our insurance company, the french Asset Liability Management team is part of one of its french entities called Investment Management.
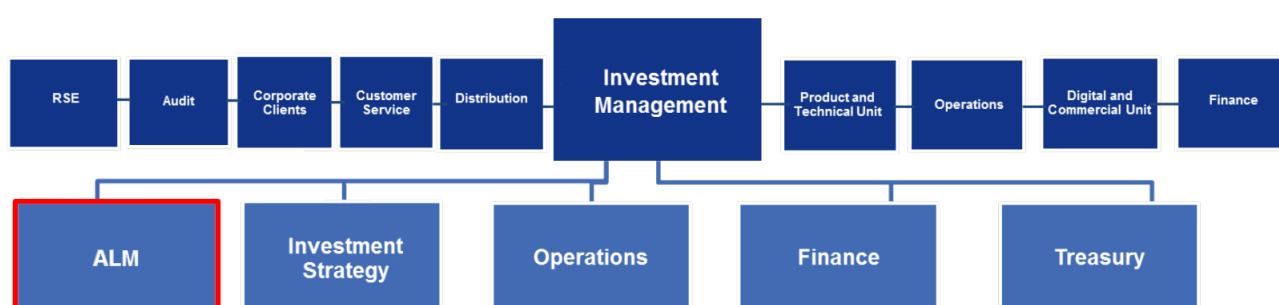

Figure 2.1: Organization chart of Investment Management in our insurance company

The ALM team operates at a company level and is in charge of defining the Strategic Asset Allocation (concept defined later in this chapter) of the company.

In our insurance company the Asset Liability Management team takes into consideration the following risks:

Market Risk	Credit Risk	Liquidity Risk
Interest Rate	Reinsurance	Cash flow hedging
Stocks	Corporate bonds	Assets liquidity
Bounds	Derivative	
Volatility	Other Assets	
Real Estate		
Inflation		
Asset-liability mismatch		

Table 2.1: The risks considered by Asset Liability Management at our insurance company

Certain risks are not taken into account,such as insurance risks (disasters, mortality...) and operational risks (frauds, errors, reputation...).

2.2 Strategic Asset Allocations (SAA)

A Strategic Asset Allocation (SAA) determines for each portfolio the target amount that should be allocated in each category of investments. An optimal SAA should maximize return while minimizing the firms risk and while respecting the solvency II regulation. It should also take into account the asset- liability interactions. The ALM team at our insurance company is in charge of determining a SAA for the following year.

SAA have some leeway and tolerance margins defined when implemented to permit a feasible Actual Asset Allocation (AAA). The ALM teams separates investments in different classes, when defining the SAA.

Fixed Incomes
Treasury and Government Related Securities
Corporate
Cash

Public Equity
Europe
North America
Asia/Pacific
Emerging Markets

Other/Alternatives
Not listed Equity (including participation)
Private Equity
Infrastructure/Renewable
Other

Real Estate
Third party use
Our insurer's use

Table 2.2: SAA asset classes defined by ALM

As an example, our insurance company overall, the 31 December 2017, totaled 86.7% Fixed Incomes, 9.1% Public Equity, 1.7% Real Estate and 2.5% off Other/Alternatives.

2.3   Replicating portfolios

As previously mentioned, our insurance company uses an internal model instead of the Solvency II standard formula. When calculating the SCR with the company's internal model it is the liabilities that take time to model, more precisely the Best Estimate Liability.

Solvency II modelling requires insurance companies to use a method called nested stochastic approach to calculate the Solvency Capital Requirement. This approach is complicated to implement as it requires a large number of simulations that leads to long computation time consistent.

As represented in the following figure this nested stochastic approach requires to test each economic scenario for the following year (for example 1000) in a market consistent way and retest each point obtained for the following years in a risk neutral way (with again 1000 scenarios).

An alternative method to nested stochastic approach is the use of replicating portfolios. It is one of the best-known proxy approach used in the insurance industry and it has been selected by our insurance company's group.
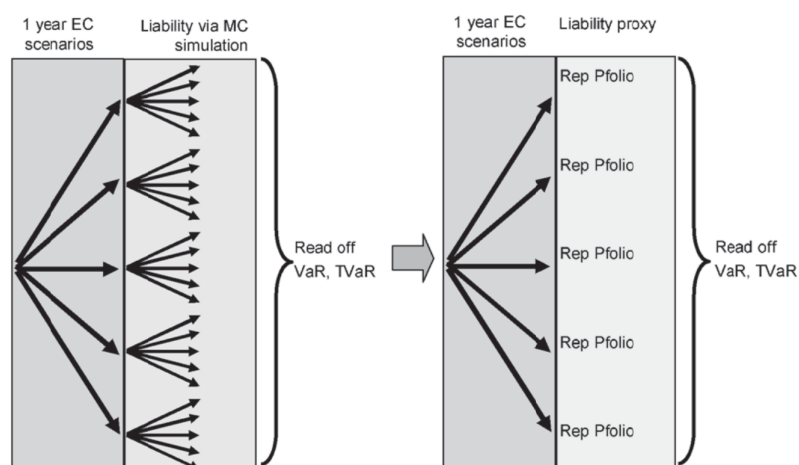

Figure 2.2: Economic Capital calculations without and with replicating portfolio

Replicating portfolios involves finding a portfolio of financial instruments standards (bonds, equities, derivatives such as vanilla options, swaption ...) whose behavior is as close as possible to the behavior of the elements we wish to value. The replicating portfolio method is a an approach that can simplify and speed up the calculation of liabilities.

Market Consistent Valuation of Insurance Liabilities*

Insurance Liabilities
No reliable market prices for exit

Replicating market
Reliable market prices for exit

Transfer the problem of valuing illiquid cash flows to a problem of valuation of Liquid financial instruments

Liability cash flow

Non hedgeable part of the cash flow

Risk Margin

Component of the cash flow that can be replicated by deeply traded financial instruments
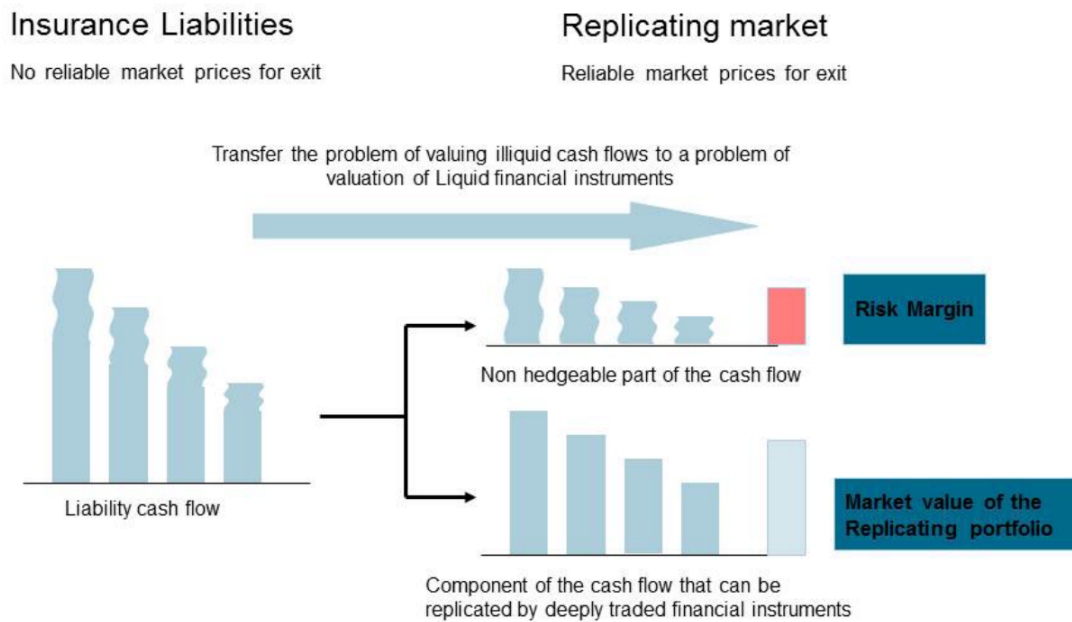
Market value of the Replicating portfolio

Figure 2.3: Diagrammatic representation of the economic valuation approach to insurance liabilities.

2.4 Financial Frame

A Financial frame is an analysis of a company's financial statements and usually the starting point for equity analysis. It is a necessary tool for measuring the variables that interest investors (cash flow, returns,...) and using those metrics to compare multiple investment opportunities.

Asset Liability Management uses the financial frame in their decision making process. One of the role of Asset Liability Management is to make sure the company can come through with its liabilities obligations.

To make sure that these obligations can be met, it helps to have liabilities and assets behaving in a similar manner to interest rate shocks. This can guarantee that the amount of asset will never decrease faster than the amount of liabilities.

Several indicators can help spot a asset-liability mismatch. Duration one of the Key Performance Indicator (KPI) used in SAA determination will be described later along is this chapter. Other indicators such as convexity and Cash Flow Matching

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2.5 Duration

To summarize duration in a synthetic way, it is the ''average actuarial duration'' of a series of cash flows. Duration represents the sensitivity to a change in interest rate, the longer the duration the higher the sensitivity.

As an example the duration of a bond, must not be confused with maturity. Maturity represents the time period until reimbursement of the value of the bond while duration could be viewed as the time period to recover the true cost of the bond.

If a bond is a zero coupon bond it has the same maturity and duration, but they differ when the bond pays some coupons. Since the investor collect his initial investment quicker the duration is shorter.
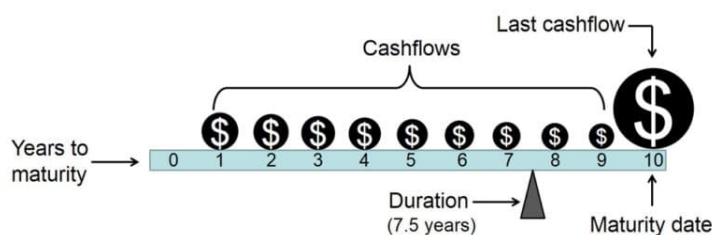

Figure 2.4: Example of the duration of a bond

The Macaulay duration that corresponds to the weighed average time to maturity of a bond can be calculated with the following formula :

$$Duration = \sum_{t=1}^{T} t \frac{C_t e^{-yt}}{\sum_{t=1}^{T} C_t e^{-yt}}$$

T : time to maturity
C_t : the bond's cash flow at time t
y : the continuously compounded bond yield

One of Asset Liability Management's goal is to match duration of assets and liabilities for each of the companies portfolios. This helps immunize a portfolio against the interest rate risk, an prevent for example assets from decreasing more then liabilities.This mismatch between asset and liabilities duration is called the duration gap.

Yet duration has limits an is not a perfect indicator. For instance it does not take into account liquidity.
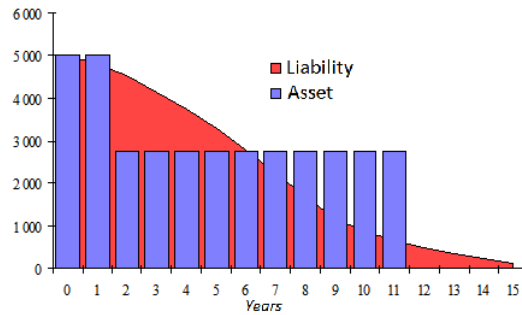

Figure 2.5: Example of portfolio with matching asset-liability duration but no liquidity match

Because of these limits in duration, the Asset Liability Management team at our insurance company does not only rely on this indicator to match assets and liabilities and also perform cash flow matching.

Duration is one of the Key Performance Indicators used by the ALM team to determine a SAA. However cash flow matching and convexity that will be presented in appendix are good indicators to complete duration but are not used as KPIs in SAA determination.

Chapter 3

Asset Liability Interaction Model (ALIM)

3.1   Key Functions of ALIM

Asset Liability Interaction Model (ALIM) is a stochastic asset / liability simulation engine developed on the MoSes platform:

It projects over several years (currently 60 years) the elements of the balance sheet (investments in assets, provisions in liabilities) and of the income statement (financial income, premiums, claims, costs) of an insurance company for one or more given financial scenarios.

The figure below schematically shows the structure of the model:
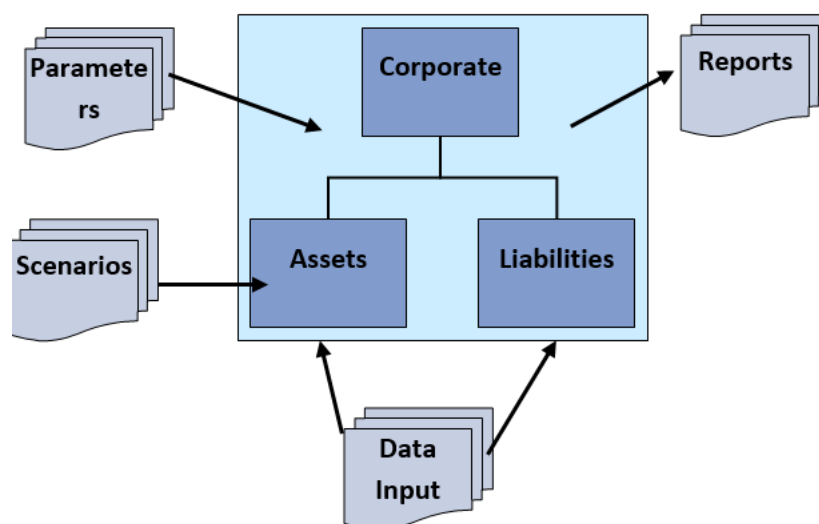
Investments and provisions in the portfolio at the valuation date (data input) are projected by the model in assets and liabilities sub-modules according to financial scenarios.

The projection also requires setting up an asset reinvestment strategy (or investment strategy) and a profit-sharing strategy (or crediting strategy - distribution of part of the technical and financial results to mathematical provisions) which make assets and liabilities interact.

As output, ALIM produces report files which summarize the results at different levels (company; portfolio; product group).

The output results consist of:

- projections of cash flows of liabilities and assets as well as their current values,

- projections of future results and their current values

ALIM is used both for needs related to the calculation of the MCEV and for needs related to the calculation of the SCR.

3.2   ALIM model architecture

ALIM (Asset and Liabilities Interaction Management) is an asset and liability projection tool used for life, health and motor annuities products.

It allows to project future asset and liability cash flows, future Profit & Loss and balance sheets items according to several economic environments. It takes also into account futures management actions (like the investment strategy, asset / liability interactions through the crediting strategy).

ALIM is used for Solvency II purposes: replication portfolio calibration, Life Non Market risk calculation, BEL calculation, etc. The model is also used for ALM, MCEV purposes.

The initial ALIM model has been developed at Group level but all the local enhancements have been developed by the insurer.

ALIM has been approved by the regulator and respects the Solvency II regulation. The focus will be made in this chapter on the aspect of ALIM used to perform Asset Liability Management and relevant to our thesis.

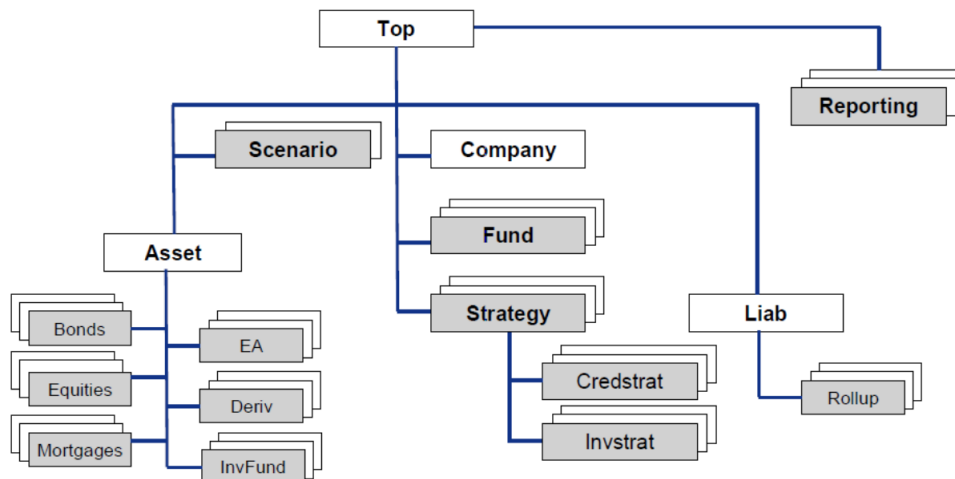ALIM possesses an model architecture presented in the diagram bellow:

Figure 3.1: ALIM Code Architecture

The main models and sub-models are:

- The top model is in charge of setting up the asset and liability interaction structure, initializing assumptions of the different underlying models and controlling the calculation order.

- The Scenario and Scenario Spread model serve as the interface to the scenario files, and associated calculations.

- The Strategy model is the core model and consists of the Credstrat model and the Invstrat model.

  – The Credstrat model performs the crediting strategy at product and company level.

  – The Invstrat model performs the buying and selling of assets according to the investment strategy (strategic asset allocation) defined at fund level.

- The Rollup model provides a well-defined interface for the liability projection at product level.

- The Fund model performs calculations for each fund of the company.

- The Company model performs calculations at company level for reporting purposes (profit and losses account, balance sheet...).

- The Asset model is the top model for all asset sub-models and provides central functionalities that the (asset) sub-models can access.
  The asset sub-models are :

    - Bond model
    - Equity model
    - Derivative model
    - Mortgages model
    - Investment Fund model
    - External Assets model
    - Spreads model

Figure 3.2: Asset classes defined by ALIM

This architecture includes several assumptions including:

- **General Assumptions:**
  mostly linked to the top model, these assumptions include initializing the model structure of assumptions and calculation rules. These assumptions also include rules at a company level and information on funds.

- **Asset assumptions:**
  They combine the modeling of assets and funds but also a table of asset types. The ALIM model divides assets in different categories included in the following table all modeled through different sub-models:

- **Liability assumptions:**
  Includes in particular several Liability modeling assumptions, and a Rollup model used for fast scenario projection.

- **Corporate Assumptions:** Contains information on the Strategic Asset Allocations of the company, and its modelling rules.

- **Scenario assumptions**
  Contains principally an Economic Scenario Generator (ESG) used to project scenarios in different environments and at different time-steps.

3.3 Investment Strategy Model

ALIM performs multiple iterations, each iteration representing a realization of a path in the financial market. Each time an iteration is executed, ALIM follows a specific calculation order.

On the one hand, the Crediting Strategy model reflects management behaviors related to profit sharing between policyholders and shareholders. The model is responsible for the realization of capital gains and the distribution of bonuses reserve. Similarly, the statutory solvency limit and net income before the strategic decision also represents the management scope handled by the Crediting Strategy model.

On the other hand, the Investment Strategy model controls and combines assets by executing purchases and sales according to defined priorities. The model will now be explained in detail.

The investment strategy model is designed to reflect in management actions taken in the practical world for the asset portfolio steering.

Usually, it evaluates assets the re-balancing mechanism (i.e. purchase and sale) required to meet the predetermined requirements for the target asset allocation. The target asset allocation and the re-balancing mechanics in ALIM correspond in the real world. A Change in Strategic Asset Allocation (SAA) is executed by the ALM team in co-operation with the investment management team in the Investment Strategy Department.

SAA is the main long-term driver of investments, therefore, the target allocation in ALIM is the principal objective of the investment strategy model.

Currently, the investment strategy model includes three main purposes:

- Control the development of the asset portfolio to ensure restrictions and limit, and at the same time follow the target asset allocation.

- To executes sales and/or strategic realization according to the defined priority;

- Controls the duration of the asset portfolio.

ALIM input allows precise parameterization of the asset types involved in the asset re-balancing and target allocation. Among other things, performing the purchase and sale and profit realization constraints can also be parameterized.

3.4   ALIM modelization process

The projections under ALIM are carried out at a company level. For each of them, a set of portfolios modelized (or "fund") has been defined. A modeled portfolio corresponds to a grouping of portfolios management or regulatory cantons.
The modeled portfolios are reported in the RAI hierarchy to allow the loading in RAI of the results obtained at the portfolio level.

3.4.1   Order of calculation

The projection step is annual. Before projection, assets and liabilities are re-balanced in ALIM for each portfolio by applying a scale factor to the assets:

- If the assets are greater than the modeled liabilities, all investments are deemed to be reduced proportionately.

- If the asset is less than the modeled liability, the asset is supplemented by a cash position (which will be then invested according to the investment strategy)

For each projection year, the model starts by projecting the assets and liabilities of each portfolio over one year.

Thus it calculates on the asset side in the Asset sub-models the annual flows (coupons, maturities, dividends, rents, interest) and the market and accounting values of the investments according to the financial scenarios.

On the liabilities side (Liability sub-models), it calculates the flows linked to insurance contracts (premiums, claims, costs) and determines the amount of mathematical provisions before profit sharing strategy.

It then applies its investment strategy to each portfolio (Investment strategy sub-model). It compares the actual allocation to the Strategic Asset Allocation and determines for each asset class the amount to buy or to sell.

The asset models are updated accordingly (adjustment of book and market values, addition assets).

In a third step, it determines the amount of profit sharing to be allocated to each product modeled according to financial and technical results, available unrealized capital gains and losses (excluding with the exception of interest rate products) and the provision for profit sharing (Crediting strategy sub-model).

In the event of a plus-and-minus additional unrealized values it updates the asset models. Profit sharing is incorporated into Mathematics Provision at the start of the following year.

Finally, he applies the minimum profit sharing strategy by adjusting the amount of the provision for profit sharing of the general portfolio.

3.4.2 Results

All results are aggregated in the fund by the portfolio sub-model. This model makes it possible to reconstitute a balance sheet and income statement per projection year and calculates the current values of the flows of liabilities and assets, which will be used to calculate the economic capital of each portfolio.

The flows taken into account to value the liabilities are claims, premiums, fees and commissions, arbitrations, retrocessions, CSG, investment management fees, terminal values (at the end of the projection) of stocks allocated to policyholders and, for the CGOS contract, cash injections and profit sharing transfers.

3.5 Best Estimate Liability calculation method

To summarize the rollup methodology, it is used to generate scenario dependent cash flows, by distortion of liabilities cash flow obtained from actuarial models.

The basic principles of the rollup approach are as follows:

- The rollup model is based on a basic approach using the deterministic liability cash flows as inputs. These inputs are based on Best Estimate assumptions and/or shocked assumptions. The deterministic liability cash flows are aggregated according to criteria defined by the Actuarial department. These criteria consist in aggregating policies according to homogeneous technical and financial features .

- Then, the rollup sub-model deforms/distorts those cash flows in order to take into account the impact of the crediting strategy/investment strategy and the different economic scenarios (given by Monte-Carlo trajectories). Furthermore policyholder behavior is also considered as scenario dependent.

When estimating the Best Estimate Liability (BEL), ALIM projects liabilities cash flows in a wild range of economic scenarios in order to determine the market value of liabilities. It uses a deterministic method consisting of deforming cash flows chronicles according to 1000 risk neutral economic scenarios.
This method permits to have a estimation of the BEL in different economic environments but is very time consuming as it needs many simulations.
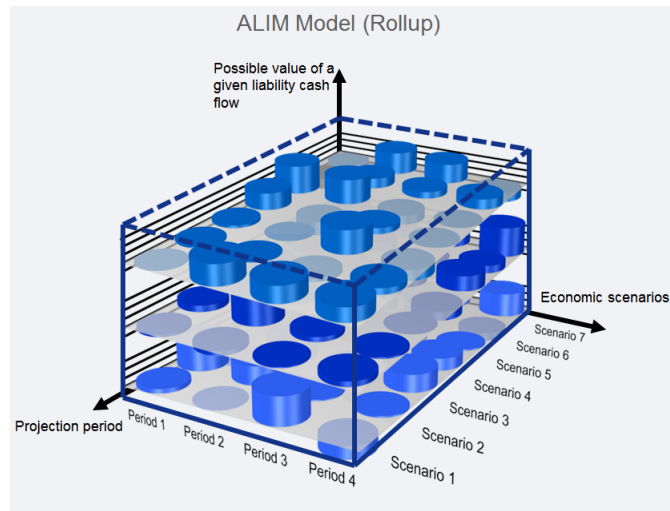
Figure 3.3: Deformation of the deterministic cash flow chronicle according to different economic scenarios

Chapter 4

Run time problematic encountered and solution

4.1 Calculation of the impact of an SAA change on the Own Funds

ALIM is an asset-liability projection tool for *assurance vie* contracts. Its main purpose is to project future cash flows from *assurance vie* products using a stochastic approach as well as tax flows.

These projections are made at the portfolio level. They include the main *euro* and *UC* portfolios of the insurance company.

A strategic asset allocation is entered as input into the model. ALIM estimates the Own Funds in one year. It takes into account the deformation of the net cash flows depending on the economic environment, dynamic redemption, and the links between products.

For a given asset allocation, ALIM projects over 60 years, according to 1000 financial scenarios the balance sheet items (investments in assets, provisions in liabilities) and by difference we obtain the solvency II Own Funds.

In order to determine efficient asset allocations, the insurer's chose the increase in the Own Funds as profitability criterion and the increase in the SCR as its risk criterion.

ALIM estimates the SCR following a change of strategic asset allocation as follows:

1. Recalculates the Own Funds after shock of each risk factor. Calculation of the difference before and after strategic asset allocation change.

2. Aggregation of risks with a correlation matrix.

3. Estimation of the impact of taxation and profit sharing.

This process is time consuming. Calculating the SCR associated with one asset allocation change takes an hour and it takes 3 weeks to test 500 Strategic Asset Allocations.

This time constrain affects the number of Strategic Asset Allocations that are able to be tested by the ALIM model. As a result the Strategic Asset Allocations chosen by the insurer is not optimal and could be more efficient.

This thesis aims to approximate the net cash flows thanks to Machine Learning algorithms. Once the Machine Algorithm is selected and trained it could calculate the new Own Funds very quickly and provide a good pre-selection of the SAA that will be tested by the ALIM model.

The Machine Learning should permit to test thousands of Strategic Asset Allocation points. We will then be able to determine the efficient asset allocations according to the insurer's profitability and risk criterions.

The best performing strategies will then be tested in the ALIM model. These Strategies should also be the ones that perform the best in the ALIM model. It should be the case if the Machine Learning algorithm approximates well the ALIM model.

Machine Learning is a powerful tool that is capable of approximating complex models. We will describe in the next section why Machine Learning was chosen as the method of approximation to replicate ALIM.

4.2   Selection of Machine Learning as a solution

When wanting to decrease the calculation time of the changes in the net cash flows estimation, Machine Learning seamed like an interesting choice. Machine Learning permits to approximate complex algorithms and could be used to approximate these cash flows.

It was one of the methods that was considered and that was selected.

When imagining the other possibilities to reduce calculation time of net cash flows, curve fitting is one of them. The principle of curve fitting is simple, calculate less points or in our cases less economic scenarios and use interpolation to estimate a fitted polynomial curve and the missing points.

A lot of other possibilities could be considered. The choice was made to Machine Learning because of its performances and its innovative aspects and many applications possibilities. More precisely, this problem can be solved with a deep learning method. Deep learning, has brought new tools and performances to Machine Learning and have permit to solve problem never solvable before with classical statistical methods.

Machine Learning can deal with a lot of data. If we imagine as an example solving a regression problem with 100 Millions Variables it is possible via Machine Learning when it would be unimaginable through classical statistics methods like for example, linear regression.

Machine Learning is a new innovative tool, getting as time passes better and better performances. Having Machine Learning implemented in the company is very useful not only for this problematic but to solve many other internal issues. Furthermore once the Machine Learning model is in place, it can always be improved as time passes and as the method improves over time.

It is a evolving technological tool that represents the future and most probably the new norm in insurance companies. Choosing Machine Learning for this problematic is not only selecting a powerful tool but also opening the company to this new technology.

Chapter 5

The Essentials of Machine Learning

5.1 Learning Algorithms

Machine Learning is an artificial intelligence technology that provides computers with the ability to automatically learn and improve from experience without being explicitly programmed for that purpose. The process of Learning is done by analyzing data and training on it.
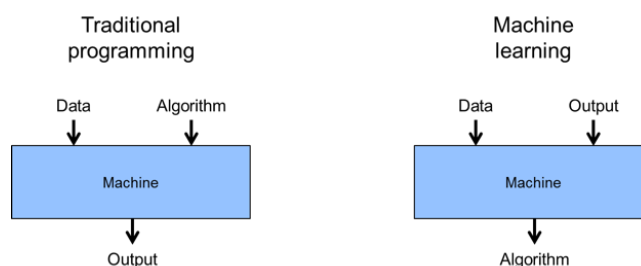

Figure 5.1: Machine Learning principles

Most Machine Learning algorithms are used to find an approximation of the algorithm that links the input data to the output. It is fundamentally different from traditional programming, where an algorithm is applied to the data to find an output.
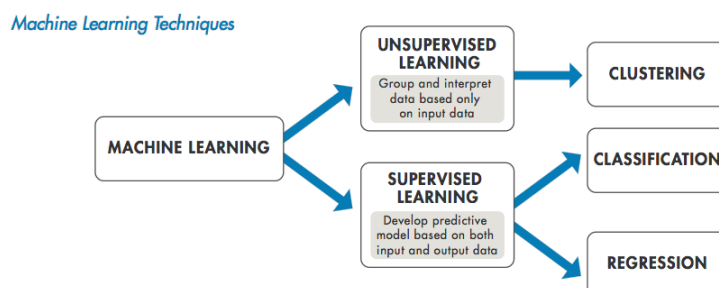

Figure 5.2: Some of Machine Learning Techniques

Supervised Learning is the most commonly used technique in practical Machine Learning. In Supervised learning you have input an output data and you use an algorithm to approximate the mapping function from the input to the output. The goal is to approximate

the mapping function with such little error, that you can now predict the output with the input data.

Unsupervised Learning methods only use the input data with no output values or labels associated. The goal of Unsupervised Learning is to Learn more from the data.

This thesis will focus on regression techniques part of Supervised Learning in order to imitate the way a model finds its values.

5.2   Validation metrics

5.2.1   Splitting Data

The first step of Machine Learning is splitting your data. You start by splitting your dataset in a train and a test set. The train set is used to calibrate the model. The test set will not be touched and will be used at the very end to evaluate our model. The model cannot be evaluated with the data used for training since it would reward models that can ''remember'' the training data.
In our case the 900 first scenarios were in the train set and the 100 left where used in the test set. This choice was made because the 100 last scenarios are a good representation of the 900 first ones.
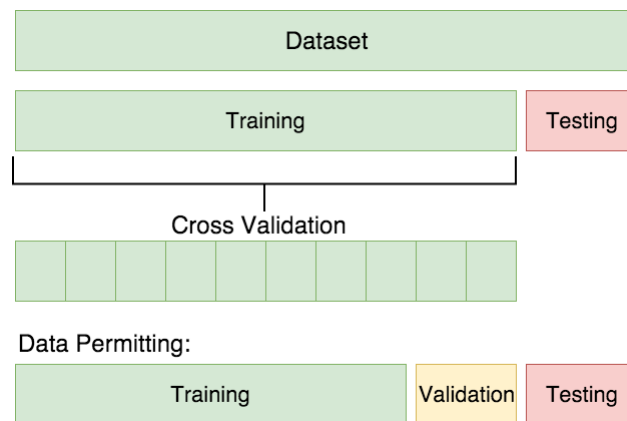

Figure 5.3: Train Test and Validation sets

As observed on the figure above, cross validation is the applied (explained further along in this paper) to our train set. The train set will be separated in a train and a validation subset. The train subset will be used to fit the model and the validation subset to validate it. Once the model is fitted it will be tested on the test set.

5.2.2 Cross-Validation

Cross-validation is used to evaluate Machine Learning models given limited data sample. In our Machine Learning K-Fold cross-validation, a type of cross-validation, was used. The principle of K-Fold cross-validation is simple. First you shuffle your train set randomly. Then you split your train set in k subsets, in our case it was 4 subsets. Select one of the subsets as the validation subsets and the remaining others will be used to train the model. Fit and evaluate your model. Repeat the operation k times, by selecting each time a different subset for validation. Summaries the evaluation of the k models, that will be the overall evaluation of your model.
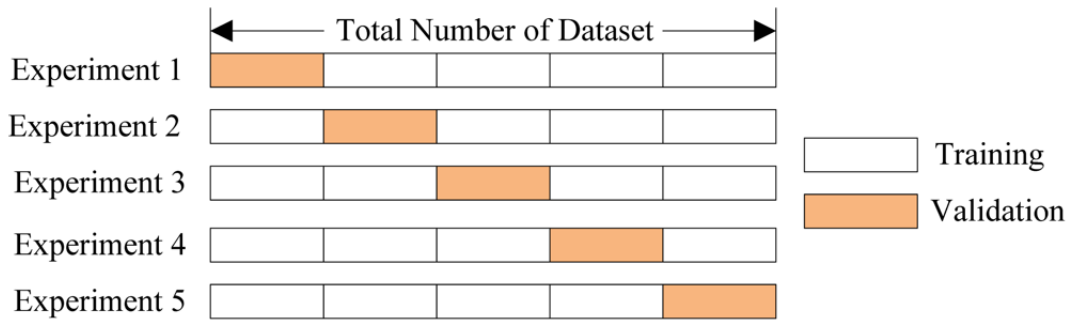
Figure 5.4: The K-Fold Cross-Validation Procedure

5.2.3 Grid Search

Using Grid search is a very effective method when wanting to test hyper-parameters. The principle of grid search is simple, it is to test all the possible combinations of hyper-parameters.

This method permits to limit error but is very time consuming. Therefore the hyper-parameters put to the test with grid search have been carefully selected beforehand and are the most likely to have an impact on the training error.

Figure 5.5: An illustration of Grid Search

5.2.4 Hyper-parameters

A hyper-parameter is a parameter that is not calibrated automatically when training the Machine Learning Model. Hyper-parameters vary depending on each of the different models. They can determine whether or not a model is performing well.

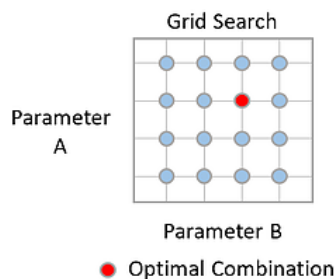They represent a specific element of the machine learning algorithm and understanding their effect can be very useful when selecting the optimal combination of hyper-parameters.

To be able to compare different Machine Learning Algorithms, the first step is to find for each one of the them the optimal combination of algorithms. The training error of these algorithms is then compared. The validation metric selected was the mean squared error. It permitted to calculate the validation error and compare the algorithms.

To selected the best combination of algorithm Grid Search was used. It is one of the classical methods to optimize hyper-parameters.

Chapter 6

Machine learning applied to Own Funds estimation

6.1 Replicating ALIM's calculations with Machine Learning

The ALM team is in charge of estimating the futures net cash flows or changes in the Own Funds.
This machine learning study aims to calibrate a model that would replicate the way ALIM calculates the net cash flows .

As referred to in chapter 10, this type of calculation takes a certain time. This Machine Learning model will permit us to obtain results in a limited amount of time, 5 seconds for one Strategic Asset Allocation.

The focus has been made on one of the insurer biggest assurance vie portfolio. This insurance company has many portfolios that behave differently in terms of liabilities. By reducing the scope, the aim is to obtain a better performing Machine Learning algorithm. Thus, the net cash flows are estimated on this portfolio and not on the insurance company as a whole.

The Machine learning model will take as inputs the data given to ALIM to perform it's calculation and the outputs given by ALIM, once the calculation is finished and it obtained it's results. The inputs of the Machine Learning model will be described more precisely in the next section.

Four different Machine Learning Algorithms where put to the test in order to find the one that gave the best results. These algorithms where part of two different categories and used very different methods that will be explained later on in this thesis.

Testing a variety of different algorithms increases the chance of obtaining better results. Machine Learning being a method that needs to be adapted to the data it aims to replicate, one method cannot be used to solve every Machine Learning problem. However there are Machine Learning algorithms known to perform well in regression and this knowledge did help preselect the algorithms that where tested in this thesis.

Once a Machine algorithm is selected we hope to obtain the closes approximation possible of ALIM results. The aim being not to replace ALIM but to have an idea beforehand of the results it could give in order to limit the number of calculation.

No decision will be made by the ALM team solely based on machine learning. The end goal for it is to increase efficiency.

The next section will describe more precisely the inputs that where used to train the Machine Learning models.

6.2   Dataset Description

This study took as entries the inputs and the outputs of ALIM when calculating the net cash flows and tried to replicate the way ALIM found those outputs.
ALIM is an Asset Liability Interaction Model referred to in section V. Many different Machine Learning methods can be used to obtain these results. Four different Machine Learning methods were tested that will be described further along in chapter 7. ALIM's inputs were aggregated with a python code in order to feed our Machine Learning model data in a form that it could process.

The Machine Learning process was separated into two different phases, Phase 1 and Phase 2:

Differences between PHASE 1 and PHASE 2

- Phase 1: changes in market scenarios

- Phase 2: changes in market scenarios + starting portfolio + Strategies

PHASE 1:

In Phase 1 of our Machine Learning study the focus was made on replicating the way ALIM calculates the net cash flows in general.

Scenario	time-step	Market Indexes
1	0	
1	⋮	
1	60	
2	0	
2	⋮	⋮ ⋮ ⋮
2	60	
3		
⋮	⋮	
1000		

(a) Scenarios Input ALIM

Scenario	time-step	cash flow
1	0	
1	⋮	
1	60	
2	0	
2	⋮	⋮ ⋮ ⋮
2	60	
3		
⋮	⋮	
1000		

(b) Output ALIM

Table 6.1: Machine Learning inputs for PHASE 1

The above figure describes the ALIM inputs and outputs used to calibrate our Machine Learning model. ALIM considers 1000 economic scenarios and studies each scenario over 60 years. It takes as input, for each scenarios and time-step, several market indicators such as the zero coupon bond and gives as output the expected net cash flows.

The Phase 1, was used in the creation of our Machine Learning model to simplify the process and obtain some first results. Our next step was to move to Phase 2 and add the SAA variable to our Machine Learning model.

PHASE 2:

In Phase 2 the goal is to estimate the net cash flows of different SAA in the same way as ALIM. The ALM team often needs to calculate the expected Own Funds for a new SAA and it was interesting for us to add this variable to our Machine Learning model.

Phase 2 is a continuation of Phase 1. It uses the same inputs but adds the variable SAA and the indicators Assets and Asset Allocation. Assets and Asset Allocation where not used in Phase 1 because they depend on the SAA of the portfolio.

SAA	Scenario	time-step	Market Indexes
1	1	0	
1	1	⋮	
1	1	60	
1	2		
1	⋮		⋮ ⋮ ⋮
1	1000	⋮	
2			
⋮	⋮		
N			

(a) Scenarios Input ALIM

SAA	Scenario	time-step	Assets
1	1	0	
1	1	⋮	
1	1	60	
1	2		
1	⋮		⋮ ⋮ ⋮
1	1000	⋮	
2			
⋮	⋮		
N			

(b) Assets Input ALIM

SAA	time-step	Asset Allocation
1	0	
1	⋮	
1	60	⋮ ⋮ ⋮
2		
⋮	⋮	
N		

(c) Asset Allocation Input ALIM

SAA	Scenario	time-step	cash flow
1	1	0	
1	1	⋮	
1	1	60	
1	2		
1	⋮		⋮
1	1000	⋮	
2			
⋮	⋮		
N			

(d) Output ALIM

Table 6.2: Machine Learning inputs for PHASE 2

As observed in the figure above, Phase 2 like Phase 1 takes into account 1000 scenarios over 60 years. However in Phase 2 each scenarios is considered for every different SAA.

The economic indicators Assets and Asset Allocation have been added to Phase 2 in comparison to Phase 1. Asset Allocation values stay constant whatever the scenario and the time-step and only evolve depending on the SAA.
However for more simplicity in the data feed to the machine learning model time-steps was still considered as a variable for Asset Allocation values.

Chapter 7

Multi-target regression models

When it comes to Machine Learning algorithms used for regression, many different choices are available. No perfect algorithm works for every regression problem. Depending on the size and the structure of the data the best performing algorithm might differ.

The correct choice of algorithm is often unclear before their performances are tested. However Machine Learning algorithms possesses characteristics that can lead us to the right choice of algorithm depending on the data.

Four algorithms were tested in total in this thesis, that deliberately use very different methods in order to extend the scope as much as possible.

The two algorithms presented in this chapter are classical machine learning algorithms using methods that make the data explainable. The two other algorithms that will be presented in the next chapter are Neural Network algorithms.

When implementing the first two ensuring algorithms, a machine learning technique was used that includes one model per time-step and is often employed for time series data. This method will be described later along in the next section of this chapter.

The reason for the use of this method for the two algorithms tested is their simplicity and therefore the fact that they where less likely to perform well with this type of complex data. Using one model per type step permits to simplify the data feed to these models and hopefully increase their performance.

This one model per time step model will now be explained in more detail.

7.1 One model for each time-step method

The two previous regression Machine Learning models are not good at predicting time series. This flaw was overcome by using a multi-target regression model that trains a separate underlying model for each time-step.

To use this method the data was arranged in a three dimensional way that includes a time-step axis and structured in the following way: scenarios x time-steps x features.

The underlying time-steps models are not trained on all of the data, witch would then mean the number of features would be inflated by a factor of n time-steps.
In fact each underlying time-steps models, is trained on the portion of the data that is relevant to that step. Input parameters also allow the inclusion of data from previous time-steps as well as predictions from previous time-steps.

This multi-target regression model that defines an underlying estimator for each time-step takes the following arguments:

- The underlying estimator to use, one at each time-step.

- The number of time-steps.

- The number of predictions of previous time-steps to use as input.

- The number of previous time-steps whose inputs will be used for training in the next time-step.

In order to grid search this multi-target model the above augments needed to be tested as well as the hyper-parameters of each underlying models. All the underlying time-steps models are identical and have the same hyper-parameters.

A new type of grid search had to be created specifically for this multi-target regression model. A parameter grid search was run with cross validation over multiple models. This made the grid search for these models very time consuming and effected the different possibilities that could be tested.

7.2   Support Vector Machine - Regression (SVR)

One of the Machine Learning model tested to solve this problem is Support Vector Machine - Regression (SVR). Support Vector Regression is a regression method that uses the same principles, with only a few minor differences, as a well known classification method called Support Vector Machine (SVM) .

This model is very intuitive and fast to implement and was interesting to put to the test as it is commonly used when solving regressions problematics. The following chapter will describe the principles of this model and how it can solve regression problem similar to the one solved in this thesis.

Support Vector Regression finds a flexible tube of minimal radius ε formed symmetrically around an estimated function approximating our data. A penalization is applied to points outside the tube, but points within the tube, either above or below the function, receive no penalty.
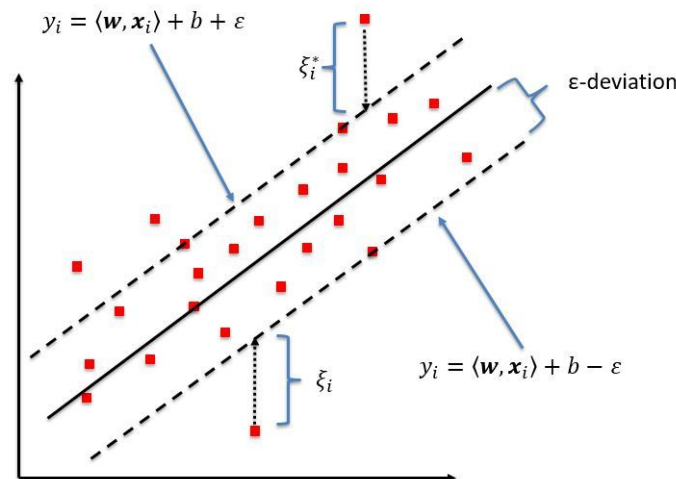

Figure 7.1: Schematic of the one-dimensional support vector regression model

Support Vector Machine most basic approach starts by tackling a problem in a linear way.

Linear SVR is an optimization problem attempting to find the narrowest linear tube explaining our data, that minimizes the prediction error or the distance between the predicted and the desired outputs.

Suppose a given training data $(x_1, y_1), ..., (x_i, y_i) \subset \mathcal{X} \times \mathbb{R}$, where $\mathcal{X}$ denotes the space of the input patterns.

The linear function explaining the data follows this equation:

$f(x) = <wx> + b$ with $w \in \mathcal{X}, b \in \mathbb{R}$.

Figure 7.1 : Schematic of the one-dimensional support vector regression model. URL: `https://www.researchgate.net/figure/Schematic-of-the-one-dimensional-support-vector-regression-SVR-model-Only-the-points_fig5_320916953/`

Formally this can be written as an optimization problem requiring :

minimize : $\frac{1}{2}||w||^2$

While respecting the constraint : $-\varepsilon \leq y_i - wx_i - b \leq \varepsilon$

To allow some errors, a soft margin loss function can be applied, the formula then becomes:

minimize : $\frac{1}{2}||w||^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*)$

While respecting the constraint : $-\varepsilon - \xi_i \leq y_i - wx_i - b \leq \varepsilon + \xi_i^*$ with $\xi_i, \xi_i^* \geq 0$

To make the function $f(x)$ flexible it is possible to change dimensions and find the dimension where the function is linear by using a kernel. A kernel function permits to project the data in a space were the data is linear without needing to look into the transformation.
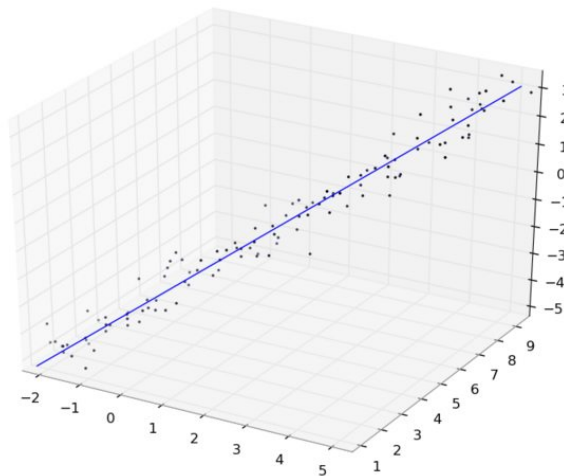

Figure 7.2: Data approximated by a linear function in a 3D space

The standard choices of kernels used with SVM are:

with x_1, x_2 vectors in the input space

Linear Kernel:

$K(x_1, x_2) = x_1 * x_2$

Polynomial Kernel of degree n:

$K(x_1, x_2) = (x_1 * x_2 + 1)^n$

Radial basis function (RBF) Kernel:

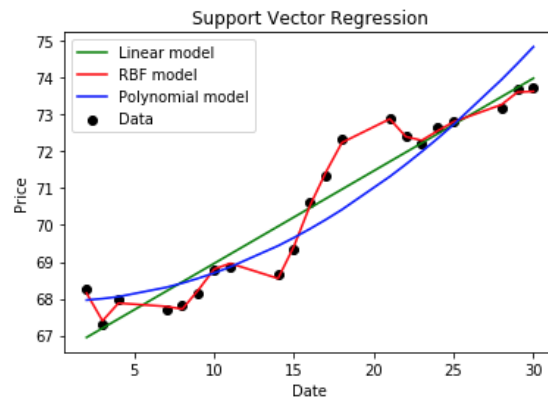$$K(x_1, x_2) = \exp^{\frac{-|x_1 - x_2|^2}{2\sigma^2}}$$

Figure 7.3: Example of one-dimensional regression using linear, polynomial and RBF kernels

Our data being non linear a RBF Kernel was used during the implementation of the SVR model. The results obtained by the SVR model will be described further along in this paper.

7.3   Gradient Boosting Machine (GBM)

() is a Machine Learning model that involves decision trees, very popular in the data science community. It was one of the model tested on our data. To be more specific, the exact algorithm tested was Light Gradient Boosting Machine (Light GBM) a type of Gradient Boosting.

The principle of a tree is to use a set of criteria to partition data. The data is the split into two nodes, with the objective of minimizing impurity. A node can have respectively zero or two ''children''. For regression trees the average of each leaf is taken and impurity is the variance of the node.
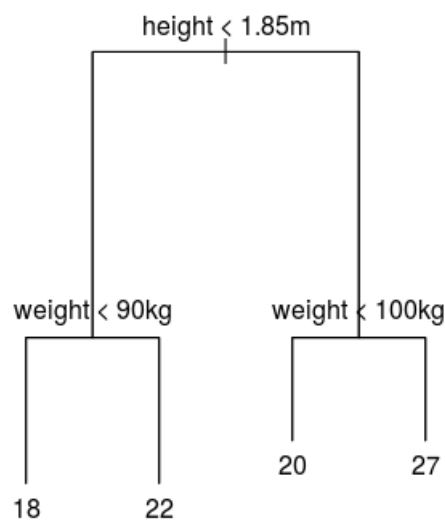

Figure 7.4: Example of regression tree

To understand Gradient Boosting, the first step is to apprehend Boosting.

Boosting is a method that combines weak learners into a strong one, in our case week regression trees. Each new tree, or new weak learner, is fitted on a modified version of the original data set. A weak learner gives sightly better results than choosing randomly.

Figure 7.4 : example of regression tree. URL: `https://www.r-bloggers.com/how-random-forests-improve-simple-regression-trees/`

Figure 7.5: Principles of Gradient Boosting

The principle of Gradient Boosting is to select the week learners used for boosting that improve the current fit and combine them in a meaningful way. The focus is made on the sample points that are not well fitted. The shortcomings of weak learners are identified by applying gradient descent to the loss function.

The general Gradient Boosting algorithm for regression step by step:

Start with some given data containing features x_i and labels y_i

Consider a tree that partitions the space of all joint predictor variable values into disjoint regions $R_{i,j} = 1, 2, ..., J$ represented by the terminal nodes of the tree.

A constant γ_j is assigned to each region, with the following predictive rule:

$$x \in R_j \Rightarrow f(x) = \gamma_j.$$

The Loss function is defined as L.

Figure 7.5: Principle of Gradient Boosting. URL: https://towardsdatascience.com/introduction-to-gradient-boosting-on-decision-trees-with-catboost-d511a9ccbd14

1. Initialize $f_0(x) = argmin_\gamma \sum_{i=1}^{N} L(y_i, \gamma)$

2. For $m = 1$ to M

   (a) For $i = 1, 2, ..., N$ compute

   $r_{im} = -\left[\frac{\partial L(y_i, f(x_i))}{\partial f(x_i)}\right]_{f=f_m-1}$

   (b) Fit a regression tree to the targets r_{im} giving terminal regions $R_{jm}, j = 1, 2, ..., J_m$.   (c) For $j = 1, 2, ..., J_m$ compute

   $\gamma_{jm} = argmin_\gamma \sum_{x_i \in R_{jm}} L(y_i, f_{m-1}(x_i) + \gamma)$

   (d) Update $f_m(x) = f_{m-1}(x) + \sum_{j=1}^{J_m} \gamma_{jm} I(x \in R_{jm}))$

Output $\hat{f}_x = f_M(x)$

The following figure compares a single decision tree regressor with an boosted algorithm combining 300 decision trees:
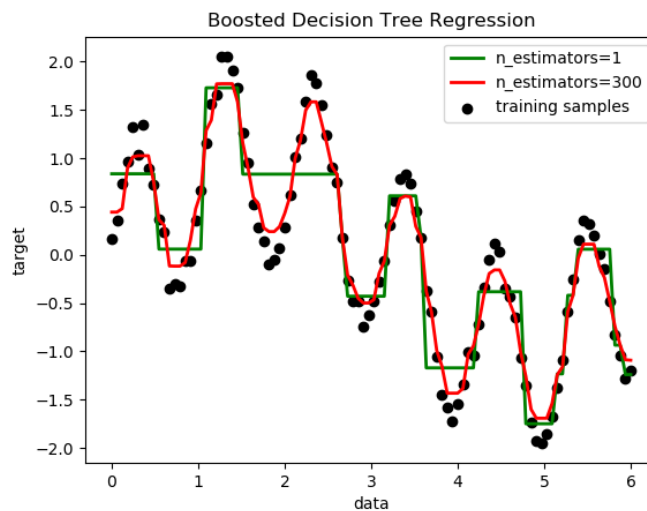

Figure 7.6: Boosted Decision Tree Regression

The Gradient Boosting algorithm considered in this thesis is Light Gradient Boosting. This particular algorithm uses a type of tree that grows Leaf-wise.
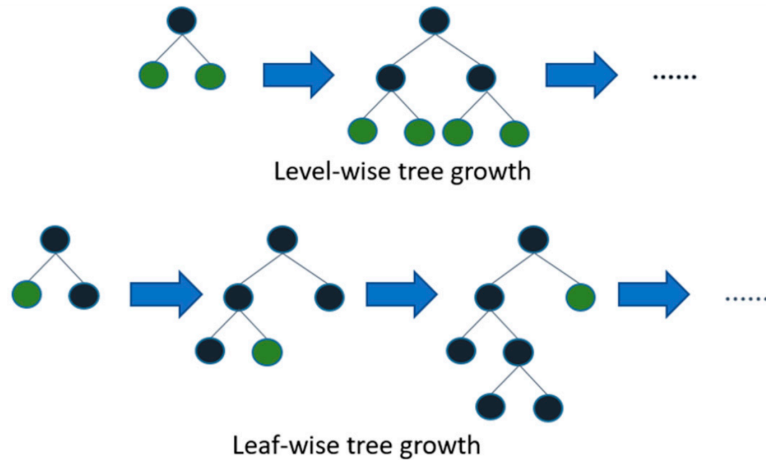

Figure 7.7: Leaf-wise and Level-wise Tree Growth

Chapter 8

Neural Network models

Neural Network model are a new type of model used for Machine Learning. They are part of what is called deep learning, a name in reference to deep neural networks.

A deep neural network is a neural network with more than one layer. This concept will be explained further in Figure 8.1.

Neural network have a particularity, they are black boxes. The way they obtain the data is not explainable. They follow no mathematical formula or rule defined by a human mind.

Neural network train themselves and adjust their parameters in a way that cannot be predicted. They adjust their parameters in order to obtain the best results, but two similar neural networks trained on the same data could adjust their parameters differently and both obtain acceptable results.

They perform well with complex data and have revolutionised several parts of our every day lives already. They perform well particularly in image recognition and every problem for with a lot of data is available.

Two types of neural networks will be presented in this chapter. A Feed Forwards Neural Network witch is the first type of neural network to be used in machine learning and Long Short Term Memory a Neural Network that drive from the first one but is more complexes and includes remembering previous information.

8.1 Feed-forward Neural Network

Feedforward Neural Network (FFNN) was one of the two Deep Learning models tested with LSTM described later along. Deep Learning is a subfield of machine learning that groups together algorithms using artificial neural networks that are inspired by human neurons.
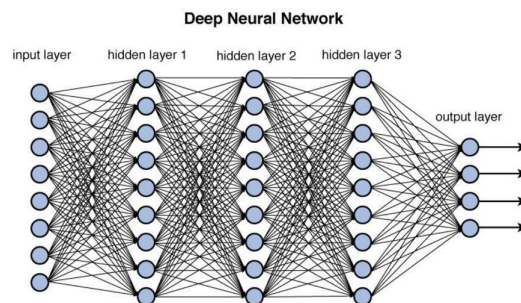

Figure 8.1: Deep network architecture with multiple layers

The architecture of a Feed-forward Neural Network is composed of a input layer of neurons, at least one hidden layer and an input layer. All the neurons of a layer are connected to all the neurons in the next layer. A deep neural network is a neural network that has more that one hidden layer. See the above figure.
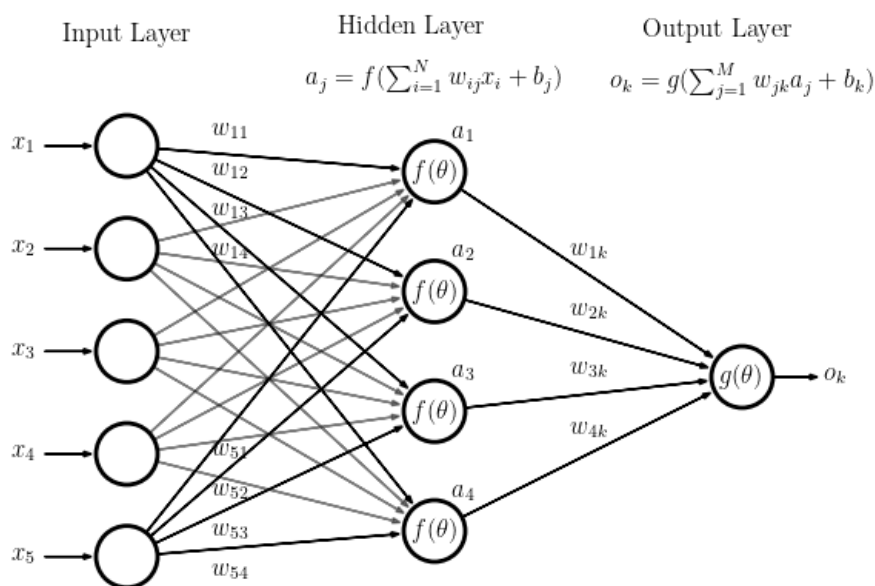

Figure 8.2: The Mathematics of Neural Networks

The above figure represents the mathematics involved with a simple neural network with one hidden layer. Every connection between neurons carries a value that is called a weight. Every neuron that is not on the input layer has a bias attached to it, and the bias, just like the weight, carries a value. Activation functions are used to transform the input value of an artificial neuron into a predictable form.

Figure 8.1 : Deep network architecture with multiple layers. URL: https://towardsdatascience.com/training-deep-neural-networks-9fdb1964b964

Figure 8.2: The Mathematics of Neural Networks. URL: http://www.datastuff.tech/machine-learning/convolutional-neural-networks-an-introduction-tensorflow-eager/

In the above figure weights are represented by w, the bias by b and the activation functions by f or g. The inputs given to the network are represented by x and the output by o.
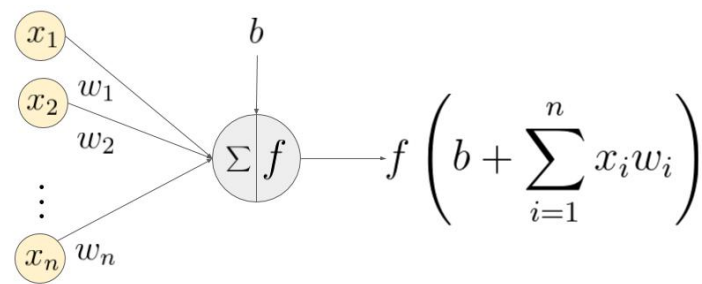
An example of a neuron showing the input (x_1 - x_n), their corresponding weights (w_1 - w_n), a bias (b) and the activation function f applied to the weighted sum of the inputs.

Figure 8.3: Artificial Neuron diagram

In the above figure representing an artificial neuron, the input vector $(x_1, \cdots, x_n)$ is multiplied with the weights $(w_1, w_2, \cdots, w_n)$. A bias b is then added. Finally, an activation function f is applied to this sum.

- f = activation function

- b = bias

- x = input to the neuron

- w = weights

- n = the number of inputs from the incoming layer
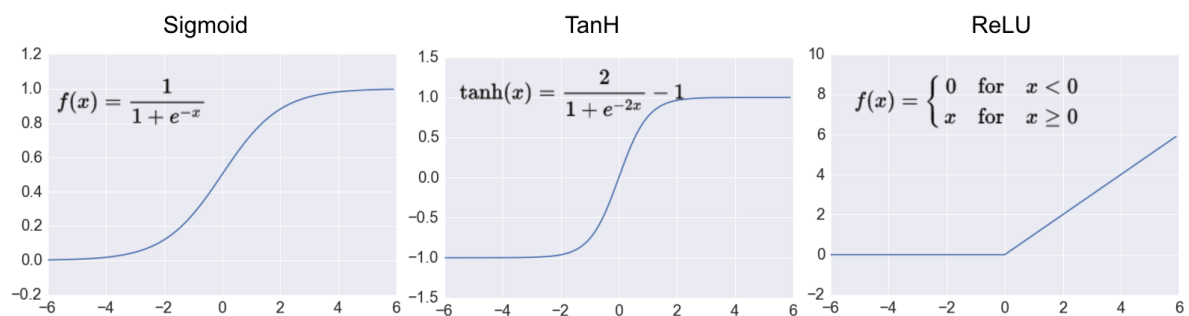
- i = a counter from 0 to n

Figure 8.4: Activation Functions

An activation function transforms the weighted sum of inputs that goes into the artificial neurons. They are used to give inputs a more predictable form. The most popular activation functions are ReLU, Sigmoid, and Tanh. ReLU is the most popular activation function in deep neural networks.

Figure 8.3 : Artificial Neuron diagram. URL: https://www.learnopencv.com/understanding-activation-functions-in-deep-learning/

Figure 8.4 : Activation Functions. URL: http://adilmoujahid.com/posts/2016/06/introduction-deep-learning-python-caffe/
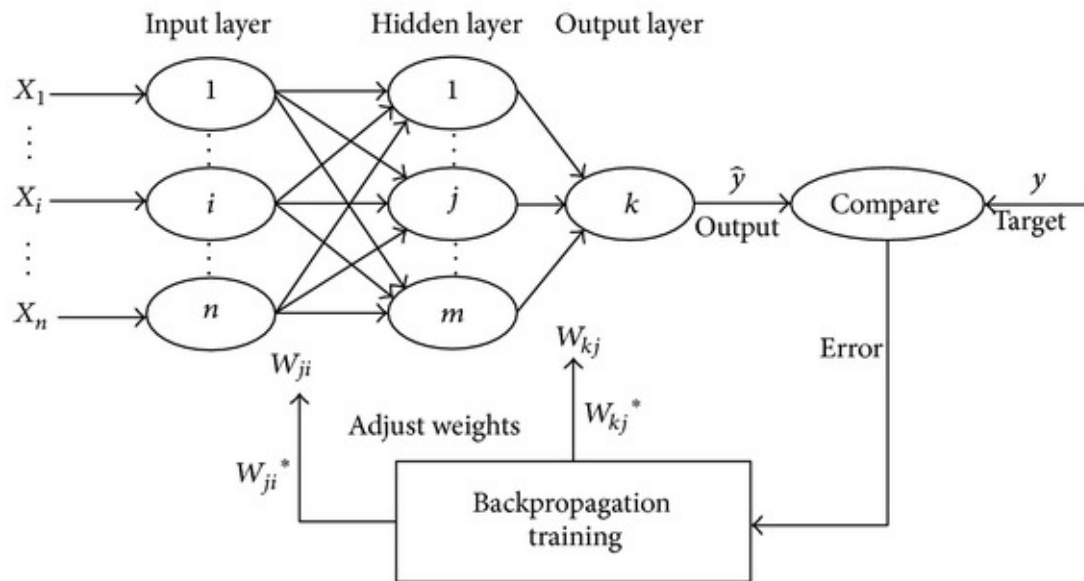
Figure 8.5: A three-layered FFNN with a backpropagation training algorithm.

- To train a Feed-forward neural network the first step is to apply randomly weights and biases to the neural network.

- Then the outputs are calculated with the calculation method explained previously in a process called forward propagation.

- The results given by the network are then compared with the target.

- Gradient decent is then used to minimise the cost function of the error. For more details on gradient decent refer to the appendixes.

- The weights of the network are then updated and forward propagation is performed again to recalculate new outputs.

- This process is redone a defined amount of time in order to train the network and have the closest results possible to the target.

8.2   Long Short-Term Memory

Long Short-Term Memory (LSTM) unlike Feedforward Neural Network is capable of remembering previous information and can face complicated tasks. LSTM is often used for task using language where some previous information needs to be remembered.The problematic needed to be resolved includes different time-steps and LSTM seemed like an obvious choice of algorithm to put to the test.

To apprehend LSTM, Feedforward Neural Networks need to be understood as they use some similar principles. These information will not be explained in his section as they were already explained in the previous section 8.1.

LSTM is a type of Recurrent Neural Network (RNN). To explain the principle that use Recurrent Neural Networks , they can be viewed as multiple copies of the same neural network passing information to their successor. This principle is illustrated in the following figure.
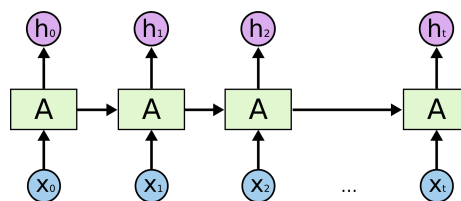
Figure 8.6: A Recurrent Neural Network

In the above diagram, A represents the neural network's hidden layer, x_t represents the inputs and h_t the outputs.

LSTM is a type of Recurrent Neural Network that uses a sophisticated system that selects the information to be kept and the one forgotten. LSTM are capable of learning long-term dependencies.

When comparing a classical Recurrent Neural Network with a LSTM, it can be observed that the first one has only one hidden layer while the second one has four interacting in a special way.

Recurrent Neural Networks structure containing a single tanh layer:

Figure 8.7: A Recurrent Neural Network module structure

While the LSTM structure can be represented in the following way:
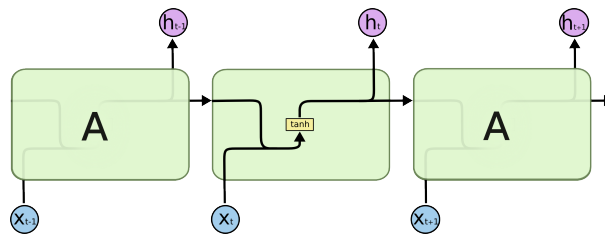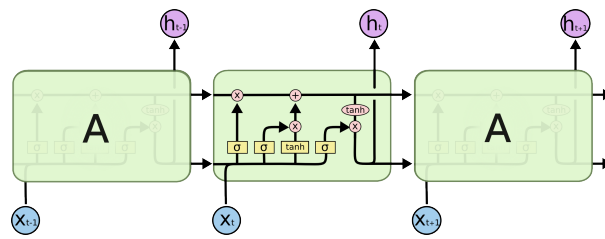

Figure 8.8: A Long Short Term Memory module structure
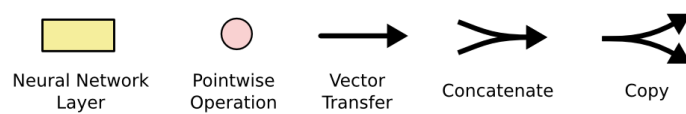
Using the following notation:

Figure 8.9

The above diagram repenting the LSTM structure will now be descried step by step.

The horizontal line running through the top of the diagram is called the cell state C_t :
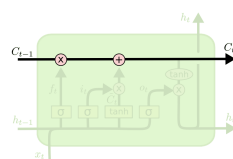
Figure 8.10

The first step of LSTM is to decide what information will be deleted from the cell state. Decision are made by a neural layer taking as input h_{t-1} and x_t to which is applied a sigmoid function σ to give a result between 0 and 1 for each number in the cell state C_{t-1}. 0 represents completely delete and one keep. The sigmoid function is defined earlier along in section 8.1.

$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] + b_f\right)$$
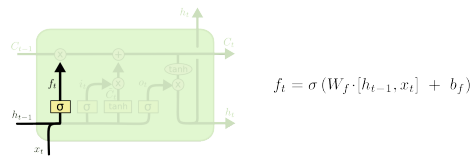
Figure 8.11

The next step is to decide what new information is going to be added to the cell state. There are two parts. First, a sigmoid function defines the value that will be updated. Next, a neural layer to with is applied the tanh function creates a new value $\tilde{C}_t$, that could be added to the state. The both are combined to create and update the cell state.

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] + b_i\right)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$
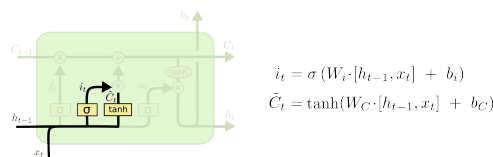
Figure 8.12

The old cell state C_{t-1} is now updated into the new cell state C_t. C_{t-1} is multiplied by ft to forget what was decided to forget in the first step. To this is then added $\tilde{C}_t$ to obtain the new cell state C_t .

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$
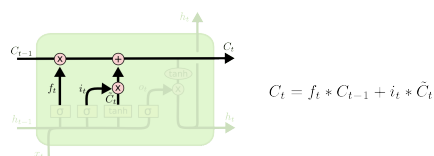
Figure 8.13

Then will be decided what is going to be output. This output will a filtered version of the cell state. First, a sigmoid layer decides what parts of the cell state is going to be output. Then, the cell state is put through a tanh layer to give results between -1 and 1 and multiplied by the outputs of the sigmoid gate to be filtered.

$$o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$
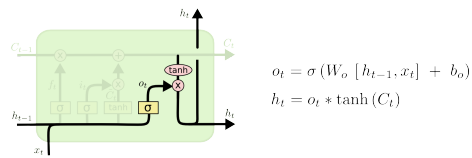
Figure 8.14

Chapter 9

Selection of the Machine learning Algorithm, limits and results

9.1 Machine Learning Strategy

9.1.1 Hyper-parameters

As explained in subsection 5.2.4 a hyper-parameter is a parameter that needs to be selected when training the Machine Learning Model it is not calibrated automatically. Hyper-parameters vary depending on model calibrated.

To be able to compare different Machine Learning Algorithms, the first step is to find for each one of the them the optimal combination of hyper-parameters. Grid Search was used to find this combination.

For each Machine Learning Models sixteen different hyper-parameters where tested.

When it comes to SVM and LGMB who use a different type of grid search as described in section 7.1 their hyper-parameters did not only include the ones associated with there Machine Learning model but also some associated with the grid search method. Such as the number of predictions of previous time-steps to use as input and the number of predictions of previous time-steps to use as input.

Eventually LSTM was selected as the best performing model and will be described further along.

The hyper-parameters involved with LSTM that where tested when performing grid search will now be analyzed :

- Learning rate : The leaning rate controls the amount of change the model will undertake in response to the estimated error each time the model weights are updated.

- Learning rate decay : The rate of decrease of the learning rate over time.

- batch size : The batch size defines the number of training data-set samples processed before the model is updated. They are part of mini-batch gradient descent, a concept described in the appendixes.

- epoch: The number of epochs, defines the number times that the learning algorithm will work through the entire training data-set.

- KFold cross validation, with four splits was used to validate the model in the grid search.

Three numbers of epochs were tested, 100 200 and 300. For each grid search combinations of hyper parameters that used 100 epochs, had a run time of 10 to 20 minutes per cross validation split depending on the parameters. This corresponds to a run time of 40 to 80 minutes per combination of hyper-parameters. This run time is multiplied by two or by three when the number of epochs is 200 or 300 respectively.

After analysis of the grid search results, it was noticed that the higher the parameter "epoch" the better the result. 300 epochs gave better results than 200 and much better than 100, but the differences were not huge in terms of results.

This brings to the conclusion that grid searching "epochs" in this case was not a good idea considering the number of time that it takes. It can be a good things to do to avoid under or over fitting but given the results when grid search will need to be performed again in case of model change the hyper-parameter "epoch" will be fixed to 300 and not grid searched.

The batch size parameter was the one that played the smallest role when improving the results. No clear tendency could be obtained when comparing the 3 different level of batch size 64,128 and 256.

Three Learning rates where tested 0.01, 0.001 and 0.0001. Learning rates was one of the hyper-parameters that had the most influence on the quality of the result. 0.0001 was very clearly a learning rate that was too low and that created over-fitting. 0.001 was part of the optimal results, but the learning rate 0.01 also provided pretty good results.

In terms of learning decay 0.0001 and 0.001 was tested. I was noticed that if overall 0.0001 gave the best results, it was ultimately the combination of a the 0.0001 decay with the 0.001 learning rate or the 0.001 decay with the 0.01 learning rate that gave the better results.

This phenomenon can be explained by the fact that a smaller decay decreases less the learning rate over time, which is less needed if the learning rate is small and vice versa.

9.1.2 Grid search results and model selection

Grid Search permits to select the best hyper-parameters for a given Model as explained in subsection 5.2.3. In our case Grid Search will need to be performed each time a model change occurs in ALIM and the training data is modified.

Grid search was performed on every machine learning algorithm to select the best hyper-parameters. We then compared these algorithms with optimal parameters to select the one that had the best scores.

While performing the grid search on the model SVM and LGBM that as described earlier in section 7.1 was created specifically for these multi-target regression models. It runs with cross validation over multiple models.

This type of grid search is very time consuming. How ever it was so long and took so many resource's that the grid search had to be stopped before the end. Therefore the parameters selected for these two models where based on the partial results obtained from the grid search.

The decision was made not too select these two algorithms. They did not obtain the best results, which could be due to the fact that we could not test as much hyper-parameters for these models. But an other reason not to select these algorithms was that since grid search will need to be performed each time the machine learning model will be updated this time constraint could be a problem in the future.

After applying Grid Search to all four of the algorithms put to the test and analyzing the results, Long Short Term Memory was selected as the best performing algorithm. It was the algorithm that obtained the best score of -0,002 using the metric Mean Squared Error.

However even if FFNN was not selected it was still kept to make a comparison on the out of sample data. This comparison, that will be described in the next chapter, strongly reinforced the choice of LSTM as the model to use as the results were highly superior.

LSTM is a Recurrent Neural Network model that is commonly used in time series. Its main characteristic is that it ''remembers'' values from previous time intervals. LSTM obtain the best score. The success of LSTM could be predicted as the data depends a lot on previous time-steps and LSTM is known to perform well on data with time dependencies.

9.2 Limits of the algorithm selected

Several limits can be pointed out when looking at the Machine Learning Algorithm selection process. The first limitation was the time necessary to perform Grid Search. The concept of Grid Search is defined earlier in subsection 5.2.3. The different types of algorithms didn't have the same type of Grid Search.

Feed Forward Neural Networks (FFNN) and Long Short Term Memory (LSTM) used a classical Grid Search method while Support Vector Machine (SVM) and Light Gradient Boosting Machine used a method of one model per time-step defined in section 7.1 that necessitated a different type of grid search that had to be remade from scratch.

Therefore testing hyper-parameters for these two algorithms was more time consuming and less possibilities could be tried out. As a consequence while LSTM was selected, it could be due to the fact that the hyper-parameters where not optimal for these two other algorithms.

The second limitation is the error metric. Mean Squared Error was selected to measure the errors. Each metrics have limitation and while Mean Squared Error is good to penalize large errors it is less intuitive, from an interpretation standpoint than for example the mean absolute error.

The third limit is the amount of data to train the model. Machine Learning always performs better with more data. Having a limited amount of data obtained to train the model, the performance of the algorithm was limited. The amount of data was important but can still be considered a limitation to Machine Learning results.

The last limit to this choice of algorithm that will be pointed out is the impact of a change in the original training data. If the Asset Liability Interaction Model (defined insection V) modelization method is changed, then the data used for training will be different. Therefore the algorithm selected could maybe not be the optimal algorithm anymore.

However the selection process will not be reiterated as LSTM showed by far the best results and it is likely that it would remain the best performing algorithm. It would also be very time consuming and even counterproductive to repeat this selection process for every model change.

It can also be noted that, if the selection process will not be repeated, grid search will still have to be performed for every model change to find the best hyper-parameters.

9.3 Results

The machine learning model was tested on four different Strategic Asset allocation. One of the strategies did not differ from the company's actual strategy. The data used was new out of sample data that had never been seen by the machine learning model before.

Stategic Asset Allocation change %					LSTM		
Derivatives	Equity	Alternative Investments	Real Estate	ALIM Output	Machine Learning Output	abs-error	%-error
0%	0%	0%	0%	5 268 562 513	5 266 663 526	1 898 987	-0,04%
0%	0%	+0,5%	0%	5 235 223 005	5 204 984 187	30 238 819	-0,58%
0%	0%	0%	+1%	5 238 179 800	5 226 166 666	12 013 135	-0,23%
0%	+0,5%	0%	0%	5 249 361 096	5 244 707 632	4 653 465	-0,09%

Figure 9.1: LSTM: Machine Learning results compared to ALIM

The LSTM Machine Learning model obtained acceptable results. It was noted that the model had a tendency to overestimate the Own Funds from +0.04% to +0.58% depending on the strategy. The best results where obtained when the strategic asset allocation did not differ giving a error of estimation of only 0.04%.

The results obtained with LSTM were overall satisfying. The algorithm can reasonably be used for asset allocations not too far removed from the original allocation. We do not recommend using it for very different allocations.

Although LSTM was selected with grid search as the best performing model, FFNN was put to the test and its results where analysed.

Stategic Asset Allocation change %					FFNN		
Derivatives	Equity	Alternative Investments	Real Estate	ALIM Output	Machine Learning Output	abs-error	%-error
0%	0%	0%	0%	5 268 562 513	5801281375	532 718 862	10,1%
0%	0%	+0,5%	0%	5 235 223 005	5770642445	535 419 440	10,2%
0%	0%	0%	+1%	5 238 179 800	5776557019	538 377 219	10,3%
0%	+0,5%	0%	0%	5 249 361 096	5778989777	529 628 681	10,1%

Figure 9.2: FFNN: Machine Learning results compared to ALIM

The FFNN model has a very high error overestimating the own funds by around 10% no matter what strategy is tested. These result confirm the choice of LSTM as the best performing algorithm.

The difference between the results of the FFNN model and the LSTM model was not as major when analysing the results on the training data but this new out of sample data revealed how much of a difference of performance these two models have.

More strategies will need to be tested, but the results of the model look promising.

The errors rate obtained with this model was acceptable in terms of results. The tolerated errors was not large as it could, even with a little percentage represent millions of dollars of the company's funds.

The results are obtained 300 times faster than with ALIM and could represent a big time gain for the company. The results for one Strategic Asset Allocation are obtained in 5 seconds compared to one hour previously.

The run time of the model is incredibly more quick and permits to test new possibilities never tried before. However it should not be forgotten that this run time does not include the training and grid search time used to calibrate the model.

Chapter 10

Asset Allocation Optimization with Machine Learning

The ALM team of our insurance company uses an approximation method to calculate the risk or Solvency Capital Requirement impacts of a SAA change with a method that has similarities to the standard formula.

Each standalone risk is calculated from a proportional ratio obtained by difference between the sensitivity before and after SAA change. A simplified correlation matrix is applied to aggregate the standalone risks. The tax relief impact is then applied.

This method requires in particular to calculate the net cash flows that derives from this SAA change. The method to estimate the change in net cash flows used by ALIM is explained previously in section 4.1.

Estimating the net cash flow that constitute the Own Funds is the part of the calculation that is time consuming. However if we approximate the net cash flows thanks to Machine Learning, it is therefore possible to estimate an approximated SCR with the machine learning process we described earlier in this thesis.

This SCR was associated to the risk of a SAA and therefore was used in SAA selection. In our case the risk metric chosen is the increase in the SCR.

This possible application of our Machine Learning Algorithm will be described further along as well as the critics that can be made about selecting a SAA by using the SCR as a risk metric.

10.1 Increased efficiency of the SAA selection processs

This project allows us, thanks to the Machine Learning tool to have a better run time when calculating the risk associated with a Strategic Asset Allocation change. The return associated with a new SAA being very quick to obtain.

By reducing this run time more Strategic Asset Allocations can be tested by the Asset Liability Management team. This Machine Learning project can therefore help improve efficiency in SAA selection.

Thanks to this Machine Learning, testing 500 Strategic Asset Allocations witch would have taken more than half a year can now be calculated in one hour.

The end goal being to test thousands of Strategic Asset Allocations and to obtain an efficient frontier of asset allocations. The Strategic Asset Allocation then would be compared by their return (increase in the Own Funds) and their risk (increase in the SCR) criterion.

Finding a more optimal Strategic Asset Allocation can have a huge impact on a company. It can potentially improve its return while reducing its risk at the same time. It is a tool that can be very useful to improve the own funds, especially in a low interest rate environment.

In the case or insurance company, if Machine Learning will be used to test more SAA, these results being approximations of the Asset Liability Interaction Model will not be taken as such. The best performing strategies in Machine Learning will then need to be tested in the model in order to obtain the actual result of the model.

The strategic asset allocation tested by the insurance company, need to be coherent with regulation and internal procedures. Every possible strategy that could be considered will not be tested for this reason.

An example of Strategic Asset Allocations obtained by the insurer will be presented in the following chapter.

10.2 Some early results

To illustrate the Machine Learning tool,an example of Strategic Asset Allocation change obtained via Machine Learning will be presented. In this example changes where applied to the actual Strategic Asset Allocation of the insurance company.

These changes affect derivatives, equity, alternative investments and real estate but it is possible for the insurance company to use machine Learning to test many other changes in other categories of investments.

The number of strategy in this example is not representative of the number of strategies necessary to obtain an efficient frontier of SAA. It only represent a small portion of the number of strategies needed for selection.

Strategic Asset Allocation change %				Impact on the SCR	Impact on the Own Funds
Derivatives	Equity	Alternative Investments	Real Estate		
0%	*0%*	*0%*	*0%*	*0*	*0,00%*
-0,5%	0%	0%	0%	-24 707 457	-0,03%
0,5%	0,5%	0,5%	1%	35 445 830	0,13%
0,5%	0%	0,5%	1%	113 117 811	0,11%
0,5%	0%	0%	0%	1 429 235	0,03%
0%	-0,5%	0%	0%	38 668 752	-0,02%
0%	0,5%	0,5%	1%	56 845 175	0,10%
0%	0,5%	0%	0%	-12 470 442	0,02%
0%	0%	-0,5%	0%	216 354 825	-0,05%
0%	0%	0,5%	0%	77 216 198	0,05%
0%	0%	0,5%	1%	130 641 070	0,08%
0%	0%	0%	-0,5%	-6 579 223	-0,02%
0%	0%	0%	-1%	-6 092 774	-0,04%
0%	0%	0%	0,5%	28 878 110	0,02%
0%	0%	0%	1%	64 900 116	0,04%

Figure 10.1: Strategic Asset Allocation change impacts obtained with Machine Learning
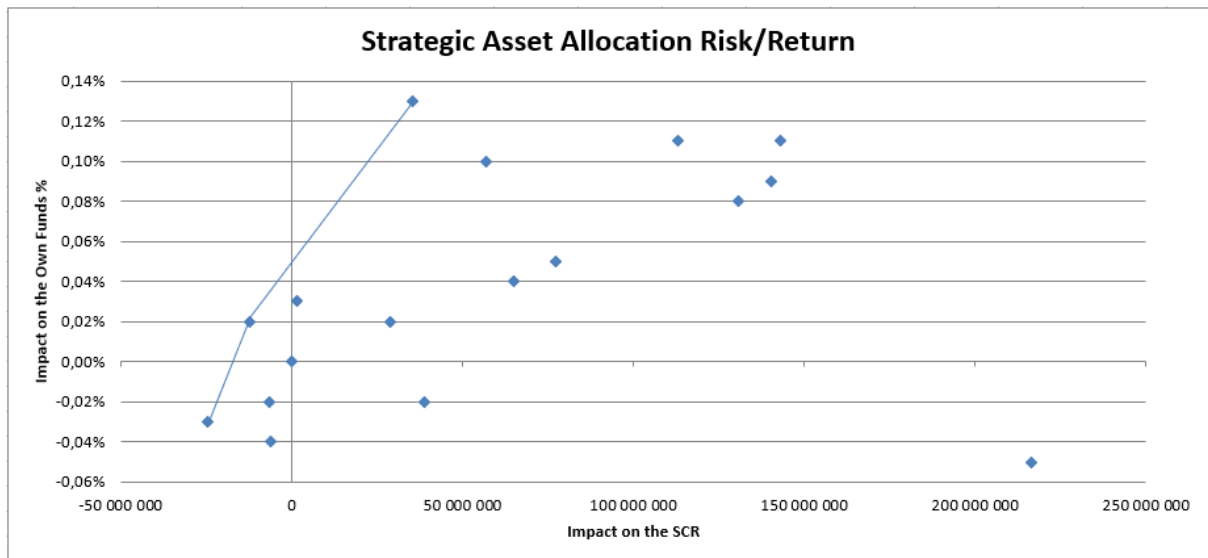
Figure 10.2: Illustration of the previous data table

The first results obtained do not yet allow us to determine an efficient frontier or to observe notable trends for the moment. Thousands of strategic asset allocations will still have to be tested before we can select the best performing ones.

By observing this example, with only a few example of values the impacts of some SAA changes can be observed.
In our results it can be observed for example that if the insurance compagny increases its asset allocation on derivatives by 0.5%, the Solvency Capital Requirement will be increased by 1.4 millions but the own funds will increase by 0,03%.

The insurance company can then decide to change its asset allocation or not this way. This tool does not select the optimal strategy but is here to bring more information when making a decision.

For the same risk taken by the company only the best performing strategy change will be considered. It is then regulation and the company's risk tolerance that will help determine the Strategic Asset Allocation of the following year.

10.3 Critique of the use of SCR as a selection metric

When selecting the optimal SAA the end goal is to maximise the Own funds while minimising the risk.

The amounts allocated to each category of investments will have an impact on both the overall return expected but also on the level of risk to which the company is exposed.

The allocation strategy thus plays an essential role in the management of the company. It has an impact on performance but also on the criteria of the company's risk appetite.

The SCR might not be the ideal risk metric to use when it comes to SAA selection. The choice of the SCR, or the variation of the SCR, as a risk metric in SII capital optimization is not obvious.

Indeed the Solvency II texts specify that the SCR is an amount of regulatory capital calculated from the effect of different risk criteria on the Own Funds not a risk metric.

If the SCR is used by several insurance companies as a risk metric for the company this method has also been disputed. The critic made for this strategy is that it can be short-termist and not be ideal in the long term view.

Certain risks are not taken into account (sovereign risk, inflation risk ...).

The risk calibration was carried out after numerous political arbitration and leads to favoring certain assets to the detriment of others. EIOPA also plans to correct some standard formula shocks in the near future to minimize interest rate risk and better deal with equities (10 to 17% increase in counter-cyclical equity shock corrections).

The SCR is very sensitive to the share portion of the portfolio. Indeed, the standard formula strongly penalizes investment in shares with a shock of 39% if they are said to be type I and 49% for type II.

The asset allocation strategy is therefore subject to a performance-risk trade-off.

On the one hand, a large allocation in shares allows for a better performance of the asset, on the other hand it generates more exposure to market risks.

It therefore appears that the larger the share allocation, the more the company deviates from respecting his risk appetite.

This is a problem long term because the company will cut itself from a big source a revenue that comes with investing in equity.

In the future the SCR could become more important due to the lack of revenue of the companies which could lead to with only the SCR as a risk metric to invest even less into equity. The impact can be sometimes counter-intuitive in a risk management logic.

An other aspect is the correlation matrix used to estimate the SCR. This matrix remains constant over time which is not a realistic approach.

Finally using the SCR can bring difficulty for the insurer to visualize the potential distortion of allowances in the event of a change. market conditions

Conclusion

This thesis presented the use of Machine Learning, to solve an Asset Liability Management run time problematic and improve operational efficiency. The goal was to approximate the way an Asset Liability Interaction Model (ALIM), calculates impact on the Own Funds, of a Strategic Asset Allocation change.

Four different Machine Learning algorithms where tested in this study, and had their performances evaluated. Grid Search was used in order to find the best hyper-parameters for each model. After that the mean squared errors of each model with the best hyper-parameters were compared.

Ultimately Long Short Term Memory (LSTM) was selected as the best performing algorithm. It is a famous Recurrent Neural Network algorithm commonly used when dealing with time series. The main characteristic of this algorithm is that it is able to remember past data and can estimate its impact on new data, in other words this algorithm uses memory.

Having LSTM as the the best performing algorithm reveals a dependency between time-steps that needed to be well modelized. The values of a cash flow, depending on the value of previous cash flows, it is understandable why this model would give the best results.

The results obtained by the LSTM algorithm were satisfying in terms of error rate and gave a good approximation of the Asset Liability Interaction Model. The algorithm can reasonably be used for asset allocations not too far removed from the original allocation. We do not recommend using it for very different allocations.

After selecting the algorithm with optimal parameters, it was then possible to estimate the change in the own funds in a much quicker way.This Machine Learning approximation model overall performed 300 times faster than the Asset Liability Management Model ALIM.

This method of machine learningwas then applied to SAA selection. The Machine Learning permits to estimate the SCR associated whith an SAA change. The SCR was used as the risk metric. This opens the door to test many SAA possibilities that could not be tested before.

Ultimately this project should lead to improve efficiency of selection by pre-selecting the SAA tested by the ALIM model with Machine Learning.

This method however has limits, one is that the Machine Learning results being approximations they need to be recalculated by the Asset Liability Interaction Model. The best performing strategies in Machine Learning are the ones that will be selected to be evaluated by the model. Machine Learning is not performing well enough to be able to replace the model completely.

An other limit is the impact of model changes. Each time they occurs, Grid Search needs to be performed again to find the new optimal hyper-parameters.

LSTM being already selected as the best performing algorithm this also mean potentially another algorithm could perform better after a model change but this possibility will not be taken into account. The algorithm takes time to be selected an this procedure will not be repeated for each model change.

Finally the use of the SCR as a metric of risk in the selection of the Strategic Asset Allocation can be disputed, as mentioned earlier in this thesis.

Other metric of risk exist. For example, the standard deviation could be the risk criterion used to determine the Strategic Asset Allocation.

This is not a new idea, in modern portfolio theory Harry Markowitz uses the standard deviation of profitability (volatility) as a risk criterion to optimize a portfolio of assets.

The volatility of the Own Funds when confronted to the financial market would be closer to the concept of risk associated with a Strategic Asset Allocation, where the SCR is an amount of regulatory capital.

Acronyms

ALIM Asset Liability Interaction Model. 35

ALM Asset Liability Management. 29

BEL Best Estimate Liability. 41

FFNN Feedforward Neural Network. 65

LSTM Long Short-Term Memory. 68

SAA Strategic Asset Allocation. 30

SCR Solvency Capital Requirement. 24

SVR Support Vector Machine - Regression. 57

Bibliography

Bernhard Schölkopf Alex J. Smola. ''A tutorial on support vector regression''. In: *Statistics and Computing* 14 (2004), pp. 199–222. URL: `https://link.springer.com/article/10.1023/B:STCO.0000035301.49549.88`.

Ferdinando M. Ametrano and Marco Bianchettiy. *Everything You Always Wanted to Know About Multiple Interest Rate Curve Bootstrapping But Were Afraid To Ask*. 2013.

Hanen Borchani; Gherardo Varando; Concha Bielza and Pedro Larra naga. *A survey on multi-output regression*. 2015.

Jerome Friedman Trevor Hastie Robert Tisbshirani. *The Elements of Statistical Learning*. 2009.

Tianqi Chen. *Introduction to Boosted Trees*. 2014.

Yuxing Yan. *Python for Finance*. 2014.

Sergio Arturo ROQUE MAYORGA. ''Dynamic Duration Steering within ALM Modeling''. MA thesis. ISFA, 2019.

Banque de France ACPR. *Le marché français de l'assurance vie en 2018*. URL: `https://acpr.banque-france.fr/sites/default/files/medias/documents/as_100_2019_le_marche_francais_de_lassurance_vie_en_2018_vf.pdf`.

Institut des actuaires. *Ressources actuarielles*. URL: `http://www.ressources-actuarielles.net`.

Insurance ERM : The online ressource for entreprise risk management. URL: `https://www.insuranceerm.com/`.

Deep Learning. URL: `http://www.deeplearning.org`.

Towards Data Science. URL: `https://towardsdatascience.com`.

Appendices

List of appendices

I Machine Learning methods

I.1 Underfitting and Overfitting

When working on calibrating a Machine Learning model, the goal is to minimize our error. However, as the error decreases overfitting can become a problem.
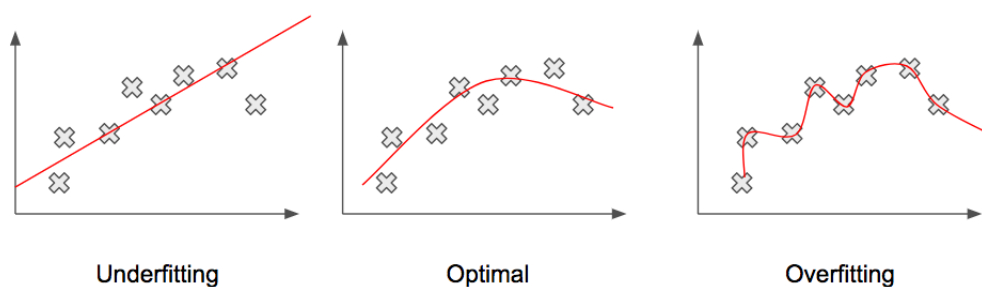

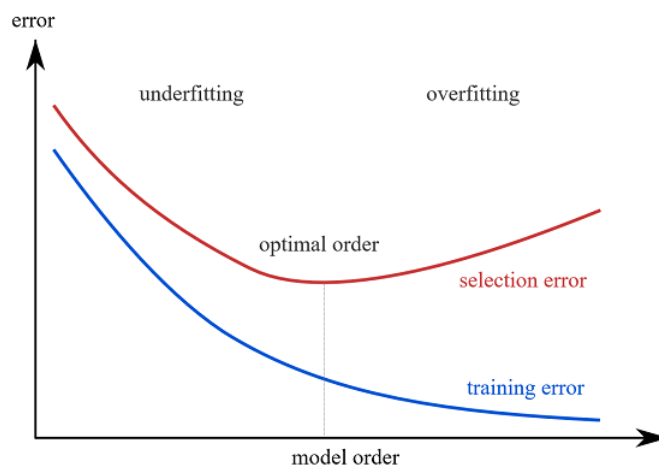Figure 3: Underfitting and Overfitting

Figure 4: Selection of the optimal model

Overfitting:

When we run the training algorithm on the dataset, we allow the total cost (that is, the distance from each point to the line) to become smaller with more iterations. Running this training algorithm for a long time can minimize the total cost.

However, this means that the line will fit all points (including noise), capturing auxiliary patterns that may not be needed for model versatility.
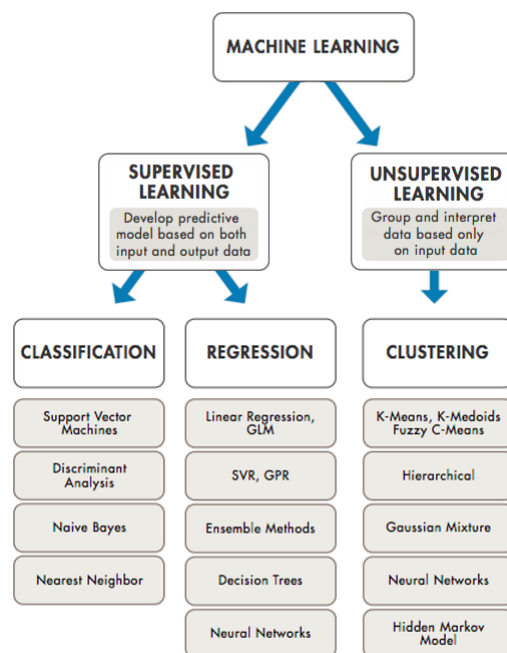
I.2 Supervised Learning

Figure 5: Supervised vs Unsupervised Machine Learning Algorithms

Supervised learning is the most common branch of machine learning today. Generally, new machine learning practitioners will begin their journey through supervised learning algorithms.

Therefore, the first of these three articles in a series will be about supervised learning. Supervised machine learning algorithms are designed to learn from examples. The name "supervised" learning stems from the idea that training this type of algorithm is like having a teacher supervise the entire process.

I.3   Gradient Descent's use in Machine Learning

Gradient descent is an optimization algorithm used in machine/deep learning algorithms. The goal of gradient descent is to use iteration to minimize the objective convex function f(x).
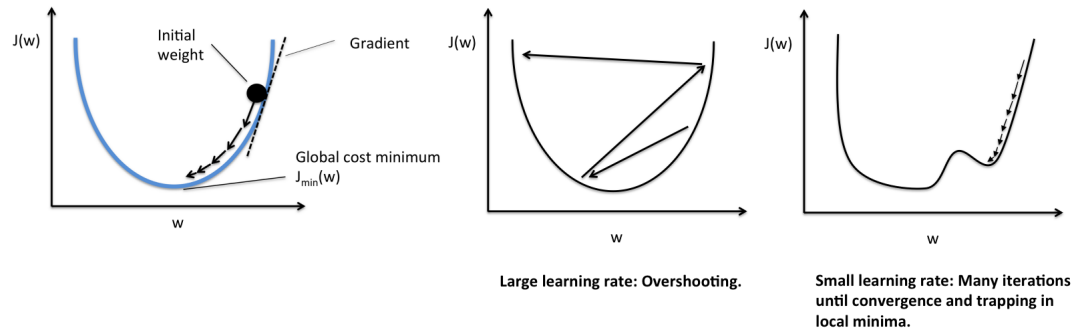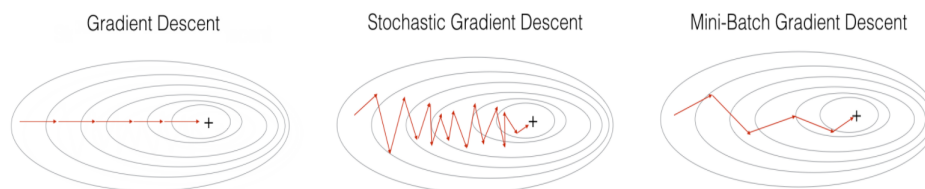

Figure 6: Gradient descent's use in error minimization

Figure 7: Batch, Stochastic and Mini-Batch Gradient Descent

II   Best Estimate Liability (BEL)

The Best Estimate Liability's goal is to measure the risk of occurrence of claims and define the fair amount of reserves necessary to cover it. The BEL is the average of expected present value of future cash-flows weighted by their probability bases on the risk-free interest rate curve.

The calculation of the BEL is based on up-to-date information and credible and realistic assumptions and uses appropriate actuarial and statistical methods.It follows a rigorous method of identification and quantification of risks impacting these cash-flows, imposed via the standard formula or the validation of an internal model.

Figure 6 : Gradient descent's use in error minimization. URL: https://hackernoon.com/gradient-descent-aynk-7cbe95a778da----https://www.hackerearth.com/blog/machine-learning/3-types-gradient-descent-algorithms-small-large-data-sets/

Figure 7 : Batch, Stochastic and Mini-Batch Gradient Descent. URL: https://engmrk.com/mini-batch-gd/

III   Convexity

Convexity measures the sensitivity of the duration of asset and liabilities to interest rate change. Lets take bonds as an example to illustrate the principle of convexity. The duration of a bond is measured as a linear relationship between bond prices and interest rates, while convexity measures this effect in a more convex way, witch is considered a more precise measurement. The ALM team at our insurance company calculates the convexity in order to have the more accurate measurement as possible.

Depending on the bonds characteristics, it can be exposed to a higher or lower convexity. The more coupons the bonds pays, the lower the convexity. Zero coupons bonds are therefore the bonds with the higher convexity.

Bond's can have negative or positive convexity's. A negative convexity means that the bonds price will be exposed to a bigger variation when interest rate rises than when it falls., a positive convexity is the opposite effect.
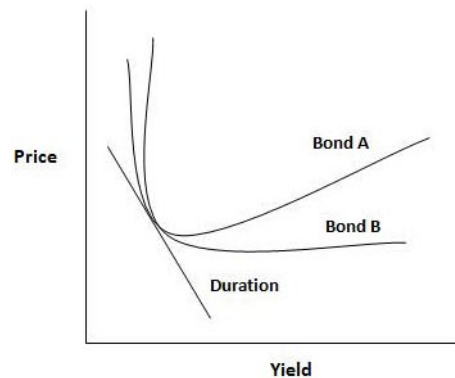

Figure 8: Example of bond convexity

Using the same notations as the ones used for duration, convexity of a bond can be defined as :

$$Convexity = \sum_{t=1}^{T} t^2 \frac{C_t e^{-yt}}{\sum_{t=1}^{T} C_t e^{-yt}}$$

IV   Cash Flow Matching

The principle of cash flow matching is simple to apprehend. It intends to ensure that each cash flow of investment income is consistent with the expected cash flow of the liability expense. It is a an other tool used to limit investment rate risk. It is often performed at a portfolio level, since different portfolios of a same company can have very different characteristics of liability. It is one of the tools used by our insurance company included in its internal model.
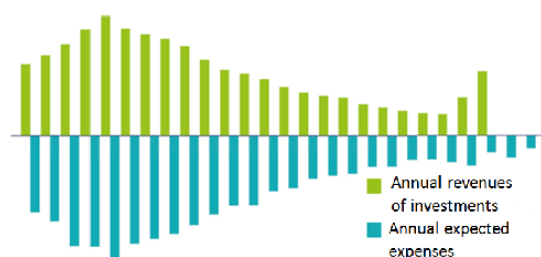

Figure 9: Cash Flow Matching

V   Asset Liability Interaction Model (ALIM)

ALIM (Asset and Liabilities Interaction Management) is an asset and liability projection tool used for life, health and motor annuities products. It allows to project future asset and liability cash flows, future PL and balance sheets items according to several economic environments. It takes also into account futures management actions (like the investment strategy, asset / liability interactions through the crediting strategy). The financial options and guarantees granted to policyholders (minimum guaranteed rates and dynamic surrender options) are also modelled.

ALIM is used for Solvency II purposes: replication portfolio calibration, Life Non Market risk calculation, BEL calculation, etc. The model is also used for ALM, MCEV purposes.

The initial ALIM model has been developed at Group level but all the local enhancements have been developed by our insurance company.

The main objectives of the company model are:

• Reporting of the aggregated values (profit and loss account, balance sheet..) at company level

• Management of simple loan interactions between the fund containing the s/h equity (referred to s/h fund) and all the other funds (referred to p/h funds)

• Projection of required capital and excess capital at company level

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Figure 9 : Cash Flow Matching. URL: https://www.agemployeebenefits.be/fr/saviez-vous-que/Pages/interview-wim-vermeir-cio-ageas-strategie-ag-insurance-branche-21.aspx

• Calculation of dividend payments and capital injections

• Calculation of the payable corporate tax and net profits

Remark: the funds can be split into ''p/h funds'' (called policyholder fund) and ''s/h'' funds (called shareholder fund). There is only one specific fund backing the s/h equity (called ''s/h''' fund). The s/h fund can just include the shareholder equity or can also include technical provisions related to the policyholder. Therefore, all other funds are considered as ''p/h'' funds which backing only the technical provisions.

Tax Tax is paid at every single period and is calculated from a flat corporate tax rate. Existing tax losses before projection start date can be taken into account (note used). During projection steps in case of negative results no taxation is applied and negative tax amount is set up as a tax loss carried forward. When profits arise in subsequent years, the tax amount is decreased by existing tax losses.

Shareholder equity Company model includes a shareholder equity strategy. It corresponds to management of the shareholder equity increased/decreased of required capital and free capital all along projection time steps.

Shareholder dividend The company model includes also a simplified shareholder dividend strategy. This dividend (after tax) is derived from the retained profit. It corresponds to the shareholder income before tax minus the increase/decrease of required capital and free capital. Then, the cash balance is impacted (after crediting step and potential capital injections) by the payments of these dividends.

• Accounting rules:

Different accountings rules have been implemented in order reflect local to reflect the local requirements for the different operating entities (OE).The main accounting methods available in ALIM are:

• Market Value Basis ("MVB")

• Cost Value Basis ("CVB")

• Exponential Value Basis ("EVB")

• Nominal Value Basis ("NVB")

• Minimum principles ("MP")

o Strong Minimum Principle ("SMP")

o Strict Minimum Principle ("SMIP")

o Moderated Minimum Principle ("MMIP")

o ALIM Minimum Principle ("AMIP")

• Investment Fund impairment principle ("IFIP")

• Investment Fund write off ("IFWO")

• Notional and rating-based ("NRB")

• Inflation Value Basis ("IVB")

Some methods and principles are only applicable for some assets. The table below gives an overview on the accounting principle applicable for each asset type.

VI   ALIM Investment strategy enhancement

VI.1   Clause CREA

For some specific funds, asset management rules require that their target allocation for equities should not depend on the evolution of the market conditions.

In order to reflect strategy, instead of considering a drop in market value, this management action has been modeled through a threshold expressed in terms of percentage of UCGL of selected asset classes.

Thus, this enhancement allows standard target asset allocation to switch to an emergency target allocation in case of equity shock (i.e. drop in equity market value). The rules exposed here are inspired by the management rules for the fund CREA.

The calculation of dynamic asset allocation depends on the level of UCGL for R332.20 assets (corresponding to equity, investment fund and real estate classes):

1. If the level of UCGL for R332.20 assets is negative, purchase of those assets is not allowed.

2. In the case where the level of UCGL for R332.20 assets is between 0% and 10% of their market value, target allocation for the corresponding asset classes is fixed at observed allocation +1

3. In the case where the level of UCGL for R332.20 assets is between 10% and 20% of their market value, target allocation will be fixed at observed allocation +2

4. In the case where the level of UCGL for R332.20 assets is higher than 20% of their market value, emergency allocation is not applied, the calculations are based on the standard investment strategy (target, minimum and maximum allocations considered in the normal asset mix input).

VI.2 Dynamic duration

Concept of the enhancement:

• Target duration input via input sheet, or via estimation of liability duration by empirical formula depending on interest rates (polynomial function should fit to historical data)

• Rebalancing between asset classes (asset class weights steering) remains untouched, as happens on a higher level

• Steering mode can be set to Active or Passive, where Passive excludes extra sales for duration management, and just uses available cash.

• Within the asset classes suitable for duration management (bonds, swaps), purchased or sold assets have to be chosen adequately to manage duration:

o Purchases happen via pre-defined buy profiles for duration increase resp. decrease. Buy profiles for bond and swap purchases can be set differently. Note that swap purchases happen market-value neutral, and, thus, can always meet duration target (no sales / no cash required)

o Bond sales happen via prioritisation by UCGLs. Additionally, a UCGL limit can be set in order not to violate PL targets. This can cause the duration target not to be met in certain time steps.

For more details please refer to asset modeling documentation table – Dynamic duration

Important remark: The dynamic duration is currently not used and has been disabled in ALIM.

VI.3 Crediting strategy enhancement

The crediting strategy module is inherited from group modeling however; all the local specifications were totally developed and parameterized by the insurer.

The Crediting Strategy reflects the decisions of the top management according two targets:

1. Profitability at company level shareholders target

2. Competitive credited rate at product level policyholders target

These two objectives are defined and implemented in a two-level mechanism: at product and company level in the asset/liability model (ALIM).

Crediting strategy at product level:

The crediting strategy at product level corresponds to the calculation of credited rate per product and the determination of the usage of the buffers (UAR and UCGL).

For traditional products, the crediting strategy is implemented in 5 steps:

1. Step 1: calculation of available buffers (UCG, UCL and UAR)

2. Step 2: calculation of minimum guaranteed amounts and if necessary, calculation of necessary UCG to realize to reach these minimum amounts. A split between minimum guaranteed rates gross and net of loadings has been introduced.

3. Step 3: calculation of the target credited rates by product depending on benchmark rates and on total buffer level in the fund.

4. Step 4: calculation of the UCGL amount to realize to reach the target credited rates.

5. Step 5: calculation of the change in UAR needed to reach the target credited rates. The usage of the UAR at product level is not authorized for the General portfolio (PGVIE) products.

For L441 products (a specific crediting strategy has been set up) implemented in 8 steps:

1. Step 1: calculation of available buffers (UCG and UCL)

2. Step 2: the minimum guaranteed rate is 0%. However, as the S/H wants to avoid a PTSC injection, UCG are realized in order to obtain a coverage ratio above 100%

3. Step 3: calculation of the target credited rate depending on the current coverage ratio (at time of calculation), a target coverage ratio and a minimum annuity revaluation that the company wants to grant

4. Step 4: calculation of the UCGL amount to realize to reach the target credited rates.

5. Step 5: not necessary because no UAR is used for L441 products

6. Step 6: check if the termination clause is activated for PREFON fund only

7. Step 7: computation of the change in the PTSC

8. Step 8: calculation of the annuity revaluation

Bonus and PTSC injection are integrated to the PTS at the beginning of the following projection period. The service value is revaluated at beginning of year with the revaluation rate calculated at the end of the previous year.

Crediting strategy at company level:

The crediting strategy at company level verifies that the regulatory minimum bonus requirement is reached and models the minimum bonus strategy driven by the top management:

• If the regulatory minimum bonus is not met (i.e. distributed amount at company level are lower than the regulatory minimum bonus), an increase in UAR is required

• If the regulatory minimum bonus is met (i.e. distributed amount at company level are higher than the regulatory minimum bonus), a decrease in UAR is needed to reach 105% of the regulatory minimum bonus. This decrease in UAR is restricted. This corresponds to a top management action also called minimum bonus strategy.

This enhancement has been performed in several times. Please refer to local technical specification documentation table:

• First version of AGF's crediting strategy (introduction in ALIM of a two level crediting strategy: [TS7]

• Modeling correction at product level credited rates to policyholders at the beginning of the projection are not consistent with past credited rates: [TS19]

• Review of profit-sharing strategy in order to develop a more sustainable model (improvement is to take into account the credited rate of the year before besides the benchmark rate that is highly reactive to market changes: [TS42]

VI.4   Rebalancing process

The rebalancing process covers the purchase and sale of assets, allowed to meet a target allocation rates as well as target duration (duration strategy).
The rebalancing process can further be subdivided as follows:

The core algorithm (before duration strategy) can be illustrated as follows:

• At a user-defined frequency, it is checked if the actual market value allocation rates of each (relevant) asset classes are within their defined tolerance limits.

• In the case that at least one asset class is out of its tolerance limits, a rebalancing is triggered, i.e. the volume of asset classes is above (below) their maximum (minimum) limits are decreased (increases) by sales (purchases).

• Results of rebalancing are linked to the rebalancing method the user has chosen. It is possible to focus on minimizing the realization of capital gains or the focus can be placed on meeting the target allocation rates.