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Titre : **Estimations of the claim reserves for the CPI: a trade-of between risks and prudence**

Confidentialité : ☐ NON ☒ OUI (Durée : ☐ 1 an ☒ 2 ans)

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Abstract

The following study develops several reserving processes to estimate the claim reserves for the Credit Protection Insurance -CPI- products under the European norm Solvency II. This reserve is one of the major reserves for insurers which covers the cash flows of the claims. The amounts to reserve as well as the assessment of the error in the estimates are reviewed for various methods. These estimations and the way they are estimated, reflect a balance between profits and prudence which depends on the risk appetite of the insurer.

Résumé

L'objet de cette étude est de comparer différentes méthodes d'estimations des réserves pour les sinistres à payer -PSAP- dans le cadre de la norme européenne Solvalité II, pour le produit d'assurance Emprunteur -CPI. La PSAP est l'une des principales réserves ou provisions pour les assureurs qui couvre les couts liés aux sinistres. Les montants des réserves ainsi que la quantification de la qualité des estimations seront mis en avant pour les différentes méthodes. Ces estimations et les choix faits dans le processus d'estimation reflétent un arbitrage entre profits et prudence qui dépend de l'appétit pour le risque de l'assureur.

Acknowledgements

I thank G.Jourdrin for letting me join the Actuarial and Planning team at BNP Paribas Cardif Japan, and V.Joseph for helping me on my daily work and answering all my questions regarding the CPI and its working. He has always been here to explain and correct me.

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Contents

Introduction

The insurance sector is featured by its counter cycle business: earnings are known and regular -at least on a yearly basis-, whereas the spending is random. From this uncertainty towards their spending, insurers have to face several constraints: from their insured, from their shareholders, from their partners, from the regulators...
In order to answer all the sides, an amount of money is put aside in their balance sheet: this sum is called a reserve or a provision. Reserves are built to ensure that the insurance companies remain solvent and comply with their commitments. One commitment leads to one reserve also called "Technical Provisions" under the Solvency II norm. The latest regulates the European insurance sector. As a result, there are many reserves which appear in the liabilities of the insurer.

Among all the reserves, the claim reserves are the main reserves. They cover all the future claim development, that is to say all the costs which emerge from the claims and that the insurer will have to pay. Indeed, not all the claims are reported and paid on the spot. It results in a delay between the occurrence and the payment. In order to capture this delay and to have a prudent vision of its balance sheet, the insurance company estimates the expected claim costs that it will have to pay. This estimation is the reserving process: the sum is estimated and then booked in the balance sheet. The norm Solvency II gives the scope of the reserving process. The Technical Provisions have to be computed in Best Estimate, plus a Risk Margin.

When it comes to the estimation process, the insurer faces several options and choices in: the methods, their parameters, ... Moreover, it wants to measure the risk in the reserving process: how reliable is the data? Can the data and the estimates be more reliable and accurate with another process or other parameters?

First, the context and the insurance product of the study will be presented.
Second, we will underline how tremendous are the reserves for insurers and what is their scope in terms of reserves, estimations...
Before calculating the amount of reserves, the issue of the data quality will be analyzed. A good estimation relies on truthful and cleaned data. In addition, a description of the data will be done.
Based on the experience of BNP Paribas Cardif Japan for the CPI insurance product, we will present the reserving process and its impact on the insurer balance sheet.
Then, we will review other methods and parameters to estimate the claim reserves. We will try to quantify their impacts on the amounts to reserve as well as the errors made in the estimation process. Assessing the reserving risk is tremendous to monitor the insurer risks.
Finally, under the Solvency II frame, the reserving risk at one-year horizon will be estimated. It helps to compute the reserving risk at one-year vision instead of an ultimate vision of the reserving risk.

The amounts have been changed from the real booked values to respect the confidentiality of BNP Paribas Cardif Japan. Some data are truncated or modify. Numbers will be shown in scientific writing with two decimals. Some information -like the prudence margin and the last month roll out- will not be detailed to respect the confidentiality issues. All the amounts are given in Japanese yens.

Chapter 1

BNP Paribas Cardif Japan and the study

1.1 BNP Paribas Cardif, a worldwide insurance company

1.1.1 BNP Paribas Cardif: its activities and results

BNP Paribas Cardif is a French insurance company which operates around the world. It is a 100% subsidiary of the French BNP Paribas group. It was created in 1973. In 2011, the unique name of BNP Paribas Cardif was chosen after the unification of three insurance businesses: "BNP Paribas Assurance", "Cardif" and "BNP Paribas". BNP Paribas Cardif offers personal and collective insurance solutions both in life insurance and non-life insurance. Its products are distributed mainly through bancassurance. The business model specificity of BNP Paribas Cardif is that it doesn't sell directly insurance products. It operates through partners such as banks, retail shops, other insurance companies... Thanks to its partnership culture, the company develops a unique approach in each country and market. Today, it has more than 150 local and global partners around the world. As a result, BNP Paribas Cardif can target each market thanks to a large and adapted spectrum of insurance products.

In 2017, BNP Paribas Cardif turnover was 29,7 billion Euros. BNP Paribas Cardif operates in 36 countries and has more than 10 000 employees around the world. It has offices in Europe, Asia and South America, above all in emerging markets or well-established markets. A 57% of its result is made outside of France, mainly in Europe. BNP Paribas Cardif is the second French insurer which operates abroad after AXA and the first in Credit Protection Insurance -CPI. Asia has been a strategic region for the development of the company. BNP Paribas Cardif has had 6 offices in 6 countries for more than 10 years. In 2017, Asia represented a 16% of BNP Paribas Cardif turnover. The main office is in Hong Kong but Japan plays a key role at the regional and global scale.

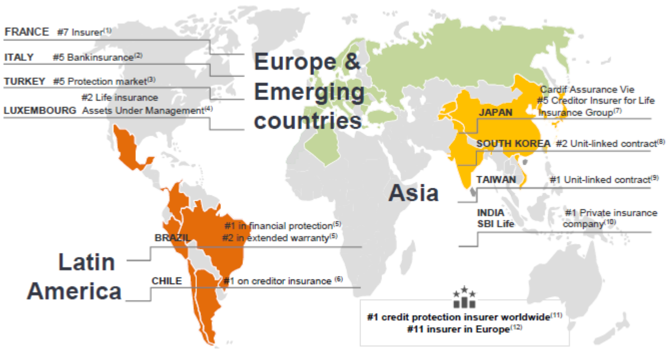


Figure 1.1: BNP Paribas Cardif around the world

1.1.2 BNP Paribas Cardif in Japan

BNP Paribas Cardif has been in Japan since 2000. It operates through its subsidiary : BNP Paribas Cardif Japan. Thanks to a culture of partnership, it sells mainly CPI insurance products under collective insurance.

The development of the branch in Japan

In 2000, BNP Paribas Cardif opened its office in Tokyo - Japan. Today, it has more than 250 employees. The business is divided between life and non-life insurance. Life Gross Written Premium -GWP, the earning of insurers- stands for a 78% of the company GWP in 2017. The split between life and non-life entities is mandatory for insurance company. It is mainly due to the differences in the underlying risks. Henceforth, BNP Paribas Cardif Japan operates through Cardif Vie -AV- and Cardif Risques Divers -RD. Only life products will be developed in the study.

Since its development in Japan, the insurance company has led an extensive strategy. First, it has built a partnership with one partner per Japanese region. This goal was achieved quickly in 2005. After 2005, the company has been gaining more partners at different levels -regional banks, national banks,... Today, it has partners all around Japan: most of them are Japanese banks and financial institutions.
In April 2018, the insurer has settled a strategic partnership with the Japanese bank, Sumitomo Trust Bank -SMTB. The latest will acquire 20% of the shares of the newly established Japanese life insurance subsidiary of BNP Paribas Cardif (appendix 6.7). SMTB is a member of the fourth largest banking group in Japan. Thanks to this partnership, BNP Paribas Cardif Japan hopes to benefit from SMTB's advanced financial services and to become more famous around the country. Indeed, the insurer suffers from a lack of visibility in a market more and more competitive. Thus, it will enable the insurer to diversify its insurance products and expand around Japan.

BNP Paribas Cardif Japan deals mainly with Creditor Protection Insurance - CPI- distributed through "bancassurance". CPI stood for a 99 % of the Life GWP and a 94% of Non-Life GWP in 2017. Bancassurance CPI is the main core of BNP Paribas Cardif business. In its expansion abroad, BNP Paribas Cardif remains an expert in the CPI and so is perceived on the Japanese market. In 2017, BNP Paribas Cardif Japan was among the fifth biggest positions on the CPI market in Japan. Thanks to its red of partners around the country, BNP Paribas Cardif Japan has been winning market shares. In addition, the insurer tries to offer new products on the CPI market. For instance, it was the first to offer credit payment protection with cancer rider on the Japanese market. This product includes a loan coverage for both death and cancer. Thanks to its initiatives and business lines, BNP Paribas Cardif Japan has a strategic position on the CPI Japanese market.

The Japanese insurance market

The Japanese insurance market is big and very competitive.

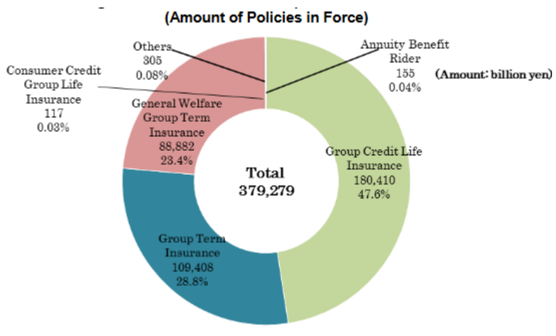


Figure 1.2: The distribution of the amount of in force policies on the Japanese market in 2016. Source: The Life Insurance Association of Japan

In 2016, life insurance premiums were 3 times higher than non-life premiums. Japan is a major market for life insurance: in 2016, it ranked second after the USA in term of GWP. Group Credit Life Insurance -collective insurance for loans- stood for a 48% of the amount of policies In Force around all Japan in 2016. It also accounts for a 48% of the new policies.

Even though GWP had trended upward in recent years, it has been decreasing for the first time in 3 years in 2017. The evolution of the loan interest rates and the real estate prices can explain the decrease in the GWP.

The evolution of the CPI insurance market is linked with the evolution of the real estate market and the evolution of the interest rates. Mortgage CPI is the main insurance product in terms of GWP in Japan.

- The Japanese mortgage market: Real estate prices are high and volatile in Japan (appendix 6.8). The acquisitions of real estate are financed mainly by banks and specialized mortgage institutions. After the Japanese financial crisis, the domestic mortgage market has been quite stable. It has increased from +1,8% between 2016-2017. In 2017, the amount of domestic mortgage stock was 190 trillion JPY -among it, a 65 % was hold by banks. Mortgage depend on the mortgage interest rates which are correlated with the Japanese Government Bond -JGB- yields. They have remained low and stable.
  One particularity of Japan is the role of the Japan Housing Finance Agency -JHFA. It is in charge of providing liquidity to the mortgage market, of the securitization support and of enhancing the housing quality of low to medium income groups. This agency is a key player on the mortgage market. For instance, it has launched the program Flat 35. This program provides mortgage with a fixed rate interest mortgage over a 35 years lifetime to households. They are distributed through the sale channels of private financial institutions. The role of the agency is to pool the risks and transfer them to the market through Mortgage Backed Securities. Flat 35 loans are among the most widely used home mortgage products in Japan. Therefore, other mortgage provided by private financial institutions rely mainly on floating or semi-floating interest rates. Indeed, it is hard for them to compete with the Flat 35 on long term fixed interest, especially in a context of low interest rates.

- Regarding interest rates, they have been stable and low since 2016. In 2018, the Japanese government has been taking actions to reduce them. As a result, credits are affordable for the Japanese. Most of the loans are a mixed of fixed and variable interest rates. Nowadays, 10-year fixed rate is 0,85%. It went under 1% for the first time in 2016. The variable rates are following the same trend.
  Loans depending on floating mortgage rates are driven by the fluctuation of the short-term interest rates. The latest change in function of the macro-economic assumptions and are also volatile.

- In addition, the trends of life parameters have to be considered -they impact the life insurance market because the guarantees rely on the life duration.
  The Japanese population is living older. In 2018, the Japanese mortality tables changed. The previous change was done in 2007. For men and women, between 2015 and 2018, the annual living improvement rate has been +1%. As a result, life expectancy at birth has risen: it is 80.98 years for women, and 86.99 years for men. Advancements in medical technology, drop in suicides and economic recovery are some of the reasons of the increasing in the Japanese life expectancy.

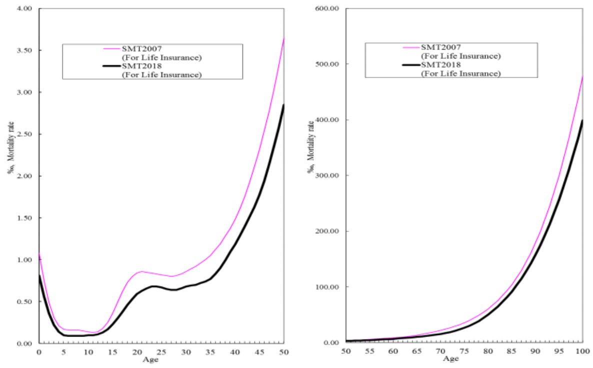


Figure 1.3: Comparison between the SMT 2018 and the SMT 2007 for male - The figure on the left shows the mortality rate for the ages under 50. On the right, the mortality rates for the ages over 50. Source: The Institute of Actuaries of Japan

> The figure above underlines the change in the mortality rates for male: mortality rates have decreased especially for the ages under 50. Women mortality rates are following the same trend. The mortality improvement results in reducing the insured's exposure to loss and, as a consequence, the insurance premiums. In particular, it applies for life insurance products as the underlying risk relies on life duration: the probability that the insurer has to pay for the losses -the mortality rate- is going down. It charges less for the coverage. As a result, BNP Paribas Cardif Japan has reviewed its pricing to integrate the change. A mitigation plan has been carried out to smooth this impact.

Finally, the insurance Japanese market is competitive. Even though BNP Paribas Cardif Japan was among the first insurance companies to offer mortgage coverage in Japan, it has many competitors. They are mainly Japanese life insurance companies: Nippon Life, Dai-ichi Life Insurance, Meiji Yasuda Life Insurance, Sumitomo Life Insurance, Mitsui Life Insurance, Japan Post Insurance. ... Thus, other French insurers are rising players on the Japanese CPI market: Credit Agricole and Axa.

Collective insurance

Most of BNP Paribas Cardif Japan contracts are collective contracts with partners. Not only the insurer's partners sell the loans, but they also sell its insurance. However, the insurance company covers the risk: in case of sinister, it pays back the loan to the lending institution. The latest is the beneficiary whereas the borrower is the insured. In case of collective contracts, the insurer has no link with the borrower - the insured: the payment is made directly to the creditor - mainly banks. Only one contract is managed by the insurance company, which results in lowering its administration expenses. In the case of CI, the premium is usually included in the monthly instalments, paid by the borrower to the bank.

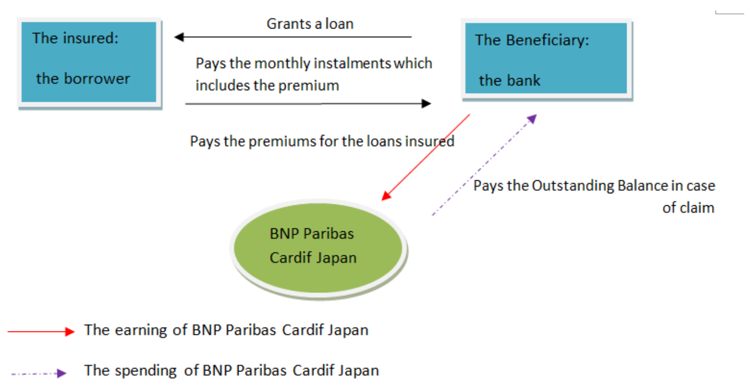


Figure 1.4: The working of collective insurance contracts for CPI

The pricing of collective contracts is made on the actuarial average features of the portfolio. It enables insurers to save administration and loadings costs but it requires homogeneous risks. Indeed, the premium is the same for all insured no matter the risk they represent. For instance, a 50-year-old man pays the same insurance premium as a 25-year-old woman even though the individual underlying risks are different. The oldest has a higher risk exposure and so, expectation that the insurer has to pay the loss.
Because of its unique premium, collective contracts are appealing for the older people: their premium is lower than in the case of Individual Protection. Thus, the contracts are easier to enter: there are less pre-conditions.

BNP Paribas Cardif Japan also has some products in Individual Protection -IP. They are individual contracts: each insured can benefit from an individual pricing. It depends on the individual features of the insured as his/her age.

1.2 The CPI product

1.2.1 The coverage

CPI is the main product of BNP Paribas Cardif Japan. CPI products are linked to personal loans: mortgage, consumer loans, revolving, credit card loans... They cover several risks for the borrower: death, accident, sickness, critical illness, unemployment... The insurer guaranties the reimbursement of the initial loan if the insured is affected by the occurrence of an accident -death, disease, temporal disability, unemployment...- covered in the contract. It can also pay back the monthly instalments the time the insured is unemployed or disabled. For the CPI products, the major guaranteed events are:

1. Death -D.
2. Temporary disability -TD.
3. Critical illness -CI.
4. Cancer.

If the insured faces one of these states, he/she can benefit from the coverage provided by his/her insurance policy. For instance, in case of death, the outstanding balance is paid by the insurer to the creditor. The outstanding balance is equal to the initial loan amount minus the amount already paid back by the borrower -the monthly-yearly instalments.
CPI insurance products are a guarantee against defaults for creditors -mainly banks or financial institutions. Not only creditors are covered, but CPI product also protects the insured's families from bearing the liabilities. All the events guaranteed trigger the well reimbursement of the loan. Indeed, they do not have to pay back the loan once the borrower has died or faces a state of incapacity to reimburse the amount. In these cases, the insurer covers the remaining monthly instalments.

Thus, in some countries, CPI coverage is mandatory for debtors when they apply for credit loan services -in particular, mortgage loans. Because their amounts are high, creditors don't want to lose the money they lend.
In Japan, only death coverage is mandatory for mortgage loans: when taking a loan out, debtors have to buy an insurance - at least to cover the risk of death of the borrower. It is sold by the bank with the mortgage. As a result, BNP Paribas Cardif Japan's policyholders are the partners -mainly- banks and financial institutions. The insured pays back the monthly instalments and the premium is included in the amount given to the debtor. The latest pays to the insurance company the total premium on the portfolio.

To benefit from the CPI coverage, the partner pays a premium to BNP Paribas Cardif Japan. One of the main features of the latest is the yearly renewal. Indeed, each year, depending on the risk evolution or business considerations, the premiums received by the insurer change. The main parameters which drive the premium amount and its evolution are:

- The risk evolution: for instance, change in mortality tables, evolution of the actuarial age, evolution of the loss ratios [1] ...
- The business evolution: for instance, the launch of a new product with a partner, a special discount on premiums to keep a partner, profit sharing...

Hence, the insurer renews the premium to cover the current risks and their changes. But, on the other hand, the renewal implies a thinner monitoring of the risks.

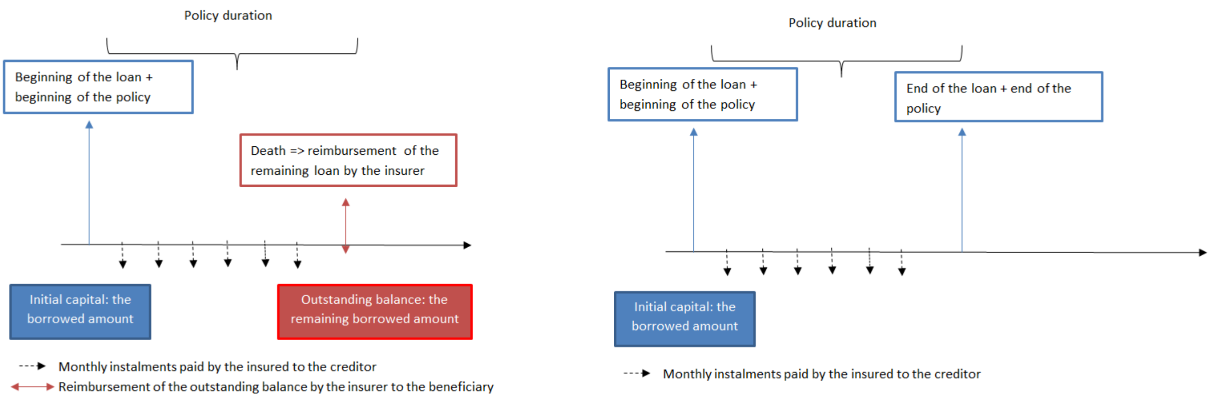


Figure 1.5: The CPI coverage: on the left when the insured dies -on the right: when the insured is still alive

The coverage and the risks for the insurer depend on the underlying loans - the initial loan amount and the loan duration- as well as the event guaranteed.

1.2.2 Different kinds of loans

CPI insurance covers different kinds of loans against several events. On each underlying market - mortgage, consumer credits...-the risks are different because the duration as well as the initial amount of the loan are not the same. Both are tremendous parameters of the CPI coverage. The sum insured is the outstanding balance amount: the initial amount borrowed minus the monthly instalments already paid back by the borrower. The duration is the period of reimbursements of the loan by the borrower. For instance, if the loan duration is 5 years, after 5 years, the loan must be paid back to the lending institution. As a result, the insurer risk exposure is longer in the case of mortgage loan than in the case of consumer credit loan.

However the risk exposure can be trigger by the possibility for the insured to buy his/her loan before the end of the loan duration. This option triggers the reimbursement scheme and, therefore, the duration of the insurance policy. If the loan is purchased before its original end, the insured leaves the policy, and is not covered any more by the insurer. However, an increase in the re-purchase is a risk for the insurer because of the lack of premiums it could have earned to cover the full duration.

The sum insured

The particularity of the CPI is the decrease in the sum insured as time runs. As the insured pays back the monthly or yearly instalments, the remaining amount decreases. Henceforth, the sum insured depends on the initial loan amount, its duration and its interest rate. The regularity in the payment of the instalments also needs to be considered: is the reimbursement on a monthly or yearly basis?

[1] Loss ratio $= \frac{\text{Claims}}{\text{Premiums}}$

Let's OB_t being the outstanding balance at time t. We assume that the payment is regular - each month for instance- and the interest rate r is constant. We have:

- a_n : the sum of the present value of n payments for 1 unit borrowed -begin Euros or Japanese Yens.

$$\forall n \in N, a_n = \frac{1}{1+r} + \frac{1}{(1+r)^2} + ... + \frac{1}{(1+r)^n} \tag{1.1}$$
$$= \sum_{k=1}^{n} \frac{1}{(1+r)^k} \tag{1.2}$$
$$= \frac{1-(1+r)^{-n}}{r} \tag{1.3}$$

  where n is the loan duration in months and r the loan interest rate -constant and different from 0.

- K : the initial loan amount.
- R : the loan annuity -the constant instalment paid each period i with $1 \leq i \leq n$.

The initial amount K is equal to the present value of the future loan annuities:

$$K = \frac{R}{1+r} + \frac{R}{(1+r)^2} + .. + \frac{R}{(1+r)^n} \tag{1.4}$$
$$= \sum_{k=1}^{n} \frac{R}{(1+r)^k} \tag{1.5}$$
$$= R \times a_n \tag{1.6}$$

$\frac{R}{1+r}$ is the amount paid back by the insured after the first month, $\frac{R}{(1+r)^n}$ is the monthly instalment after the n-th periods... And, at time t, the outstanding balance OB_t becomes: $\forall n \leq t$,

$$OB_t = R \times a_{n-t} \tag{1.7}$$

With: $R = \frac{K}{a_n}$ the instalment after n periods and OB_t is the amount insured under the CPI coverage at time t.
If the insured is still alive at the end of the next period t+1, it becomes:

$$OB_{t+1} = OB_t \times (1+r) - \frac{K}{a_n} \tag{1.8}$$

As a result, after n periods: $\forall j > n$, $OB_j = 0$. The loan has been paid back and the insured leaves the portfolio.
As time runs, the outstanding balance amount or the sum insured is reduced by the monthly instalment paid back. The time between the beginning of the policy coverage and the current date is called the seniority. For instance, a seniority of 3 years means that the insured has been covered by BNP Paribas Cardif Japan for 3 years. If the duration of its loan is 10 years, the insurer will have to cover him/her for 7 more years. When the seniority grows, the remaining capital decreases.

The main parameters of the sum insured are: the initial loan amount, the loan duration and the interest rate. Henceforth, it is key for the insurance company to monitor these parameters in order to weight the underlying risks.

Mortgage CPI

CPI mortgage is the main product of BNP Paribas Japan. This insurance product covers the loans of people when they buy real estate - flats, houses, buildings... The particularity of the mortgage CPI product is its duration as well as the high sum insured -the borrowed amount. Because of the amount borrowed, the duration covers several years during which the insured pays the monthly instalments and the premiums.
Thus, the sum insured depends on the mortgage market and the real estate market. Their trends and changes impact the sum insured and, also the insurer risk exposure. The insurer follows these parameters carefully.

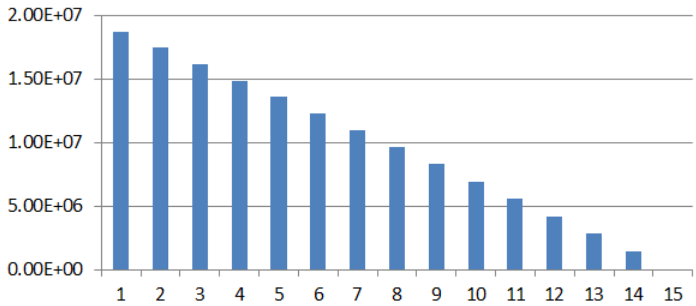


Figure 1.6: Evolution of the outstanding balance or the sum insured - for a loan of 2x10^{-7} JPY, over 15 years and with an interest rate of 0.85%.

Consumer finance CPI

Consumer finance loans are loans granted to increase the households purchasing power. The average amount of the loans as well as its duration are smaller than for mortgage loans. They represent a small part of BNP Paribas Cardif GWP.

1.2.3 The guarantees

The risks

Several risks are covered by BNP Paribas Cardif Japan in function of the different products. Usually, one policy covers one event or type of event which occurs on a random basis. When the underlying risk depend on the duration of life of the insured, the risk is categorized as a life risk. Otherwise, it is a non-life risk. The management of both categories is different. There are different levels of premiums, solvency rates, provisions...
On life side, for mortgage CPI, Death -D- is the main risk covered by BNP Paribas Cardif Japan in terms of premiums. The second main risk covered is Critical Illness -CI. The main feature of the CI insurance product is the combination of the CI and death guarantee. The study will focus on these main guarantees, categorized as life insurance.

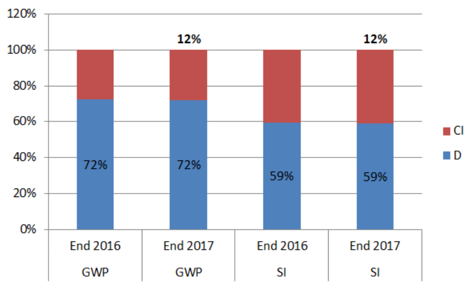


Figure 1.7: The distribution between CI and death coverage in terms of sum insured and GWP - 12% is the growth of the portfolio from 2016 to 2017

The death guarantee

On the Life side, death is the main risk covered by BNP Paribas Cardif Japan. Be it a mortgage loan or a consumer finance loan, in case of the death of the insured, the insurance company has to pay the outstanding balance to the beneficiary -the partners. In addition, the risk of Total and Permanent Disability -TPD- is also covered. That is to say that if an insured faces the state of TPD, he will benefit from the coverage. TPD is featured by a loss of independence. The insured ability to work or his/her normal mental or/and physical state are affected: it is harder to pay back the monthly instalments, mainly because of a lack of resources: less earning and an increase in the health expenses. The insurer covers the risk that the insured may not be able to pay back the loan. Therefore, there is "double" guarantee under the death coverage.

The main feature of this guarantee is that the payment to the beneficiary is made in one time. There is no waiting period. Once the death or TPD occurs, BNP Paribas Cardif Japan pays the bank or financial institution. Once the payment done, the insured leaves the portfolio.

The CI guarantee

CI coverage is a life insurance product. It guarantees both CI and death events. The CI are:

1. Cancer
2. Heart attack
3. Coronary artery bypass surgery
4. Stroke
5. Kidney failure
6. Major organ transplant
7. Paralysis/Paraplegia

If the insured is affected by one of these illnesses, he/she can benefit for the coverage. The process to benefit from this coverage is more complex than for death coverage. Once the CI has been diagnosed, a waiting period is observed. After 6 months, if the insured still suffered from the illness, the outstanding balance is paid back to the beneficiary.
Under this product, the CI guarantee comes as an acceleration of death. When the insured has been diagnosed for one of the CI, he/she is more likely to die. Therefore, in this product, two risks are guaranteed by the insurer: the risk of death and the risk of being affected by one of the CI.

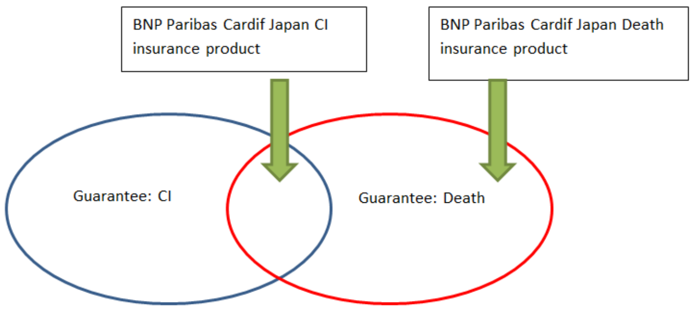


Figure 1.8: The guarantees under the death and CI insurance contract

The Death coverage and CI coverage are studied separately, even though both insurance products provide coverage in case of the death of the insured. Indeed, they show different parameters in terms of sum insured and claim features.
For instance, the average sum insured is higher for the Death guarantee than for the CI guarantee when the insured enter the portfolio -seniority 0. The difference can be explained by the different guarantees offered under the death and CI coverage. They do not attract the same borrowers, in terms of premiums and guarantees.

The more guarantees are offered, the more expensive is the premium. Thus, combining several guarantees reduces the loadings of the insurer. One contract is considered and managed, instead of two. It is also an appealing feature in terms of marketing and business: it reduces the partners' loadings and smooth its management of the contracts. But, these insurance products are more complex in terms of risk monitoring: because of the coverage of two risks, the insurer watches the evolution of the mortality risk as well as the evolution of the underlying illnesses and the evolution of the ability of the medical community to diagnose these illnesses.

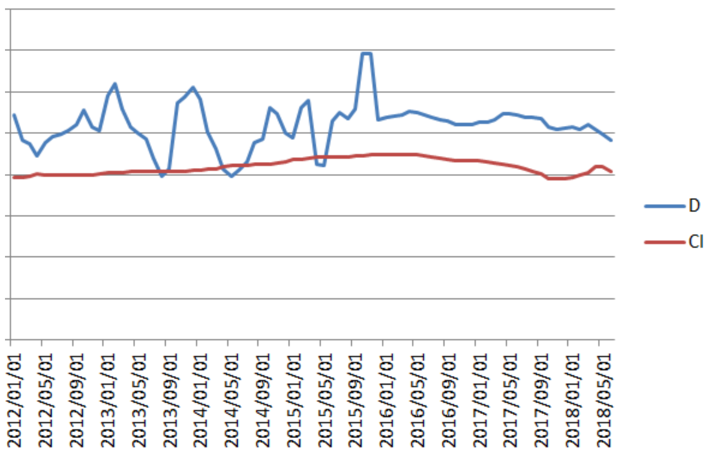


Figure 1.9: The death and CI sum insured. In blue: the death coverage. In red: the CI coverage

CI and death guarantees are different in terms of costs and risks for BNP Paribas Cardif Japan. Indeed, the risk of death does not weight the same as the risk of CI. Henceforth, it is a key issue to have various approaches for the company: by products, by guarantees, by loans,...

1.2.4 The limits and conditions of the CPI coverage

The limits of the insurance contracts

To monitor its risks, the insurer can put conditions and caps to its policies. Duration, loan amounts and age are limited.
First, the benefits -the sum insured or the outstanding balance- are limited: the insurer can choose which loans it guarantees. In addition, the coverage is limited in time. The longer is the loan, the riskier becomes the insured -especially regarding its health conditions. The loan duration is limited as well as the duration of the insurance.
Finally, the age of the insured is another limit in the contracts. When the insured gets older, his/her mortality rate increases. He-she is riskier for the insurance company even though he/she brings more profits due to a higher premium to cover the risk.

The medical check

Moreover, one other condition to benefit from the coverage is the "success" when passing the medical check.
The borrower has to go through a medical check to start the coverage. It is a standardized medical check designed by the company. Thanks to the latest, it can "select" its insured among all the borrowers and monitor their risks. Therefore, one parameter to measure the insured health conditions and their deterioration is the seniority. Indeed, seniority 0 means that the insured has just passed the medical check. She/he is less "risky" than an insured with seniority 5. Thus, the medical test enables the company to reduce the risk scope: indeed, it can "refuse" insured who will be in a bad health conditions and riskier than the other insured. Having homogeneous risks is tremendous with collective insurance contracts as their parameters rely on an actuarial weighted average.

Thanks to the medical check, the insurance company can pool the risks. They are weighted all together. It is also a way to be protected from anti-selection. Some borrowers could buy an insurance with their credit knowing that they suffer from a disease. If this disease affects their life time, then, they represent a bigger risk for the insurer. Henceforth, only the "good" risks remain in the portfolio which is assumed to be homogeneous [2].

[2] A homogeneous group risks is *"a set of insurance obligations which are managed together, and which have similar risk characteristics in terms of, for example, underwriting policy, claims settlement patterns, risk profile of policyholders likely policyholder behavior, guarantees, ..."*

Nonetheless, the medical check does not reduce the risk of moral hazard. Some insured can have riskier behavior knowing they are covered for a particular risk -death or CI for instance. If the medical check is mandatory at the beginning of the coverage, there is no other test or way to assess the evolution of the insured risks -especially regarding their health conditions.

Finally, under collective insurance, medical checks are less demanding than for individual protection. Indeed, it is designed for all the insured in the portfolio, based on the average of the portfolio and average features. It helps the insurer to select its risks in order to be able to pool them and s, being covered.

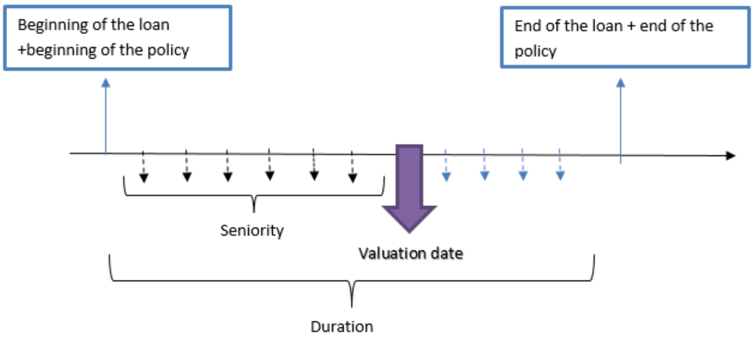


Figure 1.10: The CPI coverage: from the medical check to the end of the contract. The valuation date stands for the date when the portfolio is viewed

These limits and conditions help the insurer to reduce the risk scope, especially when dealing with collective contracts. Homogeneous risks are required to monitor the risks.

Chapter 2

The reserves: a key tool for insurers

The reserves and their allocations are keys for insurers. Because they stand for the main item on the liability side of the balance sheet, the amount reserved is carefully watched by the management of the insurance company as well as by the regulators. Indeed, the reserving process is framed. The last norm which rules this process is Solvency II. Between too much reserves and too less reserves, insurers also have to follow the process to estimate their reserves.

2.1 The importance of reserves

Due to the working of insurance companies, the amount of reserves is tremendous: it is at the crossing between prudence and profits.

2.1.1 How do insurers work?

Insurance companies have to face several constraints. They earn money from the premiums they collect paid by the insured. This income is regular: each month or year, an insured pays his/her premium to be covered by BNP Paribas Cardif Japan for a type of accident. On the other hand, insurers have to pay for the claims at a given date -when accident happens -and receives the premiums up to this date. The claim frequency is random and volatile: indeed, one cannot forecast when death, cancer or car accident happens. From this counter business cycle, insurers face several challenges.

- The first challenge for insurers is to be able to pay their insured when claims occur in order to meet their commitment: to cover the risks.
- The second challenge comes from the shareholders. They want the highest interest rate on their share. The insurer has to pay their dividends.
- Last but not least, insurance companies have to comply with regulations and laws regarding their cash flows, balance sheets and governance. Regulators aim at protecting the insured and their rights. They have a supervisory role and make sure insurers have a certain level of capital so that they remain solvent. The major regulation on the insurance field is the European norm Solvency II.

To answer all the sides, the customers, the regulators and the shareholders, insurers do some reserves or provisions. It is an amount which is "put aside" from the cash flows received. It aims at facing unexpected events -for instance a rise in the claim cost or a decrease in the premiums-, keeping their balance sheet positive, complying with the regulators and being able to pay their dividends.
The provisions are due to the counter-cycle business of the insurers. Indeed, usually companies know their spending and estimate their earnings. Regarding insurance companies, it is the contrary: insurers know their earnings -the premiums paid by their insured on a regular basis- and estimate their expenditure -the claim costs or claim charge they have to pay on a random basis.

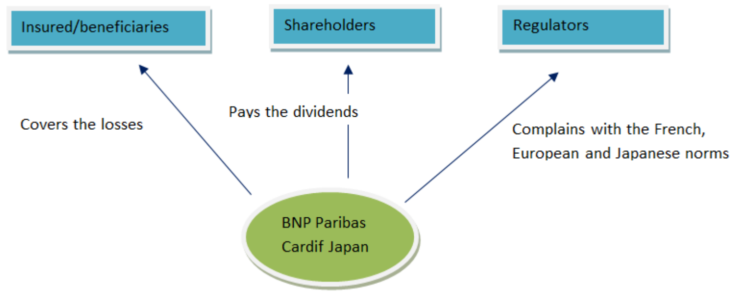


Figure 2.1: The different "constraints" of BNP Paribas Cardif Japan

Because the provisions are "put aside", they are a tremendous issue for insurer. This amount is at the core of the debate between profits and risks. Its allocation reflects the profit and risk vision of the insurer.

2.1.2 Reserving: from a profit vision

Reserving enables insurance companies to pay their future obligations towards their insured. In terms of profit, reserves are a key tool.

First, the reserves stand for a "lot of money" in the balance sheet of insurers. They are the main liability. They stand on the liability side because they are owed to "somebody": insured, partners, shareholders, regulators, ... For instance, the claim reserve is the amount put aside to cover the claim charge that is to say the commitment of the insurer towards its insured. Indeed, when selling the insurance policy, the insurer commits to cover the loss in case a precise event occurs -the event guaranteed in the contract. Other commitments can include the profit shared with the partner. It is not paid on the spot to the partner. The equivalent amount is put aside in the insurer's reserves until its payment to the partner. In the meantime, the amount is frozen, BNP Paribas Cardif Japan reserves it.

Second, from a profit vision, provisions mean less money to invest on which insurers can make profits through investments. Once the claims, expenses and running costs paid, the excess of premiums earned is invested in various financial products: stock, dividends, projects... From these investments, insurers have more earnings. Therefore, the amount of provisions is key for the management of the insurance company. If badly allocated, the reserves may stand for a high loss in the insurer balance sheet.

The reserves are also a message to the insured shareholders. If they are over-estimated -too much reserves in comparison with the real cash flows-, their dividends will be affected. On the other hand, if it is under-estimated, investors may think that the company is performing well whereas it might not be the case.

Thus, the reserves also enable the insurer to allow a current year's balance to become more accurate. That is to say that it has a complete vision of all the costs over one year. This amount captures the costs due to events which have occurred but will be charged later: for instance, due to a lack of information or justifications. Therefore, the insurer has a clearer vision of its balance sheet: what are the current and future cash flows? It can have a better allocation of its resources and go toward investments.

2.1.3 Reserving: from a risk vision

Due to their counter-cycle business, insurers have to reserve a part of their gains. Their earnings are almost deterministic whereas their spending is random and change in function of the occurred claims also called the sinistrality. To make sure they will have enough money to pay the occurred claims, to cover their insured and to remain solvent, provisions are made. They are a "tool" to cover the insurer risks and guarantee the claim payment. There are several types of provisions depending on the business of the company and the risks covered.

Because the underlying risks of the life and non-life products are different, life insurance companies do not require the same reserve allocations as non-life insurance companies. For instance, the life insurance provisions include provisions for bonuses and discounts in the contracts - these bonuses and discounts do not exist for non-life insurance, and, so, there are no reserves made to cover these costs.

Between profits and risks, provisions depend on the risk appetite of the insurer: if prudent, it will put aside more money than needed. On the contrary, the insurer can opt for low reserves and more profits with a higher risk of becoming insolvent. Nonetheless, to protect the insured, the provisions of the insurer are watched out carefully by regulators. Insurance companies have to report their reserve estimates and comply with the required ratios.

2.2 The legal framework for reserving

The insurance sector is regulated by several regulators. They aim at protecting the insured and policyholder rights as well as the sustainability of the insurance sector. Henceforth, several norms regulate this sector. Because of its international position, BNP Paribas Cardif Japan operates under the French, European and Japanese regulations. It has to meet all the requirements of the regulators and to report to all of them.
Several thresholds and indicators are computed depending on the different norms. Regarding the reserves, local norms as well as European and French regulations have to be respected. Among all the norms which define the scope of provisions, the European regulation Solvency II is the latest. It has brought lots of changes in the reserving processes.

2.2.1 The reserves in Solvency II

Solvency II: a prudential vision

Solvency II has been applied since January 2016 to European insurance companies. It is defined by 3 pillars:

- The first pillar is quantitative: it gives the requirements in terms of capital. The main indicators are the Minimal Capital Requirement -MCR- and the Solvency Capital Requirement -SCR. Both are minimal thresholds in terms of own fund capital which protect the insurer from insolvency.
  The MCR is the minimal level of capital required to operate as an insurance company on the European market. If one insurer has the level of its own capital under the MCR, the regulator has the right to act and to stop the business of the company.
  The other Solvency indicator is the SCR: the minimal amount that insurers must have to cover their commitments at one year. Thanks to this sum, the losses from exceptional and un-forecast events can be covered. The risk level is 99.5%. It means that the insurer must have enough own capital to cover 99.5% -or bi century- events. The SCR can be calculated with a standard formula specified in the regulation or through an internal model developed by the insurer and approved by the supervisor. BNP Paribas Cardif Japan uses the standard formula to provide this indicator. Unlike Solvency I, Solvency II considers all the current risks that insurers assume in calculating their Solvency capital requirements -a 0.05% risk including operational risks. The amount of reserves impacts the MCR and SCR as they are add-on margins on the liability side. Indeed, the level of reserve is set, and, the MCR and SCR are computed.
  This pillar also defines the scope for the reserving process as well as the reserve categorization used in the Solvency II balance sheet. The change in the vision of the balance sheet from an accounting to an economic vision has led to new estimations of the reserves: there are done in Best Estimate -BE. Thus, the reserves are categorized on the liability side and split between: the Best Estimate Liability -BEL- and a Risk Margin -RM. The latest aims at covering the reserves insufficiency risks so that the insurer can fulfill its obligations towards its insured in the future.

- The second pillar is qualitative: it gathers governance and risk monitoring. It includes the application of Own Risk and Solvency Assessment -ORSA. It is a set of rules and assessments to help insurers improving the knowledge they have of their own risks. Using the SCR, ORSA assesses if the need in capital is enough to keep the company solvent. Moreover, data quality is highlighted: insurers have to report the processes, the internal checks and the controls to insure the quality of the processed data. It applies for all the estimations carried out by the insurance company.

- The last pillar is about transparency and communication in terms of reporting. Insurers have to report to their insured but also to the regulators on a regular basis and with clearness. Due to its international position, BNP Paribas Cardif Japan reports to the French and European regulators-through its Head Office in Paris- and, also, to the Japanese regulators.

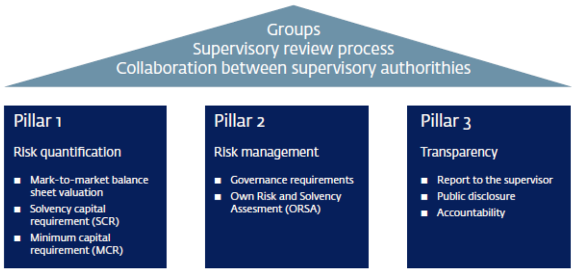


Figure 2.2: The 3 main pillars of the norm Solvency II

The 3 pillars of Solvency II give a broader scope of all the risks of insurers. Quantitative indicators as well as qualitative indicators are considered. Setting thresholds and recommendations help the insurer to follow and to monitor its risks.

Solvency II: data quality and the role of the actuarial function

Among the Solvency II recommendations in the second pillar, the issue of data quality is stressed out. Insurers must establish internal processes and procedures to ensure the appropriateness, completeness and accuracy of the data. Because data from different sources is gathered, and is processed, operational risks as well as sourcing risks appear. In particular, it applies for the calculation of the reserves. Several sources and data are used from the policy contracts of the insured, to the overheads of the company, the claim information... Several checks must be done to check their accuracy among all the steps and between the different sources: external as well as internal sources.
In addition, consistence checks and comparison with the experience have to be performed. Regarding the reserves, these consistency checks are performed on a regular basis, at least once or twice a year. For instance, the Boni-Mali study is part of these "back tests": the estimates of the claim provision are compared with the reality, that is to say with the real claim amount paid. In case of an important boni or mali, the reserving processes can be questioned, and their parameters adjusted.
These checks reduce the operational, sourcing and technical risks.

Moreover, in term of governance, Solvency II defines the role of the actuarial function. Among all its responsibilities, the person in charge of the actuarial function should coordinate the calculation of the reserves and check their appropriateness. He has to control all the steps from the data to the pertinence of the amounts. Once the reserve estimated, he certified the amounts. In case the methods are not appropriate, the actuarial function should alert the management. Thanks to his expert vision, the management weights whether or not the methods have to be revised and what are the consequences on all the cash flows. The decision to change the reserving process goes through several steps: from the actuarial department to the management. Once its approval in Japan, the Head Office in France has the last word. In each step, the actuarial function gives his expert judgment.

Solvency II: a new vision of the balance sheet

Solvency II has proposed a new vision of the insurer balance sheet. From an accounting and historical vision under the previous insurance regulation Solvency I, Solvency II gives an economic vision of the latest. It is also more prudential than under Solvency I.

First, the valuation of the items in the balance sheet is different. On the left side of the balance sheet, the assets are valued based on current market values ie on "the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction". On the right side, liability items are based on the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction [1].
In addition, there is a tremendous distinction between hedgeable and non hedgeable liability items. An hedgeable liability can be replicated through a mix of assets-liabilities with observable prices - marked to market prices. As a result, they are valued as their transferred values: the exchange value on which two willing parties would agree, having all the information available. That is to say the value which can be found on the markets.
On the non hedgeable liability side, Solvency II has brought a new split between the items as well as in their estimations. The main changes are:

1. The Own Funds are divided from the Reserves. They stand for the major part of the insurer's liabilities. All the reserves are gathered under the name : "technical provisions".

2. Reserves -or technical provisions- are valued in Best Estimate -BE- and net of reinsurance. Thus, discounts and market consistence are taken into account to give an accurate economic vision. Finally, technical provisions are computed in function of the different risks. There is a risk segmentation: for each segment[2], provisions or reserves are estimated. All the provisions make the Best Estimate Liabilities -BEL.

3. A Risk Margin -RM- is added to the BEL. It aims to ensuring that an insurer facing bankruptcy can transfer its liabilities to a third party if needed.

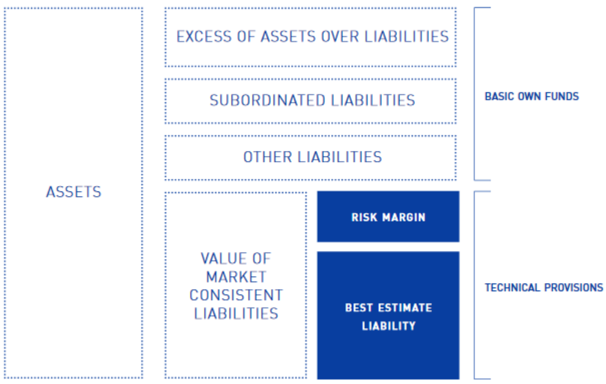


Figure 2.3: The balance sheet under Solvency II :a new vision of the assets and liabilities

2.3 The Technical Provisions

According to the Solvency II Directive, Article 77, "the value of technical provisions shall be equal to the sum of a best estimate and a risk margin". They are mandatory under the French Code of Insurance -article R331-3.

[1] Two parties with all the information on the items
[2] The definition of the segments will be provided later

2.3.1 Definition

The technical provisions stand for the insurer commitments toward its policyholders and other beneficiaries during the lifetime of the insurer's portfolio of insurance contracts. They cover all the insurance obligations be they past, present or future. Because the technical provisions have no observable market prices -and, thus, are non hedgeable-, they are computed as the sum of the Best Estimate of the Liability -BEL- and a Risk Margin -RM. They are based on estimations because they can cover future "developments" be they on the claim or premium sides.
Technical provisions are estimated net of reinsurance. Indeed, not considering the reinsurance underlines the prudential vision of the balance sheet. Reinsurance is an add-on coverage and is not reported in the Solvency II balance sheet. It is estimated separately.

2.3.2 The different kinds of technical provisions

There are several kinds of technical provisions depending on the risks covered by the insurer. Life and non-life risks have different kinds of provisions due to the different risks they bear. Thus, the categories of the technical provisions can vary between the regulations and the countries.
Under the Solvency II regulation, the technical provisions are : the BEL and the RM.

$$\text{Technical Provisions} = \text{BEL} + \text{RM}$$

For life insurance, the main items of the BEL are:

1. The claim reserves : "the expected present value of the cash flows from future payments for the claims occurred before the valuation date". It includes:
   (a) IBNR : reserve for the claims "Incurred But Not Reported".
   (b) RBNS : reserve for the claims "Reported But Not Settled" which includes:
      i. ICOP : the claims "In Course Of Payment".
      ii. NBNA : the claims "Notified But Not Accepted".
   (c) Prudence Margin : a margin added to the other claim reserve items, in case of a wrong estimation or a deviance in the sinistrality. It is optional.
   (d) The claim handling reserve : a reserve to cover claim expenses - mainly due to the claim management.
2. The premium reserves : "the present expected value of cash flows from future premiums, future claim events and ongoing administration of the in-force policies". It covers the "costs of premiums" and includes the insurer obligations within the boundary of the contracts, for all exposure to future claims events. Among the premium reserve, the following sub-reserves are found:
   (a) UPR : the Unearned Premium Reserve or "the amount of unexpired premiums on contracts as of a certain date". It is implemented when the claims and expenses incurred are superior to the earned premiums net of commissions and costs.
   (b) URR : the Unexpired Risk Reserve or the amount in excess of UPR to cover insurance liabilities arising in the following period in connection with insurance contracts in force during the current period. It is a quota share to cover the remaining duration of the contract. It includes the Differed Acquisition Cost -DAC- which is the amount put aside to cover the deferment of sales costs that are associated with acquiring a new customer over the term of the insurance contract.
   (c) The Mathematical or Actuarial reserve -MR: the difference between the expected present value of the insurer's commitment and the expected present value of the policyholder's commitment on a life by life basis. It is required for life risks. That is to say:

$$\text{MR} = \text{Expected Value of the Insurer's liabilities Expected Value of the Insured's Risk Premium}$$

Therefore, the main technical provisions become:

$$\text{Technical Provisions} = \text{BEL} + \text{RM} = \text{Claim reserve} + \text{Premium reserve} + \text{RM}$$

There are also other reserves for instance the capitalization reserve, the bonus reserve or the risk equalization reserve but they do not all apply for the CPI insurance coverage.

Each reserve is estimated under the frame of Solvency II. The European norm gives the general scope to calculate the reserve amounts.

2.3.3 The estimations of the reserves

The general frame: the risk segments and reporting

First, technical provisions are valued by homogeneous risk groups as required under Solvency II[3]. Therefore, a good knowledge of the insurance products, the underlying risks as well as the definitions of the different provisions is required. paragraph Homogeneous risk groups shows similarities in terms of: premiums, claims, claim development, risk exposure... For instance, the following risks and features have to be divided when computing the technical provisions:

* Mortality risk
* Morbidity risk : for instance, disability income, critical illness...
* Lapse risk : for instance, regular premium contracts...
* Profit-sharing or non-profit-sharing business
* Guaranteed or non-guaranteed business

Therefore, under the scope of Solvency II, the following groups are defined, and they require separated reserve estimations:

* Contracts with profit participation clauses
* Contracts where policyholder bears investment risk
* Contracts without profit participation clauses
* Accepted reinsurance

Then, a thinner division is made:

* Contracts with death as the main risk driver
* Contracts with survival as the main risk driver
* Contracts with disability or morbidity as the main risk driver
* Savings contracts

For instance, it results that the CPI death coverage and the CPI accepted reinsurance for death coverage create two reserve amounts and two lines in the balance sheet.

A first analysis of the contracts is required to have a good reserving process. From this analysis, homogeneous groups of risks can be built. The reserve estimates will be computed for each group. The following chart displays the main segments for life products.

Collective or individual protection	Risk
CPI	Critical Illness
CPI	Death
CPI	Accepted reinsurance
IP	3 Critical Illness
IP	CBM

[3] From EIOPA ie Solvency II *"Undertakings should segment their (re)insurance obligations into homogeneous risk groups, and as a minimum by line of business, when calculating technical provisions"*.

The CI coverage is split from the death coverage. In the first product, the main risk driver is disability or CI. Whereas, in the second, the main driver is death. Nonetheless, the CI guarantee comes as an acceleration of death. Divisions also apply for collective and individual protection -IP- as well as insurance and accepted reinsurance. The underlying risks and developments are different.

Finally, a segmentation per currencies is required. The currency is the one used to pay the insurer obligations -for instance, the claims. Only significant currencies will be used: in our case, Japanese Yen -JPY- and Euros -EUR.

The estimation of the BEL is at the very core of the actuarial function. It integrates several parameters and assumptions which require statistical and actuarial judgement. Moreover, data quality checks and consistence checks with the Accounting Department -the department in charge of recording and reporting the cash flows of the transactions- have to be performed.
The Actuarial function is in charge of computing the BEL and the RM from several data sources. Then, the amounts estimated are added to the Solvency II Balance Sheet by the Accounting Department. In order to provide consistent reporting between the Actuarial and the Accounting departments, a reconciliation is needed. Indeed, the same reporting scope is required to have an accurate vision of the insurer balance sheet and of its profits and losses -P& L.
The accounting data is based on the International Financial Reporting Standard -IFRS- and Generally Accepted Accounting Principles -GAAP. It is considered as the reference. The goal of the reconciliation is to check the consistency between the data -technical provisions and assets- in the IFRS GAAP Accounting Balance Sheet and the inputs -model point assumptions or others- used for the BEL calculation by the actuarial function.
In addition, local GAAP -here Japanese- data has be also reported. It is sent to the local regulator the Financial Services Agency -FSA.

The methods to estimate the provisions

Under Solvency II, technical provisions are computed in Best Estimate -BE. The BE is "the expected or mean value -probability weighted average- of the present value of future cash flows for current obligations, projected over the contract's run-off period, taking into account all up-to-date financial market and actuarial information". That is to say that the BE method is an average estimation of the future cash flows in and out of the insurance company.

$$BEL_t = E(\sum_{k>t} \frac{\delta_k}{(1+r)^k}) \tag{2.1}$$

with δ_k the variation of the cash flows out minus the cash flows in and r the actualization rate.

The BEL captures the uncertainty in :

- The timing, frequency and severity of claim events.
- The claims amounts, underlying inflation, and the reporting dates.
- The amount of expenses.
- Other parameters: demographic, legal, medical, technological, social, environmental and economic developments including inflation.
- Policyholder behavior.

All these factors impact the BE valuation of provisions.

2.3.4 The Risk Margin

Moreover, a Risk Margin -RM- is added to the BEL. This margin takes into account the costs due to the immobilization of the capital. Mainly, these costs come from Solvency II requirements -the SCR and the MCR. Indeed, from the capital of the insurer, an amount is frozen because of the requirements.

The required RM is the difference between the expected value and the value needed to achieve a given, overall, entity-wide level of confidence for all risk factors combined, including uncertainty over the assumed distributions. The RM is valued as "the cost of providing an amount of eligible own funds equal to the SCR necessary to support current obligations over their lifetime". Its valuation methodology is called a Cost of Capital -CoC- methodology: the estimate relies on the amount that a third party would accept to take over and to bear the obligations. For instance, it can be obligations regarding the insured, the partners, the running costs, the regulators...

$$\mathrm{RM} = \mathrm{CoC} \times \sum_t \frac{E(\mathrm{SCR}_i^t)}{(1 + r_t)^t} \tag{2.2}$$

With: t: the time, i: a defined risk segment of the insurer, $E(\mathrm{SCR}_i^t)$: the SCR of a given risk segment i for the year t, r_t: the free rate at year t and CoC: the immobilization costs of the capital -estimated at 6%. The SCR gathers the underwriting risks. Among them, there are the reserve and premium risks, the risks of re-insurer bankruptcy, the operational risks and the unavoidable market risks. All these risks are non hedgeable. The RM appears in the left side of the Solvency II balance sheet. It is an add on to the BEL.

2.3.5 The claim reserves - a major reserve for the company

Under the Solvency II norm, the claim provision is "the expected present value of the cash flows from future payments for the claims occurred before the valuation date". It covers all the future claim development and costs, all the future cash flows which arise from the claims already incurred. It is the main reserve for insurance companies.

All the occurred claims have to be paid. However, if we look at the claim cost by date of occurrence, it is distinct from the claim cost by date of notification or payment. Indeed, not all the claims are reported to the insurer on the spot that is to say after occurring. Then, there may be a delay in the payment. Some notifications and payments happen later due to various reasons: late notifications, administrative processes, lack of knowledge of the insurance policy... Even though some claims are reported later after their occurrence, the insurer must be able to pay them at any time. For instance, if a claim occurs in March 2016, but is reported in October 2016, the company has to be able to pay the claim amount at this date. The time between the occurrence, the notification and the payment will be called "the development" or "claim development".

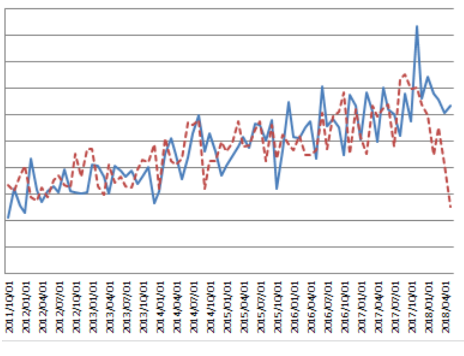


Figure 2.4: The impact of the delay in the claim reporting. In red: the claims paid by date of occurrence. In blue: the claims paid by date of payment

The claim reserves cover the claim development. This reserve does not include the claims reported and paid on the spot, because there is no need to put aside this claim charge. Nonetheless, this reserve gathers several claim statuses categorized in function of the information available for the insurer. The main category is:

- RBNS : the claims Reported But Not Settled
- IBNR : the claims Incurred But Not Reported

The IBNR claim cost is very uncertain because the claims have incurred but have not been reported to the insurer at the valuation date. The valuation date is the time when the reserves are determined and settled in the balance sheet. The main difference between RBNS and IBNR is that RBNS are reported. Their occurrence dates are known as well as an estimation of their amounts. The uncertainty in the RBNS is the status because these claims can become "accepted" or "rejected". Henceforth, IBNR are more uncertain than RBNS because they are not reported. The insurer has no knowledge of the IBNR claim costs. The following chart sums up the information available at time t for the insurer.

Type	Date of occurrence	Date of notification	Status	Claim cost	Provision?
Paid claims	known	known	known	known	no
RBNS	known	known	unknown	know (at least 99%)	yes
IBNR	unknown	unknown	unknown	unknown	yes

The claim development

The claim charge by date of occurrence does no equal the claim charge by date of payment or settlement. There is a delay in the notification of the claim by the insured to the partner. Then, the partner who sells the insurance contract with the loan, has to report to the insurance company. In addition, once reported, the insurance company needs time to proceed to the claim acceptance and, then, to the claim payment. Therefore, the claim report goes through several processes and departments: Operations, Compliance, Accountability, ... During the process time, the insurer assigns an amount in reserve which estimates the likely claim settlement amount. It is a way to make sure it will be able to pay for the claim when the claim report will reach this step.

The claim reserves represent the future obligations of the insurer towards its insured. It varies in function of the claim charge and the claim process: for instance, the closing of insurance companies at the end of the year can slow down the claim process, which increases the claim reserves. A change in the claim status between reported, settled to paid also impacts the amount of this reserve: once paid to the partner, the claim amount is "removed" from the reserve.

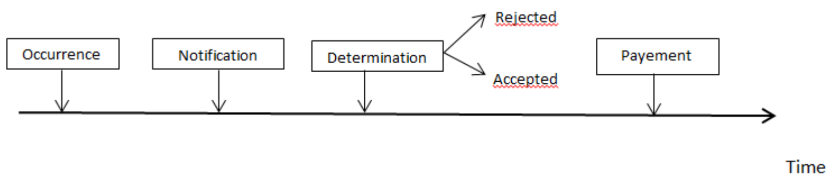


Figure 2.5: The development of the claims

Among the claim reserves, there are two main claim categories depending on the information available: the RBNS and the IBNR. The amount reserved to cover the total future claim cost is :

$$\text{Total claim reserve} = \text{RBNS} + \text{IBNR} + \text{PM}$$

PM is a Prudence Margin added by the insurer to cover risks in the claim reserve allocations. For instance, a change in the claim development pattern can increase the claim charge, without anticipation in the claim reserves. This margin is a safety margin for the insurer.

RBNS

The RBNS include all the claims "Reported But Not Settled". That is to say the claims which have been notified to the insurance company but are in the process of assessment by the company. This assessment consists in accepting or rejecting the claim. Most of the time, for the CPI coverage, the claim cost is known: it is the loan outstanding balance.

The RBNS reserve gathers two steps for the claims:

1. NBNA : the claims Notified But Not Accepted yet. These claims are under the process of being "Accepted" or "Rejected". If one document required to justify the claim -for instance, the death certificate- has not been sent to the insurer, the claim is stated as NBNA. In the meantime, the company adds it to the claim data basis -its occurrence date as well as its cost are known, and its status is "NBNA". Notified claims can be rejected for various reasons: wrong declaration, lack of justification, event no covered in the policy, claim reported twice, premium not paid, loan already paid back,...

2. ICOP : the claims In Course Of Payment. They have been approved ie "Accepted" by the Claim Department, but the payment is done in several times to the beneficiary. In the meantime, the remaining amount is put aside.

The reported claims -RBNS- are first NBNA, once accepted they become ICOP and then paid.

For the CPI coverage, the claim cost is mainly objective and fixed. There is no need of an expert to estimate the costs of the damage as it can apply for other insurance products. Thus, the payment is fast -usually, during the same month or quarter of the claim notification. It results that the ICOP amount is small when we look at the current claim status for the claims occurred before 2018. At 2018Q3, the ICOP amount stands for a 0.4% of the total CI claim charge for the claims reported in 2017. Nonetheless, when we look at all the reported claims, the ICOP stands for a 23% of the claim charge. Indeed, it is due to the last reported claims -for instance at 2018- and which are still in the claim process.

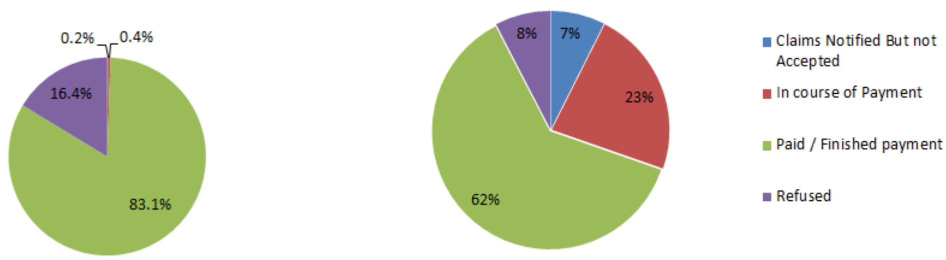


Figure 2.6: On the left: The evolution of the claims notified in 2017 at 2018Q3 and on the right: The distribution of the claim status at 2018Q3 for all the claims reported

Finally, the uncertainty in the RBNS is the status. Because the insurance company does not know if the claim will be accepted or not, it has to reserve the corresponding amount in case the payment happens. This provision depends on the acceptance rate but is determinist.

IBNR

The IBNR are the claims "Incurred But Not Reported". That is to say claims which have already occurred but have not been notified to the insurer. For instance, at date t, Mr John dies, but his close family ignores his insurance policy for death. No one reports the death to the creditor. The claim -John's death- will be notified later. The date of notification to the insurer will be t+n. But, at date t, the insurer estimates the expected cost of this claim and puts the amount aside in the IBNR reserve.

$$IBNR_t = \sum_{i=t+1}^{\infty} \hat{\text{claim}}_t^i \tag{2.3}$$

With $\hat{\text{claim}}_t^i$ the estimation of the claims occurred at date t and notified at date i - with $i \geq t+1$ and $IBNR_t$ the reserve estimate at date t. The real amount is unknown at date t, it is estimated.

The IBNR reserve is the actuarial estimate of the claim expected amount. It relies on the estimation of the future claim notifications: the claim amount, the occurrence date and the notification date are estimated. Under Solvency II regulation, the IBNR estimation is done in BE. Several methods can be used to compute the IBNR amount: Chain Ladder, GLM,... The choice of the best method depends on the available claim data, the regulations, the claim pattern and the risk appetite of the insurer.

Finally, the estimation of the reserves is key from a risk and profit monitoring vision. On the one hand, as this amount is "put aside", insurance companies cannot invest it in other financial funds. On the other hand, if badly estimate, the company can become insolvent due to a lack of resources to cover its obligations. Among all the reserves or technical provisions, the claim reserves are tremendous as they represent most of the insurer reserves. In the following, we will focus on the estimation of these reserves: the data used and the different methods to estimate them.

Chapter 3

Data: quality and description for the estimations of the claim reserves

3.1 Data quality

3.1.1 The quality of the data: which risks?

Under the scope of Solvency II, the assessment of the quality of the data has to be done before computing the reserves. It helps to monitor the risks in the estimation. Indeed, when performing an estimation, operational risk as well as technical risk trigger the estimates.

Only the data to assess the claim reserve will be studied: the claim data bases.

The main risks

The main operational risks are:

- Errors in the data source: for instance, using the wrong data basis, not using the latest available data basis...
- Manual errors: selecting the wrong parameters when a software is used for instance.
- Errors in the process: using the wrong formula, forgetting to update pivot tables...

All these errors "transform" the original data. It results in a "wrong" estimate.

Not only errors can occur when the estimate is computed, but there is also a technical risk around the method used to perform the estimations. The data cannot match the assumptions in the model used, the method can be un-adapted to the goal of the estimate....

Finally, when the person in charge of the actuarial function certifies the reserve amount, all these risks must have been checked. Checks can be implemented to reduce the risks. They can be based on the consistence of the estimates. For instance, the $IBNR_t$ can be compared with the $IBNR_{t-1}$. Thus, automatic checks can be set: adequacy between several data basis, sum, consistence with the balance sheet, with data provided from other department.... These checks have been performed during the study.

3.1.2 The claim data quality

The claim reporting process

To estimate the claim reserve, the data used follows several steps from the occurrence to the reserve estimation. When a claim occurs, there are two ways to report it to BNP Paribas Cardif Japan. The claim can be notified by:

1. telephone at the partner bank

2. directly to the insurer customer service

If the claim is notified to the partner, the latest sends a fax with the contents of the call. From the following the insurer's Claim Department fills the outstanding balance and, so the claim data basis. Then, the documents to justify the claims are asked to the customer: a medical certificate -in case of CI coverage- and a resident card with a settlement of death. Afterwards, the Claim Department asses the claim: its amount, its acceptance...based on the sent documents. Once the status is settled -accepted or rejected-, the claim is paid.

The sources

The main source which gathers claim data is the claim report: a data basis which gathers all the occurred and reported claims to BNP Paribas Cardif Japan. This basis is built by the Claim Department and, finally, used by the Actuarial and Planning Department. It is an Excel file with 15 276 rows. One row stands for one claim.
The file is provided and updated on a monthly basis. It is sent to the Actuarial and Planning department with one-month delay: for instance, at October 2018, the claim report of September 2018 is available.

From the original information, the Actuarial department processes the information to shape the information it needs. For instance, the dates are re formatted to get a quarter format.

The variables

The claim report gathers all the necessary and existing information on the claims. The following chart sums up all the variables required to compute the claim reserve ie the RBNS, IBNR and PM. All the items come from the claim report.

Variable	Description
Date of occurrence	When the claim occurs
Date of notification	When the claim is reported to the insurer
Date of payment	When the claim is paid by the insurer
Claim status	Paid, Refused, ICOP, NBNA
Claim charge	The claim cost
Risk	CI or Death

3.1.3 Checking the consistency of the data

Once the claim data available, the Actuarial and Planning department processes the information. For instance, some variables are added: the delay, the occurrence quarter, the notification quarter... At each process, the pertinence and accurateness of the data has to be checked.

Under this process, adjustments have been highlighted in the claim data basis used to compute the claim reserve at 2018Q3. Mainly, occurrence and reporting dates are not accurate. Indeed, the reporting date is inferior to the occurrence date. The claim would have been reported to the insurance company before occurring. These adjustments represent a 3% of the total claim charge at 2018Q3.
In these cases, a correction has been done: the reporting and occurrence date are set equal. This manual adjustment is based on the expert judgement of the actuarial function. These changes have a small impact on the amount of reserves. They are in line with the paid claim amounts reported by the Accounting Department in the balance sheet.
These adjustments in the dates may be due to the way the claims are reported. For instance, in case the justification of the CI is not provided by the insured, the partner still reports the claims but it does not provide the exact date of occurrence. Once the latest is known, this date is adjusted in the claim data basis. In the meantime, the delay is negative but corrected and set equals to 0 in the reserving process. This adjustment is more prudent than erasing the reported claims.

Finally, once these corrections made, the claim data is considered accurate and reliable to estimate the claim reserve. The same process is done for the previous claim data bases.

3.2 Descriptive statistics of the claim data

A first look at the sinistrality gives an idea of the insurance products, their features and so their claim reserve amount. Especially, the delay as well as the claim charge are tremendous. They will help the actuarial function to check the pertinence of the reserve amount and some deviations.

3.2.1 Evolution of the claim costs

The evolution of the claim cost gives the general trends of the claims. Are there more claims? Are they more expensive? Do they trigger the insurer loss ratio?

Among the total claim charge of the company since 2008 for the selected insurance products, the CI coverage stands for a 56%. The second product is the death coverage.
The claim cost of the CI coverage was 6% higher than the death coverage in 2017. The sum insured are different. If we look at the global cost of claims, there is a decreasing trend for both the death and CI coverage. The decrease is 20% for death and 5% for CI. Nonetheless, this trend is due

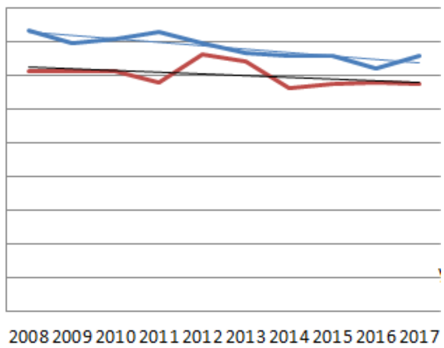


Figure 3.1: The average claim costs in time by product - in blue - the CI guarantee - in red - the death guarantee

to the decrease in the sum insured and the increase in the seniority of the portfolio. Indeed, as time is running, the sum insured -what the insurer pays back- decreases because the insured pay back their loans. Henceforth, the analysis of the evolution of the average claim cost is biased: the average claim cost decreases in time because the seniority rises and not because of a decrease in the claim cost.

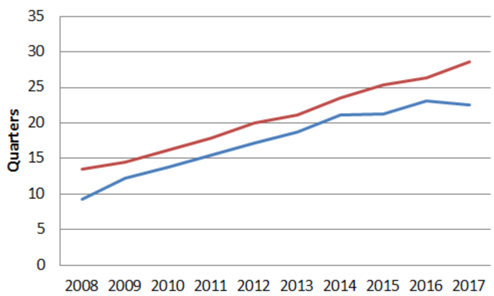


Figure 3.2: The evolution of the average claim seniority -in quarters- in time by risk segment: in red- the death coverage and -in blue- the CI coverage

The claim cost by seniority is also studied to avoid capturing the evolution of this parameter in time. Indeed, when times runs, the seniority increases. For a given seniority, the claim costs are increasing at a slow path but remain volatile.

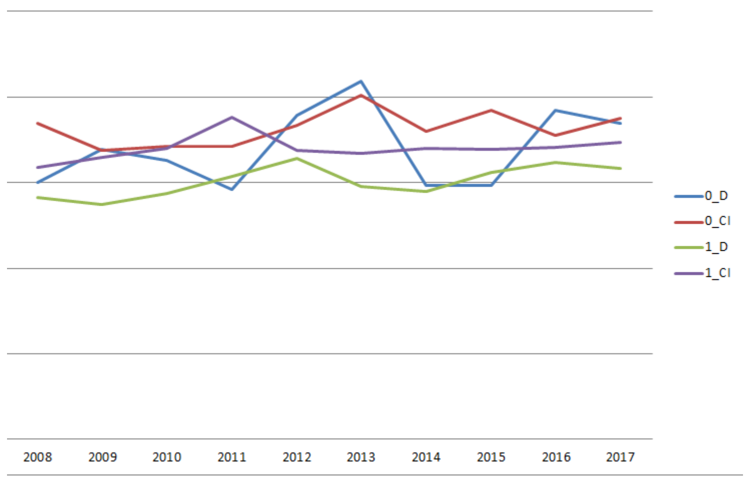


Figure 3.3: The evolution of the claim cost for the seniority 0 and 1 year by risk segment

At constant seniority -that is to say for the same outstanding balance or sum insured -, the claim costs for the CI and the death coverage stand in a small interval. The cost for the CI is a little higher +9% for the seniority 1. It is mainly due to the double guarantee -CI and death- included in this insurance product.

3.2.2 Evolution of the reporting delay

The delay in the claim reporting is a tremendous parameter to compute reserves. Most of the reserving methods relies on the way claims are reported.

In the following, the delay is defined as the time between the claim occurrence and the claim notification.

$$\text{delay=notification date-occurrence date}$$

Its unity is the quarter. Since 2005, more and more claims have been reported "faster", in the first quarters after their occurrence. Especially, when we look at the average delay of notification for the new insured -seniority 0-, it has been decreasing. In 2008, the claims for the death guarantee were reported 2 quarters after their occurrence. In 2017, it was 1.3 quarters. The decrease is more important for the CI coverage as it was launched after the death coverage and it has a higher reporting delay due to the particularity of the insurance product. Thus, the progress in the medical diagnose or in the justification of the CI have improved. It results in a smoother report and a reduction of the delay.

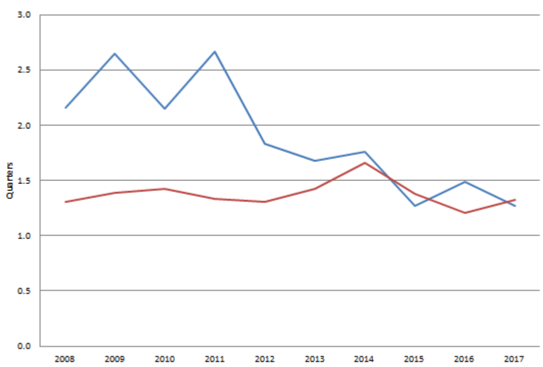


Figure 3.4: The evolution of the average delay in quarters for the new insured until 2017 -in blue- the CI guarantee and -in red- the death guarantee

Chapter 4

The provision process and its assessment for the CPI

4.1 Computing the reserves

The insurer's balance sheet gathers several items under the name of "technical provisions". They depend on the insurance products, the underlying risks and the requirements.

4.1.1 The different reserves for the CPI product

Under Solvency II

Regarding the computation of the reserves, they are done under the Solvency II scope. Non-life and life product are separated, as well as individual and collective contracts. In addition, technical provisions are divided by risks: the CI guarantee and the Death guarantee. Therefore, each risk segment leads to the estimation of reserves.
For the life balance sheet and the CPI product, the following provisions are found:

1. Claim reserves :
   (a) IBNR
   (b) ICOP
   (c) RBNS
2. Premium reserves :
   (a) UPR
3. Bonus reserve

The claim handling reserve does not appear because it is included in the overheads. This specification is due to the necessity of consolidated accounts between FGAAP and JGAPP. Indeed, under the Japan law for Life Insurance, claim expenses are not included in the paid claims but in the overheads. The claim handling reserve is 0.
There is no URR and DAC reserves because the commission rate is constant, and premiums are paid monthly. As a result, no administration cost is differed.

The particularity of the technical provisions of the insurer is the absence of Mathematical Reserve. Because, the collective insurance products -CPI- are re-priced on a yearly basis, the company does not need to have mathematical provisions. The present value of the insured and insurer commitment is the same.

$$\text{Mathematical Provisions} = \max(\text{insurer commitment-insured commitment};0)$$

The insurer commitment is to cover the claim cost. The insured commitment is to pay the premium. When the price is settled at the beginning of the contract, it is based on the average features of the portfolio and the average risk the latest will bear during the contract. When the contract is launched, the premium excesses the risk: the insurer builds its MR -represented with the number 1. in the figure below. Then, as time runs, the portfolio gets riskier and the premium equals the risk -with the number 2. in the figure below. Finally, the risk excesses the premium: if claims occur, the insurer will use its MR to cover the costs -represented with the number 3. in the figure below. However, in the case of our insurer, the premium is repriced every year. The pricing follow the evolution of the portfolio in terms of risks. Hence, there is no need to put aside an amount because the risk is always covered -the figure below on the right. This re-pricing implies a thinner risk monitoring by the insurer and more administration processes.

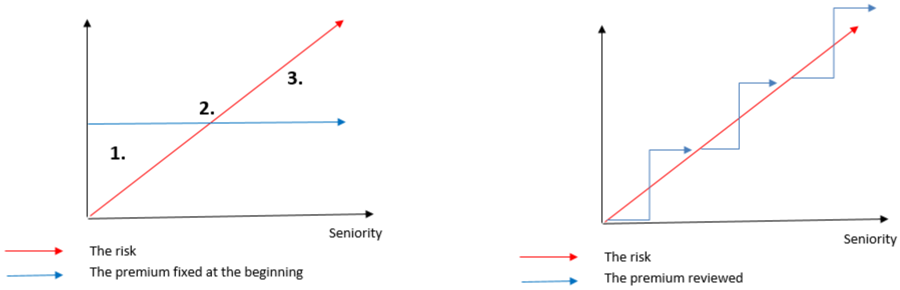


Figure 4.1: On the left : 1. : the insurer reserves the difference between the premium received and the risk amount, 2.: the premium equals the risk, 3.: the risk covered by the insurer is higher than the premium On the right: the repricing of the premium and the risk born by the insurer

Under the Japanese regulation

Due to its international position, BNP Paribas Cardif Japan is under the scope of the French-European regulator and the Japanese regulator : Financial Services Agency -FSA. Even though the Japanese insurance norms tend to follow the Solvency II main principles, there are distinctions in terms of requirements and reporting. As of today, the Japanese vision of the balance sheet remains accounting.

Regarding the reserve amount and the reserving processes, the main differences in the Japanese norms and requirements are:

- For the claim reserves:
  - NBNA are booked with an acceptance rate of 100%
  - IBNR are computed based on a loss ratio adjustment. It is the only method accepted by the regulator.
- For the premium reserves:
  - There is no URR in the Japanese GAAP
  - The UPR is calculated without the adjustment premiums
  - There is no DAC
  - There is a Contingency reserve defined by the Japanese regulator. It is the equivalent as the shareholder equity under the IFRS FGAAP.
  - The MR includes the products with a premium based on monthly instalments

The Japanese regulation tends to be more conservative and prudential than Solvency II. For instance, the NBNA are booked with an acceptance rate of 100%.

During the study, we will focus on the claim reserve under the Solvency II requirements.

4.1.2 The claim reserves - RBNS

As developed previously, the claim reserve is divided between RBNS and IBNR.

Several sources of uncertainty

When a claim is reported to the insurer, it goes through a process before its closing and settlement. There are several uncertainties regarding the claims: its amount and its acceptance. The acceptance is when the insurance company accepts to pay the claim. Otherwise, the claim is rejected: for instance, if the accident is not covered by the contract, in case of a wrong declaration, a loan already reimbursed... Once accepted, there is still the possibility to have a correction of the claim amount. Nonetheless, for the CPI insurance coverage, these corrections of amounts are low in comparison with other insurance products. As the claim charge is the outstanding balance provided and paid to the lender -the banks or financial institutions, there is a small probability of adding the wrong amount in the claim data basis. There are few errors in the outstanding balance, and, so, in the estimation of the RBNS amount.

When it comes to estimate RBNS, the uncertainty toward the acceptance can be evaluated into several ways. The RBNS reserve is the amount put aside while waiting for the claims to be accepted or rejected. The case is settled when the status is determined by the insurance company. If the status becomes accepted, the claim is paid -the amount goes from the RBNS reserve to the paid claims. If the claim is rejected, the RBNS decreases from the claim amount estimation. Henceforth, including an acceptance rate to estimate the RBNS amount enables to have a thinner estimation of the claims reported but not settled.

The acceptance rate

The acceptance rate is the probability for a claim to be accepted and, then, paid by the insurer. A low acceptance rate can be due to the launch of a new product: for instance, if some information in the insurance contract is not well defined or is ambiguous, if the claim department registers several times the claim, if the loans are already paid back, if the premiums have not been paid....

The acceptance rate is defined as follows: at time t,

$$\textbf{Acceptance rate}_t = \frac{\textbf{Number of reported and accepted claims}_t}{\textbf{Number of total reported claims}_t}$$

The previous definition applies because the payment to the beneficiary is made once. On the other hand, a recovery curve would be required to compute the acceptance rate. In our case, once the status is accepted -and changes with a low probability-, the payment is straight forward: the outstanding balance is paid back.

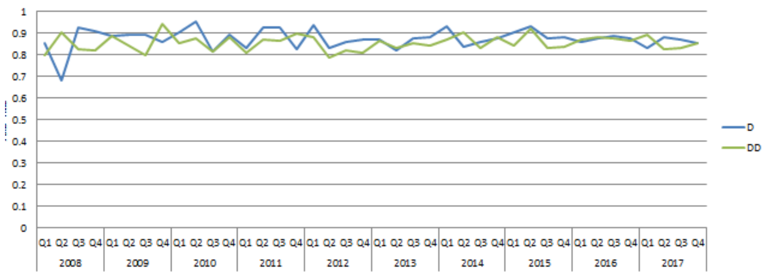


Figure 4.2: The 3-year average acceptance rate for CI and Death coverage. In blue: the death coverage. In green: the CI coverage

Since 2008, the acceptance rates have remained stable around 85%. The CI and death acceptance rates are in the same range. The death coverage has a higher acceptance rate due to the smoothness of its reporting process: only the death certificate is required to benefit from the coverage. There is less ambiguity around the acceptance or rejection. The status is objective and fast to process. Nonetheless, a deeper scale -by sub-group of risks- shows a different picture.

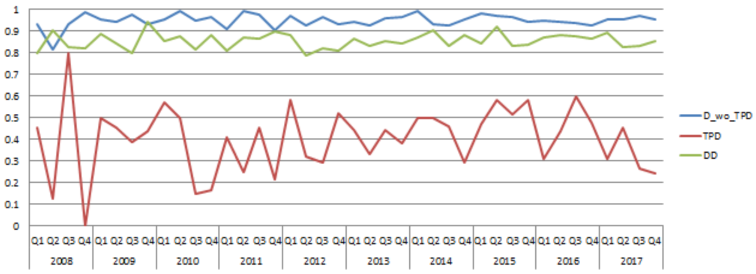


Figure 4.3: The acceptance rate by sub-groups of risks. In blue: the death without the TPD coverage. In red: the TPD coverage. In green: the CI coverage

Under the death coverage, the TPD guarantee is also included. It is a catalysis of the death. TPD is studied separately because of its low acceptance rate: it comes from a tough process when the status is made. Indeed, it is hard to diagnose a TPD -a lot of justifications are required and the medical diagnose is hard to settle. Its claim investigation is longer than for the other guarantees. The average delay -the time between the occurrence and the notification- is 1.32 quarters for the death guarantee without TPD, 1.91 quarters for the CI guarantee and 2 quarters for the TPD guarantee at 2018Q3. After being reported, a lot of reported claims are rejected for the TPD: the 3-year average acceptance rate was around 25% in 2017Q4.

Because the acceptance rate plays a key role in the RBNS estimation and differs between the sub risk groups, the choice is made to use the latest at this risk segmentation or granularity: death only, CI and TPD.

RBNS estimations with acceptance rates

The RBNS are the claims reported but not settled yet. To identify these claims, the status in the claim data basis is tracked: ICOP or NBNA. The variable "Total amount of payment" is 0: no payment has been done for these claims. The cost estimation is the loan outstanding balance. To take into account the impact of the acceptance or rejection of the claims, the acceptance rate acts as a weight to capture the uncertainty in the claim status.

The acceptance rate used is the 3-year average of the acceptance rates based on the historical claim data. It depends on the sub risk groups to capture the low acceptance rate of the TPD and, as a consequence, to be more exact. The rate is then applied to the outstanding balance -the amount that the insurer has to paid back to the beneficiary. At time t,

$$\mathbf{RBNS}_t = \mathbf{Acceptance\ rate}_t^i \times \mathbf{Outstanding\ balance}_t$$

with **Acceptance rate**$_t^i$ the acceptance rate for the sub risk group $-i \in [1,3]$. Once the RBNS are estimated at the sub risk group level, TPD and death are summed to get one RBNS amount for the death coverage. At 2018Q3, with this method, the amount of RBNS to reserve is 8.20x10^8 JPY for the death coverage and 3.46x10^9 JPY for the CI coverage.

4.1.3 The claim reserves - IBNR

There are various methods to estimate the claims which have incurred but have not been reported yet -IBNR. Their amount is based on an estimation of the incurred claim cost. The main method to estimate this amount is Mack Chain Ladder. We will develop the models and its application for the CPI product with the guarantees death and CI at 2018Q3.

The method: Mack Chain Ladder

The Mack Chain Ladder method : it is based on the past developments of the claims. It is the most famous and method used to calculate the IBNR. Its main advantages are :

- its simplicity.

- it -only- requires claim data: the occurrence dates, the notification dates and the claim costs.
- it is approved and recommended by the regulators -the European Insurance Supervisory Authority -EISA- and the French Autorité de Controle Prudentiel et de Resolution -ACPR. It answers the Solvency II norm.

Theory : The Chain Ladder method is based on the incurred claim development. It relies on the aggregated claim cost dynamic displays in the run-off triangle. Another alternative is to use the aggregated claim number dynamic. The run-off triangle is also called the development triangle. The choice is made to use the run-off triangle in function of the claim notification -it can also be displayed in function of the claim payment- and the claim amounts.
The following notations will be used:

- n: the maximum number of quarters to report all the claims occurred in the quarter i. After the quarter n, all the claims occurred during quarter i have been notified to the company at the valuation date.
- i: the quarter when the claim occurred. It is also called "the quarter of occurrence" with $1 \leq i \leq n$
- k : the quarter when the claim is reported or "developed" with $1 \leq k \leq n$. We define the development -or delay- as the time between the claim occurrence and the claim notification to the insurer. k is the development quarter.
- $Y_{i,k}$: the sum of the amounts of the claims occurred in the quarter i and reported in the quarter k. The total cost for all the claims occurred in quarter i and developed in quarter k. They are also called the incremental payments.
- $C_{i,k}$: the aggregated cost of the claims occurred in quarter i and reported at quarter k ie

$$C_{i,k} = Y_{i,1} + ... + Y_{i,k} = \sum_{j=1}^{k} Y_{i,j} \tag{4.1}$$

The development triangle displays the aggregated claim charge $C_{i,k}$ in function of the occurrence i and the development period k. Each row i corresponds to a cohort by accident period. Each column k represents a development quarter. The aim of the Mack Chain Ladder method is to fill the lower part of the triangle using the dynamic of the upper part. That is to say, we want to estimate $\hat{C}_{i,k}$ for $i + k \geq n + 2$ from the available information $\hat{C}_{i,k}$ for $i + k \leq n + 1$. The triangle is filled once the Ultimate Charge $\hat{C_{i,n}}$ is estimated -the last column. From the latest, the IBNR reserve is estimated at each occurrence quarter i $\hat{R}_i$.

Tocc	1	2	3	4	5	6	7	8	9	10	11	12	Provisions
2010/09/01	C(1,1)	C(1,2)	...	...	...	...	...	...	...	...	...	C(1,12)	R(1)
2010/12/01	C(2,1)										C(2,11)	C(2,12)	R(2)
2011/03/01	...									C(3,10)	C(3,11)	C(3,12)	R(3)
2011/06/01	...								C(4,9)			C(4,12)	
2011/09/01	...			...	...	...		C(5,8)				...	
2011/12/01	...						C(6,7)					...	
2012/03/01	...					C(7,6)						...	
2012/06/01	...				C(8,5)							...	
2012/09/01	...			C(9,4)		...	...	...	...	...	...	...	
2012/12/01	...		C(10,3)	C(10,4)								...	
2013/03/01	...	C(11,2)	C(11,3)									C(11,12)	R(11)
2013/06/01	C(12,1)	C(12,2)	C(12,3)	C(12,4)	C(12,5)	C(12,6)	...	...	...	...	C(12,11)	C(12,12)	R(12)
													R(TOTAL)

Figure 4.4: The run-off or development triangle

Assumptions: Chain Ladder relies on tremendous assumptions. Let's F_n being the information available when the reserves are estimated, the filtration generated by the aggregated claim costs ie F_n ={$C_{i,j}$ with $i + j \leq n + 1$, $j \leq n$ }. It is assumed that:

1. H1 : There is no calendar effect. All the occurrence periods i are independent. It means that the claim occurrences are fully independent ie $\forall i \neq j$, $(C_{i,1}, C_{i,2}, ..., C_{i,n})$ is independent with $(C_{j,1}, C_{j,2}, ..., C_{j,n})$. It results that:

$$\forall i \neq k, E(C_{i,j}|C_{k,j}) = E(C_{i,j}) \tag{4.2}$$

The developments per occurrence period are fully independent.

2. H2 : $\forall i \in [1, n]$, $\forall j \in [1, n-1]$, $\exists\ \lambda_j \in R$ so that:

$$E(C_{i,j+1}|F_{i+j-1}) = E(C_{i,j+1}|C_{i,1}, ..., C_{i,j}) \tag{4.3}$$
$$= \lambda_j \times C_{i,j} \tag{4.4}$$

   $(\lambda_j)_j$ is called the Loss Development Factor -LDF. The past development quarters explain the development of the future claims ie $E(C_{i,j+1}|C_{i,j}) = \lambda_j \times C_{i,j}$. As a result, a big and homogeneous claim data basis is required, without extreme events. H2 implies a regularity in the development for all the occurrence quarters. For instance, the claim development from first quarter to second quarter is the same for the occurrence quarter 2018Q2 and 2018Q1.

3. H3 : $\forall i \in [1, n]$, $\forall j \in [1, n-1]$, $\exists\ \sigma_j > 0$ so that:

$$Var(C_{i,j+1}|F_{i+j-1}) = Var(C_{i,j+1}|C_{i,1}...C_{i,j}) \tag{4.5}$$
$$= \sigma_j^2 \times C_{i,j} \tag{4.6}$$

   The variance of the future claim charge for the occurrence quarter i $C_{i,j+1}$ is proportional to the previous claim charge for the same occurrence quarter $C_{i,j}$.

The LDF can be estimated thanks to the following formula:

$$\forall j \in [1, n-1], \hat{\lambda_j} = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}} \tag{4.7}$$

$\hat{\lambda_j}$ is a "weighted average" of all the claims occurred and developed between quarter j and j+1. These estimates summarize the past claim history and the claim development between both quarters. Thanks to these estimates, the lower part of the triangle can be estimated.
The estimation of the ultimate charge $\hat{C_{i,n}}$ is this amount to put aside to cover the full claim development for the occurrence quarter i. It is estimated as follows:

$$\forall i \in [2, n], \hat{C_{i,n}} = \hat{\lambda_{n+1-i}}...\hat{\lambda_{n-1}} C_{i,n+1-i} \tag{4.8}$$
$$= C_{i,n-i+1} \prod_{k=n-i+1}^{n-1} \hat{\lambda_k} \tag{4.9}$$

with $\hat{C}_{1,n} = C_{1,n}$, $C_{i,n+1-i}$ the anti-diagonal of the run off triangle which is known and $\hat{\lambda_k}$ the estimates of the LDF.

For each quarter of occurrence i, the insurer has to put aside the difference between the amount of the claims occurred at quarter i and developed at quarter n -the ultimate charge or the estimation of the claim charge after the last development n- and the amount of the claims occurred at time i and already notified at quarter n+1-i. The claims occurred at time i and developed at time n+1-i are the amounts in the anti-diagonal of the development triangle. Therefore, the estimation of the amount to reserve for the occurrence quarter i $\hat{R_i}$ is:

$$\forall i \in [2, n], \hat{R_i} = \hat{C_{i,n}} - C_{i,n+1-i} \tag{4.10}$$

with $\hat{C_{i,n}}$ the estimation of the ultimate charge and $C_{i,n+1-i}$ the anti-diagonal of the run off triangle which is known. $\hat{R_1} = 0$ because the total development for the first occurrence quarter is known, there is no need to reserve an amount to cover the claim development.

At the valuation date, the total IBNR reserve is the sum of the provisions made for each quarter of occurrence i. As a result, the total amount of provisions $\hat{R}_{total}$ to cover all the occurrence quarters i becomes:

$$\hat{R}_{total} = \sum_{i=1}^{n} \hat{R_i} \tag{4.11}$$

Tocc	1	2	3	4	5	6	7	8	9	10	11	12	Provisions
2010/09/01	C(1,1)	C(1,2)	...	...	...	...	...	...	...	...	...	C(1,12)	R(1)
2010/12/01	C(2,1)										C(2,11)	C(2,12)	R(2)
2011/03/01	...									C(3,10)	C(3,11)	C(3,12)	R(3)
2011/06/01	...								C(4,9)			C(4,12)	
2011/09/01	...			...	...	...		C(5,8)				...	
2011/12/01	...						C(6,7)					...	
2012/03/01	...					C(7,6)						...	
2012/06/01	...				C(8,5)							...	
2012/09/01	...			C(9,4)		...	...	...	...	...	...	...	
2012/12/01	...		C(10,3)	C(10,4)								...	
2013/03/01	...	C(11,2)	C(11,3)									C(11,12)	R(11)
2013/06/01	C(12,1)	C(12,2)	C(12,3)	C(12,4)	C(12,5)	C(12,6)	...	...	...	...	C(12,11)	C(12,12)	R(12)
													R(TOTAL)

Figure 4.5: The full development triangle. In yellow: the ultimate charge. In orange: the amounts of provisions for each occurrence quarter i. In this example n=12

From the Chain Ladder method, T.Mack [1] develops a stochastic version of the previous method. He adds indicators on the quality of the Chain Ladder estimates. Henceforth, the variability of the estimate of the ultimate charge $\hat{C_{i,n}}$ and of the provisions estimated $\hat{R_i}$ can be measured. Quantifying the variability of the estimates is tremendous to assess the risk in the estimations and the deviations which may occur. The risk around the estimations of the provisions $\hat{R}_{total}$ is captured by the variability of the ultimate charge estimate $\hat{C_{i,n}}$ -because $C_{i,n-i+1}$ is known at the valuation date. The main findings of T. Mack are developed below.

First, the estimate of the ultimate charge is unbiased [2]. We consider $i \in [1, n]$. We have:

$$E(\hat{C_{i,n}}) = E(C_{i,n-i+1} \times \prod_{k=n-i+1}^{n-1} \hat{\lambda_k}) \tag{4.12}$$

Because $C_{i,n-i+1}$ is known -the anti-diagonal of the run-off triangle- and the λ_k are not correlated [3] -from H2- detailed in appendix 6.3.2. It becomes:

$$E(\hat{C_{i,n}}) = C_{i,n-i+1} \times \prod_{k=n-i+1}^{n-1} E(\hat{\lambda_k}) \tag{4.13}$$

Finally, using the fact that the (λ_k) are unbiased estimates -details in appendix 6.30-, we have:

$$E(\hat{C_{i,n}}) = C_{i,n-i+1} \times \prod_{k=n-i+1}^{n-1} \lambda_k \tag{4.14}$$

$$= C_{i,n} \tag{4.15}$$

Hence, the estimation of the ultimate charges has a "good quality": its value is close to the real unknown value $C_{i,n}$.

In addition, the variance of the future claim charge $C_{i,j}$ can also be estimated thanks to the following formula:

$$\hat{Var}(C_{i,j+1}) = \hat{\sigma_j^2} C_{i,j} \tag{4.16}$$

where $\forall j \in [1, n-2]$,

$$\hat{\sigma_j}^2 = \frac{1}{n-j-1} \times \sum_{k=1}^{n-j-1} C_{k,j} \times (\frac{C_{k,j+1}}{C_{k,j}} - \hat{\lambda_j})^2 \tag{4.17}$$

and

$$\hat{\sigma_{n-1}}^2 = min(\frac{\hat{\sigma}_{n-2}^4}{\hat{\sigma}_{n-3}^2}, min(\hat{\sigma}_{n-3}^2, \hat{\sigma}_{n-2}^2)) \tag{4.18}$$

[1] T.MACK . 1993 *Measuring the Variability of Chain Ladder Reserve Estimates*
[2] $E(\hat{\beta}) = \beta$
[3] $E(\lambda_j \times \lambda_k) = E(\lambda_j) \times E(\lambda_k)\ \forall j \neq k$

In particular, we are interested in estimating the variance of the ultimate charge and so, of the provisions.

$$\hat{Var}(R_i) = \hat{Var}(C_{i,n} - C_{i,n+1-i}) \tag{4.19}$$
$$= \hat{Var}(C_{i,n}) \tag{4.20}$$
$$= \hat{\sigma^2_{n-1}} C_{i,n-1} \tag{4.21}$$

The smallest is the variance, the best is the estimation of the ultimate charges and, so, the quality of the estimations of the provisions. Indeed, in this case, the estimated values are a little spread around the real values: the estimations are close to the reality.

Application:
the claim data historic is long enough to perform the Chain Ladder method. The full historic of claims is available for the CPI insurance product at 2018Q3.
Before starting the estimations, the consistence of the data is checked. As previously, when the occurrence date is higher than the reported date, they are set equal with a manual adjustment. The statistical software R is used. Several choices are made:

1. The development triangle is based on the sum of the incurred claims. The columns stand for the development periods.

2. From the historical claim data basis, the current maximum development quarter -chosen by the insurer- is 40 quarters -around 10 years of claim development. In addition, it is assumed that there is no development after the quarter 32. Therefore, the development triangle is 40x32. The reasons for such a triangle will be developed later.

3. Death and CI IBNR are computed separately. Indeed, under Solvency II, the IBNR estimations have to be performed according to a risk segmentation.

At 2018Q3, the amount to reserve is IBNR_{2018Q3}=5.71x10^9 JPY -resp 1.09x10^9 JPY- to cover the full development of claims which have occurred before 2018Q3 for CI -resp death coverage.

Risk	IBNR_{2018Q3}
CI	5.71x10^9
Death	1.09x10^9

To the IBNR amount, a PM is added in the balance sheet.

4.1.4 The Prudence Margin

A Prudence Margin -PM- is added to the IBNR_t amount to guarantee that the insurer will cover the claim cost no matter the events. It is an additional reserve which captures the volatility and considers almost all the possible future scenarios. It acts as a buffer against possible exceptional claims: for instance, late claims or an unexpected rise in the claim costs. These late claims or events create volatility in the claim development. Therefore, the PM amount takes into account this volatility and insures with a risk of X% that the insurer will remain solvent in all cases. The level of risk X is chosen by the insurer once a year. The amount to reserve under the PM is then adjusted to meet the desired risk level.

Theory

The PM is computed from the distribution of the IBNR provisions to consider its volatility. It is the difference between the quantile at X% of a theoretical parametric probability distribution and the mean of the provision distribution issued from this distribution. The quartile at X% helps to capture the volatility of the data. It gives a "certain" -X- level of confidence for the IBNR_t estimation. X means that the insurer takes a risk of 100-X% when it reserves IBNR_t. The latest will be covered in X% of the events. Usually, the quartiles are 90%, 95% or 99.5% ie $X \in [90, 95, 99.5]$.

A parametric theoretical distribution is used to model the overall claim reserve distribution. Nonetheless, only IBNR are taken into account because RBNS are determinist. There is no uncertainty in their estimations ie SE(RBNS)=$\sqrt{Var(\text{RBNS})}$=0 with SE(X) the standard error of the variable X and Var denotes the variance.
The theoretical law used is a lognormal featured by

$$\text{Shape} = \hat{R} \text{ and Location} = \frac{\sqrt{M\hat{SEP}(\hat{R})}}{\hat{R}}$$

where $\hat{R}$ is the estimate of the IBNR provision valued at t [4] and $M\hat{SEP}(\hat{R})$ will be developed later. From the distribution of the theoretical probability law, the quantile at X% is chosen. The PM becomes :

$$\text{PM} = \text{IBNR}^{X\%} - \bar{\hat{R}}_n \tag{4.22}$$

$$= \text{IBNR}^{BE,X\%} - \frac{1}{n} \times \sum_{k=1}^{n} \hat{R}_k \tag{4.23}$$

with $\text{IBNR}^{X\%}$ the quantile at X% for the given a priori distribution of the IBNR at valuation t and $\hat{R}_k$ the IBNR estimate.

Finally, the total booked amount for the IBNR claim reserve is: at time t,

$$\text{IBNR claim reserve}_t = \text{IBNR}_t + \text{PM}_t \tag{4.24}$$

And, the total booked amount in the claim reserves is: at time t,

$$\text{Claim reserves}_t = \text{RBNS}_t + \text{IBNR}_t + \text{PM}_t \tag{4.25}$$

Application

From the previous part, the $\hat{R}_i$ are kept. They will be used to compute the parameters μ and σ^2 of the a priori IBNR distribution. The chosen distribution is a lognormal. Thanks to the statistical software R, its distribution is simulated.

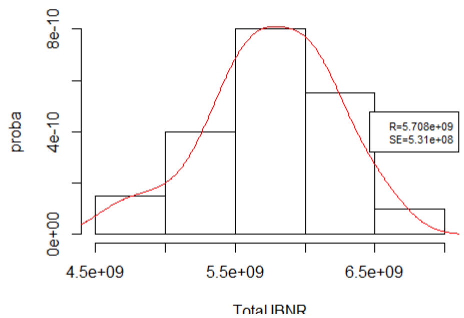


Figure 4.6: The histogram of the IBNR $\hat{R}_i$ from the Mack Chain Ladder method and the density of the theoretical lognormal chosen a priori for the CI coverage at 2018Q3

If Shape = μ and Location = σ^2 ,the mean of the lognormal distribution is $E(X) = exp(\mu + 1/2 \times \sigma^2)$ and its variance is $Var(X) = exp(2 \times \mu + \sigma^2) \times (exp(\sigma^2) - 1)$.
For the quantile at X% -X is not given for confidentiality issues-, the PM is calculated with

$$\hat{\mu} = ln(\bar{R}) - \frac{\hat{\sigma}^2}{2} \text{ and } \hat{\sigma}^2 = ln(1 + \frac{M\hat{SEP}(\hat{R})}{(\hat{R})^2})$$

Where $\hat{R}$ the IBNR amount estimated for the total period.
At 2018Q3, we have -the PM is given in % of IBNR_{2018Q3}:

X%	$PM_{X\%}$ CI	$PM_{X\%}$ Death
90%	13.76	23.27
95%	19.76	30.28
99%	23.40	35.76

[4]It can be the provision for one occurrence quarter i or for the total period

From the previous chart, it results that:

1. The less risk the company wants to take, the higher is the PM -the amount to put aside in addition of $IBNR_t$. To reduce the risk by 5% and be covered in 95% of the events, the insurer needs to increase its PM by almost 2 in comparison with the PM at 90%. If it wants the lowest risks -1%-, the add-on amount is 23.40% of the $IBNR_t$ which has to be reserved.

2. The PM for CI and Death coverage are different. The death coverage requires a higher PM amount. Indeed, as shown previously (3.3), its claim cost is more volatile than the CI claim cost. To be covered against this volatility, the insurance company needs to add a bigger margin than for the CI coverage.

The PM measures the uncertainty of the reserve estimates. The choice of the level of risk depends on the risk appetite of the insurer, When the risk in the estimates is reduced, the amount to put aside in addition of the IBNR is higher. As a consequence, the PM reflects the balance between prudence and profits.

4.2 Back testing the claim reserves

When estimating the reserves, several options and methods are available. To perform an efficient reserving estimation, the expert -the person in charge of the reserving process- has to be aware of various parameters: the underlying risks, the claim process, the data available... He also has to know the updates and changes made, for instance in the claim process. Once calculated, the amount to put aside is then approved by the management of the insurance company. It wants to put aside enough money to cover all its commitments while keeping the remaining to make investments.
To back test the chosen method and its accurateness, the expert can perform a boni/mali study. This study checks the reserve adequacy with the reality.

4.2.1 The boni/mali study: a comparison with the experience of the insurer

The boni/mali study goal is to assess the reserving process and its accurateness with the reality also called the experience. This comparison is requested by the norm Solvency II -article 82.

Boni mali study: method and issues

During the study, the insurance company compares the reserve estimates done in the past years with the current claim data. Indeed, as time runs, more information arrives. More incurred claims are reported and settled. It is a comparison between the claim reserve estimations and the claim experience at the date of the study also called the valuation date. For instance, we compare the amount of claim reserve calculated in 2017Q4 using 2017Q4 data with the amount of claim reserve calculated in 2018Q3 for 2017Q4 using 2018Q3 data. The focus is done on the RBNS and IBNR claim reserves.

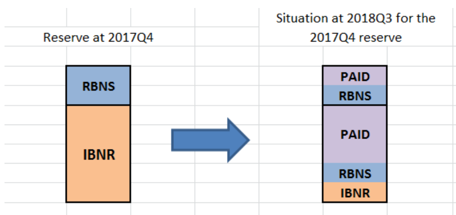


Figure 4.7: The boni/mali study principles: the RBNS estimated at 2017Q4 could have been paid or are still RBNS at 2018Q3 - the IBNR estimated at 2017Q4 could have been paid, have become RBNS or are still IBNR at 2018Q3

At t-1 -for instance 2017Q4- the claim reserves have to cover all the claim occurred before t-1: the claims already reported but not accepted yet, and the claims which will be reported after t-1. At t, new information has arrived: claims have occurred, have been accepted, rejected or notified. This information is compared with the previous claim reserves amount estimated at t-1. If the amount of reserves at t-1 is superior to the current claim charge at t, there is a Boni. If the contrary, there is a Mali.

$$\beta_t = \hat{R}_{t-1}^{t-1} - R_t^{t-1} \tag{4.26}$$

with $\hat{R}_t^{t-1}$ the provision amount computed at the valuation date t-1 with the data available at t-1 and R_t the real amount of reported claims for the period t-1 at t.
The best situation is a boni/mali of zero: the reserving process estimates "perfectly" the future claim development one quarter or several quarters after the valuation date. There is not "too much" or "too less" money in the reserve. In the case of high boni/mali, the efficiency of the reserving methods can be questioned.

To evaluate the boni or mali on the claim reserve amount -RBNS and IBNR-, the comparison with the current claim data has to be done using the same methodology to compute the reserves. The methodology used for this study is:

- RBNS are computed with an acceptance rate as developed previously
- IBNR are estimated with a 40x32 incurred claim development triangle

The first step in the study is to re compute the claim reserves for the past periods with the claim data bases used at that time but with this methodology. The comparison between the different periods is done at constant estimation methodology. Only the changes in the claim costs are captured, and the method is back tested with several data bases. They are based on the past claim data.

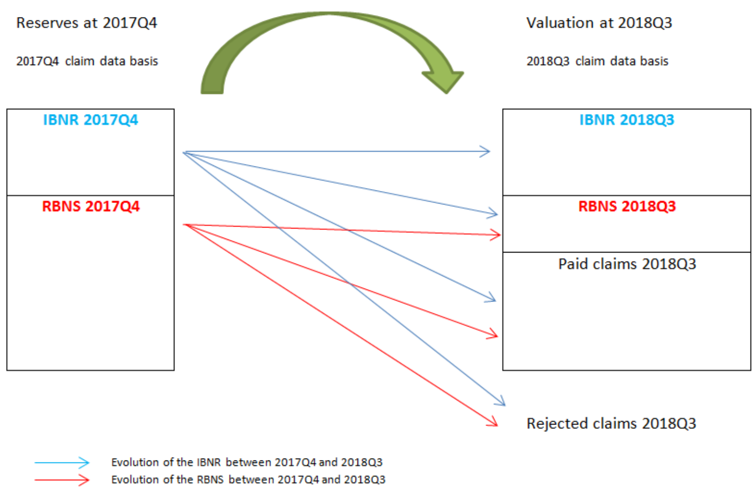


Figure 4.8: Following the claim evolution and back testing the claim reserves

To back test the reserving process, a split is done between RBNS and IBNR.

Boni mali study on RBNS

Work on data: To compare the evolution of the RBNS reserve amounts, the past claim data bases are used. For instance, to compare with 2017, the claim data basis used in 2017Q4 to estimate the RBNS reserve at 2017Q4 is considered. From the latest, we use the last method to compute the RBNS including the acceptance rate.

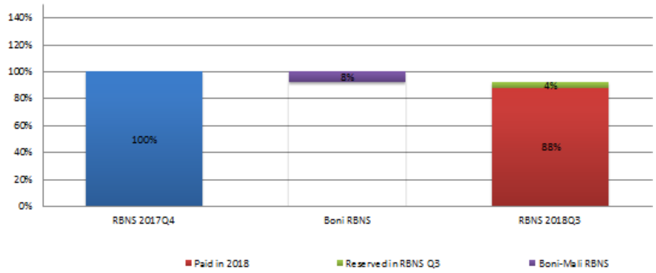


Figure 4.9: The decomposition of the RBNS 2017Q4 boni at 2018Q3 for the CI coverage

Boni/mali study: From the claims stated "RBNS" at 2017Q4, an 88% has been settled and paid at 2018Q3. A 4% is still in the RBNS reserve: these claims are still in the process of being accepted, rejected or are in course of payment by the insurer. Therefore, at 2018Q3, the 2017Q4 RBNS amount -the RBNS in reserve at 2018Q3 for the claims occurred before 2017Q4 and still not "accepted" or "rejected" is 1.44x10^8. From the 2017Q4 RBNS, the paid claims at 2018Q3 have to be subtracted: 2.89x10^9. Finally, the boni-mali on RBNS is: at t=2018Q3 in comparison with 2017Q4,

$$\text{Boni}^{2017Q4} = \hat{\text{RBNS}}_{2017Q4} - \text{Paid claims}_t^{2017Q4} - \text{RBNS}_t^{2017Q4} \tag{4.27}$$

with $\hat{\text{RBNS}}_{2017Q4}$ the RBNS estimated at 2017Q4, Paid claims$_t^{2017Q4}$ the RBNS at 2017Q4 which have be settled and paid at 2018Q3 and RBNS$_t^{2017Q4}$ the RBNS at 2017Q4 which are still RBNS at 2018Q3.
For 2017Q4, there is a boni of 1.53x10^9 that is to say an 8% of the 2017Q4 RBNS amount.

Data date	Valuation Date	RBNS
2018Q3	2017Q4	1.44x10^8
2017Q4	2017Q4	3.29x10^9
2017Q4B	2017Q4	3.75x10^9

At 2017Q4, with the claim data of 2017Q4, the RBNS amount was 3.29x10^9 when computing with the acceptance rate. It is 3.75x10^9 when computing with the 100% acceptance rate. As it captures the possibility for a claim to be rejected -and no paid by the company-, using the acceptance rate reduces the RBNS amount.

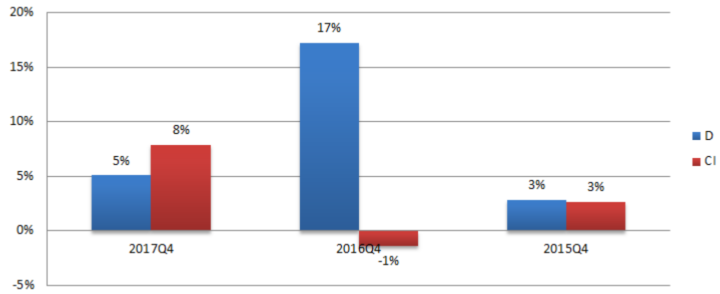


Figure 4.10: The evolution of the Boni-mali on RBNS at 2018Q3 with the acceptance rate in comparison with the previous periods

The figure above shows a constant boni on RBNS for both CI and death -except in 2016 for the death guarantee. High boni or mali have to investigated: are they due to volatility or to a change in the acceptance process, in the reported claims...? From the RBNS reserved in 2015Q4, as off 2018Q3, there is almost no claims to cover: most of them have been paid. Nonetheless, there is a high boni for 2016Q4. It means that "too much" money has been reserved to cover the future claim cost of the claims reported before 2017Q4. The high-boni on the death product is due to the over estimation of the acceptance rate. Indeed, as shown before, the acceptance rate for death is around 80%. A lower acceptance rate due to more claim rejections results in a high boni when compared with the 2018Q3 claim data.

Boni mali study on IBNR

To assess the evolution of the IBNR reserve, the previous claim data bases are used as references. We compare the amount of IBNR estimated in the previous quarters and the current IBNR estimation for these quarters with the latest claim data basis.
The RBNS estimations are assumed to be the same. Their estimations include the 3-year average acceptance rate. As a consequence, only the evolution in the IBNR amounts will be capture
Because of the way the IBNR are estimated, assessing the boni/mali on IBNR leads to the comparison of two development triangles. The first triangle is computed thanks to -for instance- the 2017Q3 claim data basis. Using the Mack Chain Ladder method, it gives the ultimate charge and the amounts of provisions. The second triangle is based on the latest claim data basis -2018Q3.

tocc	1	2	3	4	5	6	7	8	9	10	11	12	provision
2006	1.6.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	1.9.E+09	2.0.E+09	2.0.E+09	0.0.E+00
2007	2.1.E+09	2.6.E+09	2.7.E+09	2.7.E+09	2.8.E+09	2.8.E+09	2.8.E+09	2.8.E+09	2.8.E+09	2.8.E+09	2.8.E+09	2.8.E+09	0.0.E+00
2008	3.5.E+09	4.0.E+09	4.1.E+09	4.3.E+09	4.3.E+09	4.3.E+09	4.3.E+09	4.3.E+09	4.3.E+09	4.3.E+09	4.4.E+09	4.4.E+09	9.3.E+06
2009	4.1.E+09	4.6.E+09	4.8.E+09	4.8.E+09	4.8.E+09	4.8.E+09	4.9.E+09	4.9.E+09	4.9.E+09	4.9.E+09	4.9.E+09	4.9.E+09	1.1.E+07
2010	4.8.E+09	5.8.E+09	5.9.E+09	5.9.E+09	6.0.E+09	6.0.E+09	6.1.E+09	6.1.E+09	6.2.E+09	6.2.E+09	6.2.E+09	6.2.E+09	4.7.E+07
2011	5.7.E+09	6.8.E+09	7.0.E+09	7.2.E+09	7.3.E+09	7.4.E+09	7.4.E+09	7.5.E+09	7.5.E+09	7.5.E+09	7.5.E+09	7.5.E+09	8.2.E+07
2012	7.0.E+09	8.2.E+09	8.4.E+09	8.6.E+09	8.7.E+09	8.7.E+09	8.8.E+09	8.8.E+09	8.9.E+09	8.9.E+09	8.9.E+09	8.9.E+09	1.5.E+08
2013	7.5.E+09	8.8.E+09	9.1.E+09	9.2.E+09	9.3.E+09	9.3.E+09	9.4.E+09	9.4.E+09	9.5.E+09	9.5.E+09	9.5.E+09	9.5.E+09	2.3.E+08
2014	9.4.E+09	1.1.E+10	1.1.E+10	1.1.E+10	1.1.E+10	1.1.E+10	1.1.E+10	1.2.E+10	1.2.E+10	1.2.E+10	1.2.E+10	1.2.E+10	3.7.E+08
2015	1.1.E+10	1.2.E+10	1.2.E+10	1.2.E+10	1.2.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	6.1.E+08
2016	1.1.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.3.E+10	1.4.E+10	1.4.E+10	1.4.E+10	1.4.E+10	1.4.E+10	1.4.E+10	9.8.E+08
2017	***1.1.E+10***	***1.3.E+10***	***1.3.E+10***	***1.3.E+10***	***1.3.E+10***	***1.4.E+10***	***1.4.E+10***	***1.4.E+10***	***1.4.E+10***	***1.4.E+10***	***1.4.E+10***	***1.4.E+10***	***2.7.E+09***

tocc		1	2	3	4	5	6	7	8	9	10	11	provision
2007	2.14E+09	2.61E+09	2.71E+09	2.75E+09	2.78E+09	2.81E+09	2.81E+09	2.81E+09	2.85E+09	2.85E+09	2.85E+09	2.9.E+09	0.0.E+00
2008	3.54E+09	3.99E+09	4.13E+09	4.28E+09	4.29E+09	4.31E+09	4.32E+09	4.34E+09	4.34E+09	4.36E+09	4.36E+09	4.4.E+09	0.0.E+00
2009	4.06E+09	4.61E+09	4.75E+09	4.78E+09	4.8E+09	4.84E+09	4.86E+09	4.88E+09	4.93E+09	4.93E+09	4.93E+09	4.9.E+09	5.0.E+06
2010	4.84E+09	5.79E+09	5.92E+09	5.94E+09	5.98E+09	6.02E+09	6.12E+09	6.14E+09	6.15E+09	6.16E+09	6.17E+09	6.2.E+09	1.4.E+07
2011	5.72E+09	6.8E+09	7.05E+09	7.23E+09	7.33E+09	7.42E+09	7.47E+09	7.48E+09	7.52E+09	7.53E+09	7.54E+09	7.5.E+09	5.7.E+07
2012	7.05E+09	8.18E+09	8.41E+09	8.58E+09	8.71E+09	8.72E+09	8.73E+09	8.76E+09	8.8E+09	8.81E+09	8.82E+09	8.8.E+09	9.3.E+07
2013	7.52E+09	8.84E+09	9.06E+09	9.23E+09	9.28E+09	9.32E+09	9.37E+09	9.4E+09	9.45E+09	9.46E+09	9.47E+09	9.5.E+09	1.5.E+08
2014	9.36E+09	1.07E+10	1.11E+10	1.12E+10	1.12E+10	1.13E+10	1.14E+10	1.14E+10	1.15E+10	1.15E+10	1.15E+10	1.1.E+10	2.6.E+08
2015	1.06E+10	1.21E+10	1.22E+10	1.22E+10	1.23E+10	1.24E+10	1.25E+10	1.25E+10	1.26E+10	1.26E+10	1.26E+10	1.3.E+10	3.7.E+08
2016	1.14E+10	1.28E+10	1.29E+10	1.31E+10	1.32E+10	1.33E+10	1.33E+10	1.34E+10	1.35E+10	1.35E+10	1.35E+10	1.3.E+10	5.8.E+08
2017	***1.44E+10***	***1.56E+10***	***1.6E+10***	***1.62E+10***	***1.63E+10***	***1.64E+10***	***1.65E+10***	***1.66E+10***	***1.67E+10***	***1.67E+10***	***1.67E+10***	***1.7.E+10***	***1.1.E+09***
2018	7.45E+09	8.49E+09	8.69E+09	8.82E+09	8.88E+09	8.94E+09	8.99E+09	9.01E+09	9.06E+09	9.07E+09	9.08E+09	9.1.E+09	1.6.E+09

Figure 4.11: Decomposition of the 2017Q4 IBNR at 2018Q3 - a 11x11 development triangle is used as an example

At 2017Q4, the information available is represented in blue: the claim costs for the occurred and reported claims. In white, the estimated information $\hat{C_{i,j}}$ with the ultimate charge $\hat{C_{i,n}}$ in red. From the following, the insurer can compute its provisions for each occurrence quarter:

$$\hat{R_i} = \hat{C_{i,n}} - C_{i,n+1-i} \tag{4.28}$$

with $C_{i,n+1-i}$ the anti-diagonal of the development triangle.
Between 2017Q4 and 2018Q3, new information on the claims has arrived: claims have been reported to the insurer, some paid, some settled... Hence, the estimated claim amounts -the anti-diagonal below the blue anti-diagonal- is known. The new information impacts the development and, as a result, the estimation of the ultimate charge. For instance, at 2017Q4, the IBNR provision to cover the claims occurred during the quarter 2017Q3 was $2.28 \text{x} 10^9$ -with a 40x32 triangle for the CI coverage. At 2018Q3, it becomes $4.41 \text{x} 10^8$. The future costs for the occurrence quarter 2017Q3 were overestimated in 2017Q4. After one year of claim development, the 2017Q3 IBNR provision has been reduced by 80%.
Henceforth, if the total IBNR is considered, we have:

Valuation date	Occurrence quarter	IBNR
2018Q3	2017Q3	$2.56 \text{x} 10^9$
2017Q4	2017Q3	$5.73 \text{x} 10^9$
2017Q4B	2017Q3	$5.96 \text{x} 10^9$

The valuation date is the period when the reserves are computed with the last claim data basis available. The IBNR for the quarter 2017Q3 means that this amount has to cover all the claims occurred before 2017Q3 and reported after 2017Q3 for the IBNR part -resp. occurred and reported before 2017Q3 but settled after 2017Q3 for the RBNS. The 2017Q4B refers to an estimation based on the 40x32 triangle but with an acceptance rate of 100%. Indeed, the RBNS amount estimated impacts the IBNR estimation: RBNS appears in the blue part of the triangle. Hence, when they are reserved with an acceptance rate, their amount decreases which increases the IBNR estimate -the link between IBNR and RBNS will be developed later.

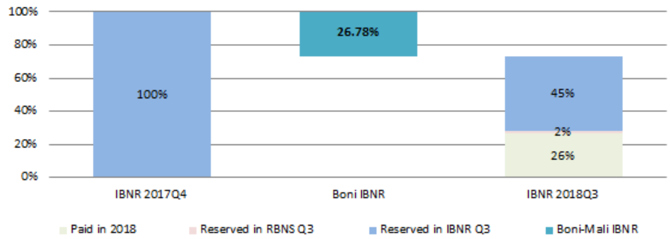


Figure 4.12: Decomposition of the 2017Q4 IBNR at 2018Q3: claims have been paid, reported or are still IBNR

To assess the boni/mali for 2017Q4 at time t=2018Q3, we have:

$$\begin{aligned}\beta^{2017Q4} = \hat{\text{IBNR}}^{2017Q4}_{2017Q4} - \text{Paid claims}^{2017Q4}_{t} \\ - \text{RBNS}^{2017Q4}_{t} - \text{IBNR}^{2017Q4}_{t}\end{aligned}$$

with $\hat{\text{IBNR}}^{2017Q4}_{2017Q4}$ the IBNR amount estimated at 2017Q4 for 2017Q4, Paid claims$^{2017Q4}_{t}$ the paid claims at t from the IBNR estimated at 2017Q4, RBNS$^{2017Q4}_{t}$ the RBNS at t from the IBNR estimated at 2017Q4, and IBNR$^{2017Q4}_{t}$ the IBNR from 2017Q4 still IBNR at 2018Q3.
At 2018Q3, the IBNR boni is 27% in comparison with 2017Q4: indeed, a 45% of the claims occurred in 2017Q4 are still IBNR, a 2% have become RBNS and a 26% have been paid. This percentage is high: it should be around 0.

Boni mali study on the total claim reserves

As seen previously, the boni/mali is the result of a comparison between two development triangles and two different periods. The first development triangle is the one computed with the data from t-1=2017Q4. It highlights the expected claim development for the claims occurred before 2017Q4. The expected development is "measured" through the loss development factors -LDF- and then the ultimate charge $\hat{C}_{i,n}$. The second development triangle used is the one computed with the data from t=2018Q3. In comparison with the previous run-off triangle, it shows another development: new claims have occurred some paid, others in the process of being accepted, others in reserves... New information has arrived in the running time between 2017Q4 and 2018Q3 which impacts the claim development and, hence, the claim reserves.
The claim reserves have to cover the full development of the claims. Hence, its estimation is reliable and accurate if the insurer has enough provisions trough all these developments. To assess the efficiency and accurateness of the claim development estimations -the reserving process-, these 2 triangles are compared. They have to be built using the same methodology: the same RBNS model and the same IBNR model. In addition, because of the operational adjustments spotted in the claim data bases [5], their impacts on the IBNR reserve is also quantified in the total boni/mali study. paragraph When we look at the 3 development triangles on the figure below -3 with the adjustments of the occurrence dates mentioned in the part on the data quality -, 4 factors which impact the back test of the claim reserve have been highlighted. They are all represented with different colors on the figure above. They are:

1. The operational adjustments: mainly, they are adjustments in the previous claim data bases[6]. They explain a small percentage of the boni -0.5%. For 2017Q4, they reduce the boni amount. They are the differences among the A and B triangles represented in blue.

2. The RBNS adjustments: this factor gathers the changes in the real reported claim amount. Between 2017Q4, and 2018Q3, the adjustments in the RBNS amounts can be due to:

    (a) Claim rejections

[5] Highlighted in the chapter on data quality
[6] Also seen in the current claim data basis as underlined in the chapter on data quality

(b) Change in the RBNS amount - for instance, the revision or re-estimation of the claim amount before being settled as "accepted" or "rejected".

They are represented in orange in the C triangle.

3. The IBNR corrections: this factor captures the difference in the reserve estimations done in the past -2017Q4- with the current claim amount. It is a comparison between the real claim charge and their previous estimations as done in the boni/mali study on IBNR. As time runs, the last claim charge estimations have become known because these claims have been reported. Hence, the estimates are corrected, replaced by the real amount. The real amount is not completely real and complete as more claims could be reported in the future, after 2018Q3 for the claims occurred before 2017Q4. This factor stands for the highest part of the boni in 2017Q4- 32%. It is in purple in the last development triangle.

4. Other factors: late claim can impact the development triangle. In addition, new information modifies the development and, as a result, the estimates. This category is in green. This information cannot be back tested as it is still unknown at 2018Q3.

A

	CI - 2017Q4	1	2	3	4	5	6	7	8	9
1	09/2015	1,471.37	1,759.78	1,842.41	1,859.51	1,860.55	1,866.11	1,867.30	1,867.30	1,867.30
2	12/2015	1,366.91	2,076.74	2,112.60	2,114.00	2,139.56	2,143.63	2,143.63	2,143.63	2,144.34
3	03/2016	1,534.29	2,201.92	2,262.63	2,287.87	2,287.87	2,287.87	2,293.42	2,297.26	2,298.03
4	06/2016	1,332.50	1,932.95	1,934.02	1,934.62	1,934.62	1,940.11	1,941.85	1,945.10	1,945.75
5	09/2016	1,131.24	1,931.29	1,963.17	1,966.66	1,973.59	1,978.20	1,979.97	1,983.28	1,983.94
6	12/2016	1,582.59	2,359.14	2,396.97	2,396.97	2,403.77	2,409.38	2,411.54	2,415.58	2,416.38
7	03/2017	1,623.26	2,218.64	2,251.89	2,260.47	2,266.88	2,272.17	2,274.21	2,278.02	2,278.77
8	06/2017	1,741.49	2,576.76	2,621.50	2,631.49	2,638.95	2,645.11	2,647.48	2,651.91	2,652.79
9	09/2017	2,063.59	2,903.30	2,953.71	2,964.97	2,973.37	2,980.32	2,982.98	2,987.98	2,988.97

B

	CI - 2017Q4	1	2	3	4	5	6	7	8	9
1	09/2015	1,471.37	1,759.78	1,842.41	1,859.51	1,860.55	1,866.11	1,867.30	1,867.30	1,867.30
2	12/2015	1,366.91	2,076.74	2,112.60	2,114.00	2,139.56	2,143.63	2,143.63	2,143.63	2,144.34
3	03/2016	1,534.29	2,201.92	2,262.63	2,287.87	2,287.87	2,287.87	2,293.42	2,297.26	2,298.03
4	06/2016	1,332.50	1,932.95	1,934.02	1,934.62	1,934.62	1,940.11	1,941.85	1,945.10	1,945.75
5	09/2016	1,131.24	1,931.29	1,963.17	1,963.17	1,970.11	1,974.71	1,976.47	1,979.78	1,980.44
6	12/2016	1,582.59	2,359.14	2,396.97	2,396.97	2,403.77	2,409.38	2,411.54	2,415.58	2,416.38
7	03/2017	1,623.26	2,215.62	2,245.13	2,253.56	2,259.95	2,265.23	2,267.25	2,271.05	2,271.81
8	06/2017	1,736.81	2,572.08	2,616.58	2,626.41	2,633.85	2,640.01	2,642.37	2,646.79	2,647.67
9	09/2017	2,056.05	2,892.64	2,942.69	2,953.75	2,962.12	2,969.04	2,971.69	2,976.67	2,977.66

C

	CI - 2018Q3	1	2	3	4	5	6	7	8	9
1	09/2015	1,467.29	1,755.70	1,838.33	1,855.43	1,856.48	1,862.03	1,863.23	1,863.23	1,863.23
2	12/2015	1,366.91	2,076.74	2,112.60	2,114.00	2,139.56	2,143.63	2,143.63	2,143.63	2,143.63
3	03/2016	1,534.29	2,201.92	2,262.63	2,287.87	2,287.87	2,287.87	2,293.42	2,293.42	2,293.42
4	06/2016	1,332.50	1,932.95	1,934.02	1,934.62	1,934.62	1,940.11	1,943.13	1,943.13	1,943.13
5	09/2016	1,131.70	1,931.75	1,963.64	1,963.64	1,963.64	1,963.64	1,980.01	1,980.01	1,980.62
6	12/2016	1,582.59	2,350.76	2,388.59	2,388.59	2,388.59	2,388.59	2,388.77	2,392.52	2,393.26
7	03/2017	1,616.32	2,196.45	2,225.97	2,237.11	2,241.93	2,241.93	2,244.54	2,248.07	2,248.76
8	06/2017	1,720.74	2,547.54	2,569.84	2,608.72	2,608.72	2,614.03	2,617.07	2,621.18	2,621.99
9	09/2017	1,988.27	2,474.13	2,488.07	2,491.76	2,497.31	2,502.39	2,505.31	2,509.24	2,510.01
	12/2017	2,291.38	2,927.05	3,006.74	3,019.66	3,026.38	3,032.54	3,036.07	3,040.83	3,041.77
	03/2018	2,455.49	3,328.03	3,383.59	3,398.12	3,405.69	3,412.62	3,416.59	3,421.95	3,423.01
	06/2018	1,543.28	2,164.54	2,200.67	2,210.12	2,215.05	2,219.55	2,222.14	2,225.63	2,226.31

Figure 4.13: The comparison between all the development triangles to decompose the boni/mali - the numbers are illustrative. A: the original development triangle. B: the original development triangle with the adjustment of the dates. C: the development triangle at the valuation date

The same decomposition is made for the death risk. The main explanation of the boni/mali remains the factor (3): "the correction of the estimations with the real" that is to say the corrections of the IBNR estimates.

The risks in the estimations

The boni/mali study has revealed a "bad" adjustment of the reserve estimations when they are compared with the reality. It applies especially for the CI risk. A well-adjusted reserving process would show a boni/mali around 0. This study highlights the accurateness of the claim reserves: it can reveal reserving processes in line with the reality or, on the other hand, far from the real claim charge.

Provisions are not without technical and operational risks. The methods used can be inappropriate depending on the data, the evolution of the underlying risks, the evolution of the business -a growing portfolio, new products,- or the evolution of the insurance products -for instance, new guarantees. Thus, other parameters can affect the determination of the reserves: the legislation, the economic context, the increase in longevity, the change in social behavior, the medical progress... The back test of the claim reserves has to take into account all the evolutions of all these factors. They can explain a high or unusual boni/mali amount.

Not only is it tremendous to have a good and reliable estimation of the claim reserves, but their allocation is a key tool for the management of the insurance company. A high amount of reserves reflects a prudential vision -toward risks- but may be financial gap because the money "put aside" is not invested. Therefore, the reserving process is key for insurance companies. It reflects their risk appetite or risk aversion.

The results of the back-test study on IBNR and RBNS have underlined the over-estimation of the claim reserves. Henceforth, in the following parts, several aspects of the reserving methods will be highlighted. The parameters of the current Mack Chain Ladder method will be chocked to measure their impacts on the claim reserves. Other reserving methods will be presented. At each step, a boni/mali study will be performed to assess the reserve amount: is there enough to cover the claim development based on historical data?

Chapter 5

Issues and methods to estimate the claim reserves

There are various methods to compute the claim reserves. They can be divided between the determinist and the stochastic methods. The first are easier but do not provide statistical estimates to assess the quality of the provision estimations. This piece of information is important for the insurer when it wants to monitor its risks -especially the technical risk. Indeed, the amount to put aside is reliable if it is well estimated.

Because of the conclusions of the previous boni/mali study, several reserving methodologies will be developed and back tested. Moreover, the statistical parameters to measure the quality of the estimates will be displayed. Confidence intervals are also studied because they help the insurers to reduce the scope of the likely estimations: with a risk of X%, the IBNR estimate is between the bounds of the confidence interval.

5.1 Statistical indicators to quantify the quality of the estimations

Not only the values of the amount to reserve are important, but the reliability of the estimates is tremendous. The risks in the estimates can be estimated: this indicator helps insurers to monitor their risks. A good estimate is unbiased and shows a low variance. It has to be the closest possible to the real value of the parameter.

5.1.1 Theory

Stochastic reserving methods -including Mack Chain Ladder- enable its user to compute statistical estimates to assess the reliability of the reserve estimations. The estimate is "good" and reliable when it has a small uncertainty that is to say when there is a small risk in the estimation. The actuarial function is interested to know how far the estimates from the reality are. How volatile is the data and how much uncertainty contains the estimate? Indeed, for the actuarial function, monitoring the risks helps him to give the most accurate advice to the management.

There are statistical estimates to quantify and assess the efficiency and reliability of the estimations. The Mean Square Error of Prediction -MSEP- is the main indicator to determine the quality of the estimations. It measures the uncertainty in the estimates as the mean of the difference between the projections and the real values.
T.Mack in *The standard error of chain ladder reserve estimates: recursive calculation and inclusion of a tail factor* demonstrates the use of this estimator to quantify the uncertainty in the total IBNR amount $\hat{R}_{total}$ through the Chain Ladder methodology.
From the Chain Ladder computation, all the uncertainty in the estimates come from the ultimate charge. The errors made when estimating the ultimate charge $\hat{C_{i,n}}$ capture most of the total error when $\hat{R}_{total}$ is estimated. We consider $i \in [2, n]$ as $\hat{C_{1,n}}$ is known and $R_1 = 0$. For the estimate of

the ultimate charge $\hat{C_{i,n}}$ and with $F_n = (C_{i,j}|i+j \leq n+1)$, we have:

$$MSEP(\hat{C_{i,n}}) = E((C_{i,n} - \hat{C}_{i,n})^2|F_n) \tag{5.1}$$

with $MSEP(\hat{C_{1,n}}) = 0$.
Using $E((X-\theta)^2) = Var(X) + (E(X)-\theta)^2$ with θ a real[1], the MSEP can be decomposed:

$$MSEP(\hat{C_{i,n}}) = V(C_{i,n}|F_n) + (E(C_{i,n}|F_n) - \hat{C}_{i,n})^2 \tag{5.2}$$

where X : $C_{i,n}|F_n$ and $\theta = \hat{C_{i,n}}$ and$\hat{C_{i,n}}$ is known when F_n is known. Therefore, two terms appear in the decomposition of the MSEP:

- $Var(C_{i,n}|F_n)$: gives the process variance that is to say the error in the assessment due to the variability of the process.
- $(E(C_{i,n}|F_n) - \hat{C_{i,n}})^2$: captures the error due to the estimations of the factors. It is the "parameter estimation error".

Knowing that $\hat{R}_i = \hat{C}_{i,n} - C_{i,n+1-i}$ and with the properties of the mean, the MSEP for the provisions $(\hat{R}_i)_i$ can be calculated:

$$\begin{aligned}
MSEP(\hat{R}_i) &= E((R_i - \hat{R}_i)^2|F_n) && (5.3)\\
&= E((R_i - (\hat{C}_{i,n} - C_{i,n+1-i})^2|F_n) && (5.4)\\
&= E(((C_{i,n} - C_{i,n+1-i}) - (\hat{C}_{i,n} - C_{i,n+1-i}))^2|F_n) && (5.5)\\
&= E((C_{i,n} - \hat{C}_{i,n})^2|F_n) && (5.6)\\
&= MSEP(\hat{C}_{i,n}) && (5.7)
\end{aligned}$$

Under the Chain Ladder assumptions H1, H2 and H3, the MSEP can be estimated for each quarter of occurrence i.[2].

$$\begin{aligned}
M\hat{S}EP(\hat{R}_i) &= M\hat{S}EP(\hat{C}_{i,n}) && (5.8)\\
&= \hat{C}^2_{i,n} \times \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma_k}^2}{\hat{\lambda_k}^2} \times \left(\frac{1}{\hat{C}_{i,n}} + \frac{1}{\sum_{j=1}^{n-k} C_{j,k}}\right) && (5.9)
\end{aligned}$$

with the estimation of the variance coefficients provided by T.Mack:

$$\forall j \in [1, n-2], \hat{\sigma_j}^2 = \frac{1}{n-j} \times \sum_{k=1}^{n-j-1} C_{k,j} \times \left(\frac{C_{k,j+1}}{C_{k,j}} - \hat{\lambda_j}\right)^2 \tag{5.10}$$

and

$$\hat{\sigma_{n-1}}^2 = min\left(\frac{\hat{\sigma}^4_{n-2}}{\hat{\sigma}^2_{n-3}}, min(\hat{\sigma}^2_{n-3}, \hat{\sigma}^2_{n-2})\right) \tag{5.11}$$

The estimations of the amounts of provisions by quarter of occurrence i $(\hat{R}_i)_i$ are correlated because they all come from the same estimation of the LDF $\hat{\lambda_i}$. It results that the estimation of the MSEP for the total amount to reserve is not the sum of the estimations of the MSEP by quarter of occurrence i:

$$M\hat{S}EP(\hat{R}_{total}) \neq \sum_{i=1}^{n} M\hat{S}EP(\hat{R}_i) \tag{5.12}$$

The "error" estimated on the total amount to reserve is:

$$M\hat{S}EP(\hat{R}_{total}) = \sum_{i=2}^{n} M\hat{S}EP(\hat{R}_i)^2 + \hat{C}_{i,n} \times \left(\sum_{k=1+i}^{n} \hat{C}_{k,n}\right) \times \sum_{k=n+1-i}^{n-1} \left(\frac{2 \times \frac{\hat{\sigma_k}^2}{\hat{\lambda_k}^2}}{\sum_{t=1}^{n-k} C_{t,k}}\right)) \tag{5.13}$$

[1] $E((X-\theta)^2) = Var(X-\theta) + E((X-\theta))^2 = Var(X) + E((X-\theta))^2 = Var(X) + (E(X)-\theta)^2$, with θ a real
[2] All the demonstrations have been shown by T.Mack in *The standard error of chain ladder reserve estimates: recursive calculation and inclusion of a tail factor*

It gives the variability and the uncertainty of the total IBNR amount. From the latest, the Relative Standard Error -RSE- is computed:

$$RSE_i = \frac{\sqrt{M\hat{S}EP(\hat{R}_i)}}{\hat{R}_i} \tag{5.14}$$

With $i \in [1,n] \cup \{\text{total}\}$, $\hat{R}_i$ the amount of provisions for the occurrence quarter i and $\hat{R}_{total} = \sum_{k=1}^{n} \hat{R}_k$. The RSE is the Standard Error in terms of the total IBNR amount.

Finally, to "compare" the "quality" of the different methods, we will add the analysis of the RSE. The amounts of provisions as well as this parameter will be reviewed. The smaller the RSE is, the best is the model in terms of estimations: the estimates are close to the reality.

From the information added by T.Mack and the estimation of the MSEP, a confidence interval for the estimation of the IBNR at 1-α% can be estimated. It can have two shapes depending on the theoretical distribution assumed for the IBNR. If we consider $i \in [1,n] \cup \{\text{total}\}$.

1. Based on the central limit theorem and the large number of data, it can be assumed that the IBNR follow a normal distribution $(\hat{R}_i, M\hat{S}EP(\hat{R}_i))$. The confidence interval is:

$$\hat{IC}_{1-\alpha} = [\hat{R}_i - \pm q_{1-\frac{\alpha}{2}} \times \sqrt{M\hat{S}EP(\hat{R}_i)}] \tag{5.15}$$

   with $q_{1-\frac{\alpha}{2}}$ the quantile of the Normal (0,1) and the MSEP defined previously. For a risk at 5%: $q_{97.5} = 1.81$

2. Because the normal distribution can lead to negative bounds, it can also be assumed that the IBNR follow a lognormal distribution, featured by the following parameters $(\hat{\mu}_i, \hat{\sigma}_i^2)$.

$$\hat{IC}_{1-\alpha} = [exp(\hat{\mu}_i \pm q_{1-\frac{\alpha}{2}} \times \hat{\sigma}_i)] \tag{5.16}$$

   The parameters are estimated thanks to the following relation :

$$(\hat{\mu}_i, \hat{\sigma}_i^2) = (ln(\hat{R}_i) - \frac{\hat{\sigma}_i^2}{2}, ln(1 + (\frac{M\hat{S}EP(\hat{R}_i)}{\hat{R}_i^2})) \tag{5.17}$$

The details can be found in appendixes 6.3.2 with n=1.

5.1.2 Application

At 2018Q3, the total amount of provision -IBNR$_{2018Q3}$ = $\hat{R}_{total}$, we have:

Risk	RSE in % of the IBNR$_{2018Q3}$
CI	9.30%
Death	21.20%

The RSE is higher for the death product because it is more volatile. There are less claims for this coverage. The smaller number of data points increases the variability and reduces the quality of the reserve estimates. Hence the IBNR estimations for the death risk are more likely to be wrong -or far from the real value- than for the CI coverage.

Moreover, the RSE increases sharply for the last occurrence quarters. Its estimate is mainly based on estimates. In particular, the last occurrence quarter $\hat{C}_{n,n}$ only relies on the real amount $C_{1,n}$. As a consequence, these estimates are riskier, and they show a high RSE. The RSE is 5.26% for the occurrence quarter 40 vs 2.03% for the quarter 39 for the CI coverage at 2018Q3. The figure below shows the rise in the RSE in function of the occurrence quarter.

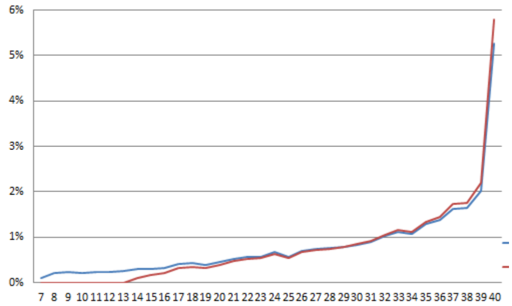


Figure 5.1: The RSE in function of the occurrence quarter for the CI coverage for a triangle 40x40 -in blue- and 40x32 -in red

To build the confidence interval for the total amount to reserve, both the normal and lognormal are considered. We have : i=total to consider the total amount of provisions for the period and n=1.
The confidence interval at 95% is for the CI risk:

Risk	Lower bound	Upper bound
Normal	$4.67\text{x}10^9$	$6.75\text{x}10^9$
Lognormal	$4.74\text{x}10^9$	$6.82\text{x}10^9$

Lower bound refers to the lower bound of the confidence interval. In our case, as the amounts of IBNR are positive, the bounds estimated with a normal law remain positive.
For the death coverage, it becomes:

Risk	Lower bound	Upper bound
Normal	$6.40\text{x}10^8$	$1.55\text{x}10^9$
Lognormal	$7.10\text{x}10^8$	$1.61\text{x}10^9$

The confidence interval for the death coverage is larger than for the CI coverage: indeed, it is more volatile and it has a higher MSEP.

5.2 Determinist methods

RBNS and IBNR can be computed thanks to several methods and assumptions. The choice of the method depends on the data available, the expert advice and the decision of the management. Several options will be reviewed to assess the impacts on the provision amounts. The Mack Chain Ladder method is reviewed in this paragraph even though, T. Mack has developed the stochastic method to compute the RSE.

5.2.1 The acceptance rate and the RBNS

The main alternative in the RBNS estimation is the inclusion or not of the acceptance rate. The latest is less prudent than including all the reported but not settled claims.

Including all the claims

The RBNS reserve is the amount put aside while waiting for the claims to be accepted or rejected. To compute the RBNS amount, all the reported claims can be taken into account. The acceptance rate is assumed to be 100%: all the claims reported are assumed to be accepted and, the reserve amount covers this cost. No recovery nor change in the status of the claims is included because the payment is made once for both the death and CI insurance coverage.

$$\mathbf{RBNS}_t = 100\% \times \mathbf{Outstanding\ balance}_t$$

The alternative method -including the 3-year average acceptance rate- reduces the amount of RBNS by 11% for CI and by 25% for Death at 2018Q3. For the previous years, the 100% acceptance rate leads to more RBNS reserve. The impact of the new acceptance rate is bigger for the death coverage because of the low acceptance rate for the guarantee TPD included in the death coverage.

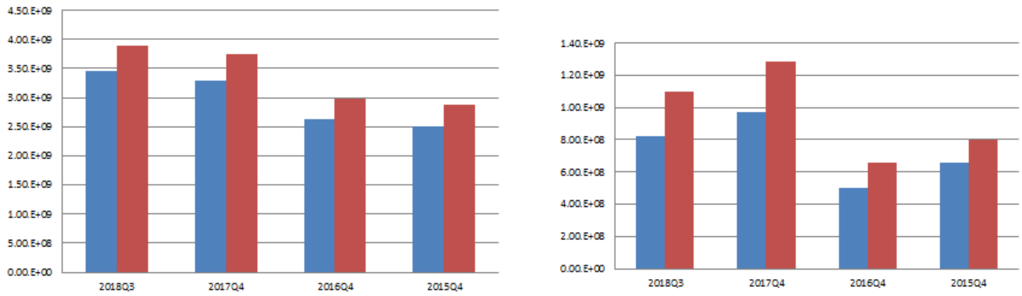


Figure 5.2: On the left: the comparison of the RBNS amount with the different methods for the CI coverage by valuation dates and on the right: the comparison of the RBNS amount with the different methods for the Death coverage by valuation dates. In red: a 100% acceptance. In blue: a 3-year average acceptance

The RBNS valuation with an acceptance rate of 100% is more prudent: indeed, more money than "really needed" to cover the claim cost of the reported but not settled claims is put aside. This sum covers all the notified claims, but some are to be rejected, and so, will reduce the real claim cost. The acceptance rate helps to capture the claim rejection, but it reduces the margin taken by the insurer in its estimations. If there is a deviation in the development of the claims in the future -for instance, a higher acceptance rate- the amount of RBNS might not be enough to cover the reported but not settled claims.

Applying an acceptance rate to the RBNS amount helps to better capture the rejection or acceptance of the reported claims. However, it includes more uncertainty in the RBNS amount because the acceptance rate is estimated: the 3-year average acceptance rate based on historical data. It might deviate from the real claim status and not capture the current trend.

The impact of RBNS on IBNR

The computation of the RBNS has some consequences on the IBNR amount. The IBNR estimates issued from the Mack Chain Ladder method is based on the claim information available at the valuation date: all the reported claims and their development. This information is summed up in F_n or the upper part of the development triangle. The inclusion of the average acceptance rate modifies this part of the triangle and so, the claim development.

When an acceptance rate of 100% is used, all the reported claims appear in the development triangle. Nonetheless, the rejection of some claims modifies the aggregated claim costs $\hat{C}_{i,j}, \forall j+i \leq n+2$ and, as consequence the LDF. When we compare the LDF for the same run off triangle with the acceptance rate 100% and the average, the LDF with the last method are smoother than with the first method. Indeed, the first method -100% acceptance rate- leads to "add" and then "subtract", the claim costs a few development quarters later. On the other hand -an average acceptance rate-, the LDF are smooth: the claim cost does not change. The claim cost $C_{i,j}$ is weighted by the acceptance rate.

Acceptance rate	Occurrence quarter	IBNR
3-year average	2017Q3	5.73x10^9
100%	2017Q3	5.96x10^9

If the 3-year average acceptance rate reduces the RBNS amount, it has a small impact on the IBNR. For instance, at 2017Q4, the amount of IBNR with a 100% acceptance rate is 4% higher than the IBNR amount with the 3-year average acceptance rate. This small change is due to the short development of the CPI coverage: the delay between the occurrence and the development is short. Hence, the changes in the RBNS amounts are mixed with the information on the claim costs $C_{i,j}$

Finally, the RBNS method with the average acceptance rate or a 100% acceptance rate, reflects the risk appetite of the insurer. This choice has a small impact on the IBNR estimates in comparison with other parameters.

5.2.2 The Mack Chain Ladder parameters

The model of Mack Chain Ladder relies on several parameters. To select these parameters, the actuarial function in charge of estimating the claim reserves has to make choices. To do so, it is key to know the impacts of these parameters on the claim reserve amount as well as on the errors in the estimations.

The main parameters of Mack Chain Ladder method

When applying a method, the underlying assumptions have to be checked. In addition, the parameters of the model need to be chosen by the actuarial function.
Regarding Mack Chain Ladder, the expert in charge of estimating the reserves has to select several parameters:

1. The run-off triangle based on the incurred claims or the paid claims
2. The development pattern : is it monthly, quarterly, or yearly?
3. The maximum period of development : it gives the size of the run-off triangle. A larger triangle captures more late claims but does not always check the three hypotheses of the model.
4. The risk segmentation : what are the risk segments and sub risk groups?
5. The theoretical law to compute the PM : the parameters of the IBNR_t probability distribution.

The choices are made in function of the available data, the risks in the estimations and the expert judgement.

Testing the Mack Chain Ladder assumptions

Three assumptions have to be checked to estimate the IBNR with the Chain Ladder method.

1. H1 : The individual LDF -$\hat{\lambda}_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$- must be constant in the occurrence quarter i. It implies that:
$$\forall j \in [1, n-1], \forall i \in [1, n-1], \hat{\lambda}_{i,j} \approx \hat{\lambda}_{.,j} \tag{5.18}$$
with $\hat{\lambda}_{.,j} = \frac{1}{n} \times \sum_{k=1}^{n} \hat{\lambda}_{k,j}$, the average of the individual LDF by occurrence quarter i.
Henceforth, the factors $\hat{\lambda}_{i,j}$ should vary around the mean $\hat{\lambda}_{.,j}$ when they are plotted in function of the development quarter j. In addition, no trend should appear.

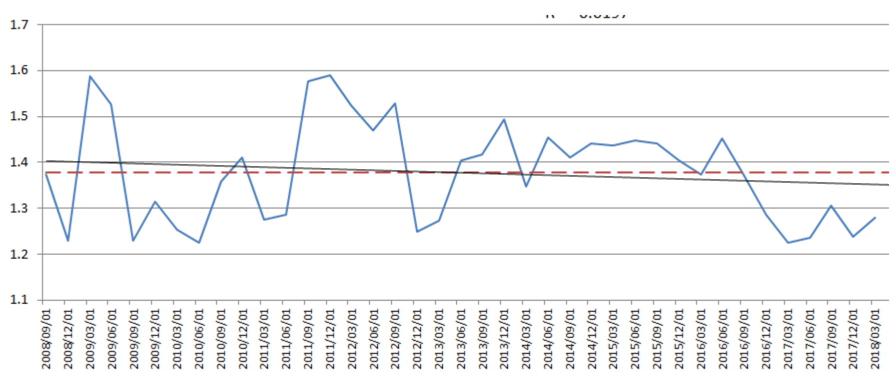


Figure 5.3: The individual LDF for the first development quarter $\hat{\lambda}_{i,1}$ in function of the occurrence quarters for the 40x32 CI development triangle at 2018Q3 -in red dotted lines the average $\hat{\lambda}_{.,1}$ -in dark- the regression line

Application: From the claim data basis at 2018Q3, the individual factors are computed and then plotted in function of the occurrence quarters i with $1 \leq i \leq 40$. The graph below shows the result for the first development quarter ie j=1. The $(\hat{\lambda}_{i,1})_i$ vary around the mean -in red-, with a lot of deviations. It seems that a decreasing trend appear when we look at the linear regression slope, driving by the last occurrence quarters after 2016.
The line equation -in dark- is : $\lambda_{i,j} = a_i \times i + b_i$ for a given quarter of development j -on the figure, j=1. To assess its accurateness, we perform a statistical test. We set: H_0: $a = 0$ vs H_1: $a \neq 0$. At 5 %, we do not reject H_0. The p value of the test is higher than the settled risk of 5%. Therefore, they might be a trend, but our conclusion might be wrong. However, with a risk of 10%, H_0 is rejected. The coefficient of the slope is not equal to 0, there is a decreasing trend. The estimate of the slope is 2.69x10^{-3}: the decreasing trend is small. However, this decrease leads to an over estimation of the LDF and thus, of the IBNR_{2018Q3}. The same analysis is performed for the other development quarters (appendix 6.3) and the death coverage (appendix 6.3.2). For the latest, a trend appears with a risk of 10%.

Thus, to complete the analysis based on the figure, an analytical test is performed following ideas of T.Mack in the Appendix G -Testing for Correlations between Subsequent Development Factors [3]. In the following $j \in [2, n-1]$ and $i \in [1, n]$.
H1 implies that the individual LDF $\hat{\lambda}_{i,j}$ are not correlated. Indeed, we have:

$$E(\frac{C_{i,j}}{C_{i,j-1}} \times \frac{C_{i,j+1}}{C_{i,j}}) = E(\frac{C_{i,j}}{C_{i,j-1}}) \times E(\frac{C_{i,j+1}}{C_{i,j}}) \tag{5.19}$$

which is equivalent to:

$$E(\hat{\lambda}_{i,j-1} \times \hat{\lambda}_{i,j}) = E(\hat{\lambda}_{i,j-1})E(\hat{\lambda}_{i,j}) \tag{5.20}$$

To test this hypothesis, a Spearman test can be implemented[4]. Several steps are required to perform the test following the demonstration made by T.Mack. It is based on a statistical test with:

H_0: the LDF are not correlated vs. H_1

Based on the incurred development triangle, we compute the individual LDF : $(\frac{C_{i,j}}{C_{i,j-1}})$ and $(\frac{C_{i,j+1}}{C_{i,j}})$.

Ci,k+1/Ci,k	1	2	3	4	5	6	7	8	9	10
2008	1.126	1.036	1.037	1.002	1.003	1.003	1.006	1.000	1.004	1.000
2009	1.136	1.030	1.007	1.003	1.008	1.006	1.003	1.010	1.000	-
2010	1.196	1.023	1.002	1.006	1.008	1.017	1.003	1.002	-	n
2011	1.189	1.036	1.026	1.014	1.013	1.006	1.002	-	n	n
2012	1.161	1.029	1.019	1.016	1.001	1.001	-	n	n	n
2013	1.176	1.025	1.018	1.005	1.005	-	n	n	n	n
2014	1.147	1.030	1.015	1.001	-	n	n	n	n	n
2015	1.137	1.012	1.002	-	n	n	n	n	n	n
2016	1.114	1.011	-	n	n	n	n	n	n	n
2017	1.080	-	n	n	n	n	n	n	n	n

Ci,k/Ci,k-1	1	2	3	4	5	6	7	8	9	10	11
2008		1.126	1.036	1.037	1.002	1.003	1.003	1.006	1.000	1.004	1.000
2009		1.136	1.030	1.007	1.003	1.008	1.006	1.003	1.010	1.000	-
2010		$\boxed{1.196}$	1.023	1.002	1.006	1.008	1.017	1.003	1.002	-	n
2011		1.189	1.036	1.026	1.014	1.013	1.006	1.002	-	n	n
2012		1.161	1.029	1.019	1.016	1.001	1.001	-	n	n	n
2013		1.176	1.025	1.018	1.005	1.005	-	n	n	n	n
2014		1.147	1.030	1.015	1.001	-	n	n	n	n	n
2015		1.137	1.012	1.002	-	n	n	n	n	n	n
2016		1.114	1.011	-	n	n	n	n	n	n	n
2017		1.080	-	n	n	n	n	n	n	n	n

Figure 5.4: The individual LDF -$(\frac{C_{i,j}}{C_{i,j-1}})$ and $(\frac{C_{i,j+1}}{C_{i,j}})$ for the CI coverage with a 11x11 development triangle -used as an illustration

Then, for each "new" triangle, the coefficients are ranked from the smallest to the highest, by development quarter j. We have $r_{i,j}$ the rank of the $(\frac{C_{i,j}}{C_{i,j-1}})$ with $1 \leq r_{i,j} \leq$ n-j -on the left. The same is done with the other triangle with $s_{i,j}$ the rank of the $(\frac{C_{i,j+1}}{C_{i,j}})$ with $1 \leq r_{i,j}$ $\leq$ n-j -on the right. Then, the Spearman's rank correlation coefficient T_j is computed:

$$\forall j \in [2, n-1], T_j = 1 - 6 \times \sum_{i=1}^{n-j} \frac{(r_{i,j} - s_{i,j})^2}{((n-j)^3 - n + j)} \tag{5.21}$$

[3] *Measuring the variability of Chain Ladder reserve estimates*
[4] Following the Appendix G -Testing for Correlations between Subsequent Development Factors -in *Measuring the variability of Chain Ladder reserve estimates* by T.Mack

With $E(T_j) = 0$ and $Var(T_j) = \frac{1}{n-j-1}$ under H_0. Thus, we compute the statistic T:

$$T = \frac{\sum_{k=2}^{n-2}(n-k-1) \times T_k}{\sum_{k=2}^{n-2}(n-k-1)} \tag{5.22}$$

with $E(T) = 0$ and $Var(T) = \frac{2}{(n-2)(n-3)}$ under H_0. Using the lack of correlation between the $(T_j)_j$ and assuming that they follow a normal distribution, the statistic T follows a normal law. H_0 is rejected when the test statistic T is outside of the confidence at 50% [5]. Henceforth, H_0 is not rejected if

$$T \in [\frac{-0.67}{\sqrt{\frac{(n-2)(n-3)}{2}}}; \frac{0.67}{\sqrt{\frac{(n-2)(n-3)}{2}}}] \tag{5.23}$$

with 0.67 the quantile at 50% of a normal (0,1) distribution.

Application: the previous test is performed for the 40x32 development triangle for both the CI and death coverage. n=40

$$T = -0.386 \notin [-4.77 \times 10^{-4}; 4.77 \times 10^{-4}] \tag{5.24}$$

For the CI risk, the statistic T is outside the 50% confidence interval. Therefore, we reject the assumption of having uncorrelated LDF. As a result, the first hypothesis of Chain Ladder might not hold. The same conclusion is done for the death coverage.

Henceforth, H1 is not checked by the current development triangle for the CI and death coverage.

2. H2: It assumes that there is a linear relation between the aggregated claim costs by period of occurrence ie $\forall j \in [1, n-1], \forall i \in [1, n-j]$, $C_{i,j+1} = \hat{\lambda}_j \times C_{i,j}$. Therefore, the couples $(C_{i,j+1}, C_{i,j})$ are aligned along a line of slope $\hat{\lambda}_j$, which starts from the origin. It has no intercept. To assess the quality of the adjustment, the R^2 should be around 1. This statistic measures the adjustment of the slope with the data points -here the couples $(C_{i,j+1}, C_{i,j})$.

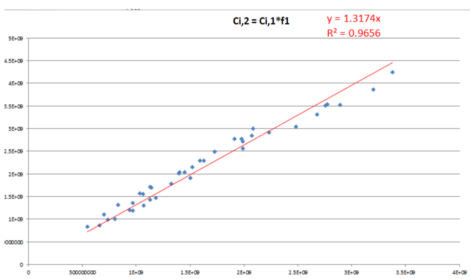


Figure 5.5: The regression line and its equation $C_{i,j+1} = \hat{\lambda}_j \times C_{i,j}$ with j=1

Application: The R^2 is high -almost 1- and the points seem to be aligned along the red line with equation $C_{i,j+1} = \lambda_j \times C_{i,j}$ for the first development quarter j=1. The same analysis is done for the next development quarters as well as the death coverage (appendix 6.3.2). This hypothesis holds.

3. H3: The last assumption implies that there is no trend in the weighted residuals when they are plotted against $C_{i,j}$. They are defined as followed: $r_{i,j}^{MCL} = \frac{C_{i,j+1} - \hat{\lambda}_j \times C_{i,j}}{\sqrt{C_{i,j}}}$.

For the death risk, the distribution seems to be random and spread around 0. On the other hand, for the CI risk, the randomness of the weighted residuals can be questioned. A trend appears for the last occurrence quarters.

[5] The risk level advised by T.Mack

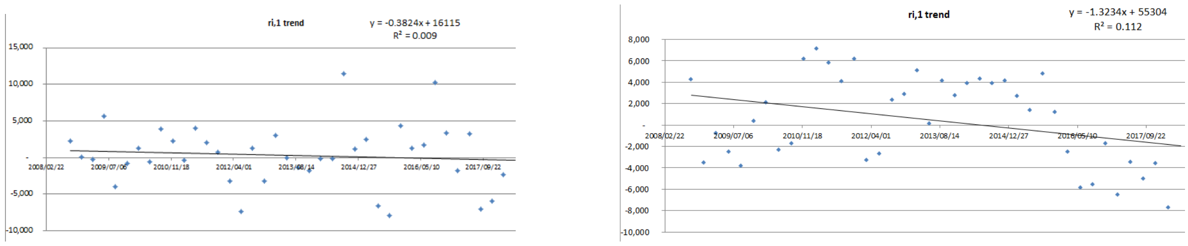


Figure 5.6: On the left:$r_{i,1}^{MCL}$ for the death coverage - On the right :$r_{i,1}^{MCL}$ for the CI coverage

Even though not all the hypotheses of the Mack Chain Ladder method are checked, it is used to compute the IBNR_t. Thanks to the previous checks, the insurer is aware of the risks in the estimations. As the data does not check the hypothesis, the estimates may be wrong and risky. During the study, before using a development triangle, the previous conditions H1, H2 and H3 will be checked.

The cadence

The first parameter to choose is the cadence - that is to say the periodicity of the development triangle. It can be computed by years, quarters or months. This choice depends on the claim data available as well as the expert view.

The claim data basis gathers all the reported claims for death coverage and for CI coverage. Therefore, the run-off triangle can be displayed whether by years, quarters, months... In the study, the initial development model relies on a quarterly development pattern.
To measure the changes between quarterly and yearly basis, we have computed the yearly development triangle with the same claim data basis. The triangle is a 11x11. Regarding the hypothesis of the Mack Chain Ladder method, H3 and H1 are not checked by the 11x11 development triangle with the claim data at 2018Q3. A trend in the weighted residuals appears as well as in the individual LDF. The test proposed by T.Mack is not checked neither.

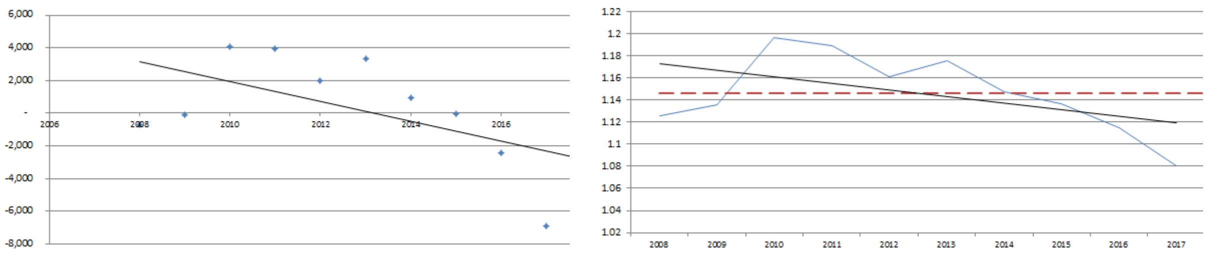


Table 5.1: On the left:H3 for the CI coverage with a 11x11 development triangle- On the right : H1 for the CI coverage with a 11x11 development triangle

For the CI coverage, the provisions are estimated at 2018Q3: the amount to reserve is lower -28% lower- than with the 40x32 run-off triangle. However, the RSE is higher: 16.29% in comparison with 9.30% for the 40x32 development triangle. There is more volatility in the 11x11 triangle due to less data points. The rise in the PM to add, reflects this increase in the volatility: it growth from 30.28 to 23.13 -in % of the IBNR with a risk of 5%.

Size	IBNR_{2018Q3} (CI)	RSE
40x32	$5.71\text{x}10^9$	9.30%
11x11	$4.06\text{x}10^9$	16.29%

For the death coverage, the new triangle reduces the provisions by 33%. Nonetheless, the RSE remains in the same range -21.49% for the 11x11 vs 21.20% for the 40x32- of the IBNR amount in comparison with the 40x32 triangle. The decrease in the RSE is less clear for the death coverage, than for the CI coverage. Hence, the PM changes a little: from 30.28 % to 30.71% with the yearly development. The hypotheses of Mack Chain Ladder method are still no validated by the 11x11 death development triangle -H1 and H3- at 2018Q3.

Size	IBNR_{2018Q3} (death)	RSE
40x32	$1.09\text{x}10^9$	21.20%
11x11	$7.36\text{x}10^8$	21.49%

Because the yearly development increases the RSE, this method seems to be riskier than the quarterly development triangle. It appears clearly for the CI risk. To back test the change in the cadence, we compare the IBNR estimations from a 11x11 with the 2017 claim data with the real development. We observe a mali of -33% for the CI. Indeed, the IBNR estimate at 2017Q4 is $5.05\text{x}10^9$ and the real claim costs as of 2018Q3, is $7.23\text{x}10^9$. It results that the company would have become insolvent if it had to cover the full claim charge at 2017Q4. Hence, the yearly development is riskier than the quarterly development: it under-estimates the claim development.

The number of claim developments

The maximum development quarter n has to be chosen. It gives the size of the development triangle n. After n quarters of development, it is assumed that no more claims will be developed and impact the development triangle. The choice of n depends on the claim development.
In order to choose n, a study of the claim data basis can be done: from the historic, how long does it take for a claim to be reported after its occurrence? At 2018Q3, on average, the delay is 8 quarters for the CI between the occurrence and the notification. It is smaller for the death coverage: 5 quarters. The claim process and the justification required explain the difference between the two guarantees.

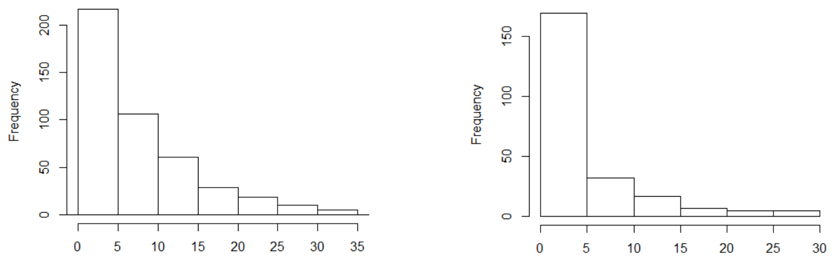


Figure 5.7: Repartition of the reporting delay for the CI and death coverage with the 2018Q3 claim data basis

n shapes the size of the development triangle. The IBNR -the provision- is a function of the past and present exposure. Under the Mack Chain Ladder model, it is assumed that the claim development is stable in time -H2- that is to say that the claims for the quarter of development j -with $1 \leq j \leq n$ develop in the same way at i and i+1 -2018Q1 and 2015Q1, for instance. It is a strong assumption for the insurers because they are affected by several events. For instance, the working of the company -the number of employees in charge of the claims, the seasonality, the claim process, the change in the technology used to report the claims...- can modify the "constant" development pattern. Henceforth, when the size of the development triangle is chosen, the company has to assess if changes have occurred in the different steps used by the claims for all the occurrence quarters i with $1 \leq i \leq n$. Is it the same amount of claims notified after j quarters? Inflation and economic indicators can also modify the regularity of the development pattern.

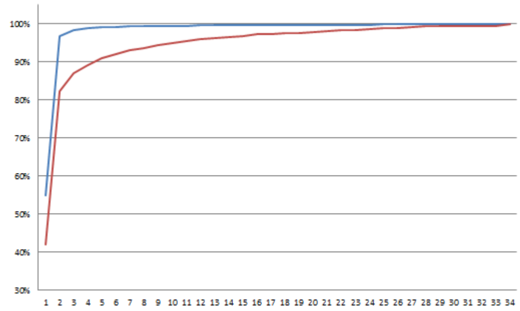


Figure 5.8: The percentage of claims reported after quarter i -in blue: the death coverage. In red: the CI coverage

From the 2018Q3 claim data basis, we can see that the claim development is fast during the first development quarters. After 3 quarters, 87% -resp.98%- of the CI -resp death- claims have been notified. The development is faster for the Death risk, because the reporting process is easier. As a result, the assumption of 40 quarters is realistic: almost all the claims have been reported. Thus, n=40 enables to capture late claims: there are claims which are reported "a long time after their occurrence". Claims are late in reporting when their development period is more than 18 quarters. 18 quarters is the quantile at 90% of the development delay distribution for the CI. Finally, getting more development quarters allows to have more data and therefore, it improves the quality of the estimates. In addition, it enables the insurer to capture late claims which may trigger its solvency if not considered. The latest could have reserved not enough money to face these unexpected costs.

What are the impacts of a change in the development triangle size? We will compare a triangle 24x24, a 40x32 triangle -current size, and a 40x40 development triangle with the 2018Q3 data. In JYP:

Size	$IBNR_{2018Q3}$ - CI	$IBNR_{2018Q3}$ - death
40x40	6.44×10^9	1.09×10^9
40x32	5.71×10^9	1.09×10^9
24x24	4.26×10^9	9.56×10^8

For the death coverage, all the claims are reported before 32 quarters of development. As a result, the IBNR amount is the same with the 40x40 and 40x32 run-off triangle.
The higher is the size, the highest is the provision. Indeed, more claims are captured, and the triangle covers more developments. For instance, the 40x40 triangle covers the future costs of the claims occurred from 2008Q3, whereas the 24x24 only covers the occurrence dates from 2012Q3. Although, the 24x24 triangle reduces the IBNR amount at 2018Q3 -by 34% for the CI and 23% for the death-,it is more "dangerous" than the 40x40 or 40x32 development triangle because of the late claims. The insurer may not capture them in its reserves and it may face difficulties to pay them once reported.

In terms of the quality of the estimations, the RSE is:

Size	RSE_{2018Q3} CI	RSE_{2018Q3} death
40x40	10.14	21.20
40x32	9.3	21.20
24x24	11.28	24.16

Regarding the 40x32, the RSE is "biased" because there is no uncertainty for the 8 last development quarters. The LDF $\hat{\lambda}_j$ are fixed equal to 1. The 40x40 development triangle shows a little less volatility for both risks than the 24x24 triangle.

Henceforth, there is a trade-off between the amount to reserve, the risks in the estimate and the threat of late claims.

5.2.3 The Mack Chain Ladder limits

Once the parameters chosen to estimate the IBNR reserve, the method uses the past claim development to forecast the future claim development. Even though it is friendly user, there are several limits. The main drawbacks of Mack Chain Ladder are:

- The estimates can be biased due to a lack of data -for instance, in the case of the ultimate quarter. Indeed, the estimate of the maximum development relies on a small number of data. It is very volatile and stands for a lot in the total IBNR amount.
- The claim development pattern may not be constant in time. A tremendous hypothesis is the constant development pattern for all the occurrence quarters i. It is determined by the past claim development. Indeed, the claim settlement is assumed to be unchanged -no speed-up or slow-down in the reporting periods.
- The occurrence of an exceptional event, for instance late claims.
- Irregular payment development.

The volatility of the last occurrence quarter

When it comes to estimate the claim charge for the last occurrence quarter $\hat{R}_n$, Mack Chain Ladder shows its limits.

The IBNR provision for the last occurrence quarter is very volatile and stands for a lot in the total IBNR estimation. Indeed, this amount is higher as it has to cover all the claims occurred at quarter n and which will be notified in the future. It covers a development period from 2 to n quarters ie 39 development quarters. In addition, this estimates $\hat{R}_n$ is "very" risky.

To measure the risk in the estimation of the ultimate charge of the last occurrence quarter $\hat{C}_{n,n}$ the RSE [6] is used. It grows in function of the occurrence quarter. Between the occurrence quarters n and n-1, the RSE increases by +61% to reach 50% -which stand for half of the total RSE at 2018Q3.
The RSE increases in the occurrence time as the ultimate charge $\hat{C}_{i,n}$ relies on more estimations. That is to say that the most recent is the occurrence quarter, the worse is this estimation. Indeed, the ultimate charge of the last occurrence quarter relies almost on the estimates -$\hat{\lambda}_k$ and the known claim charge for the first development quarter $C_{n,1}$.

$$\hat{C_{n,n}} = C_{n,1} \times \prod_{k=1}^{n-1} \hat{\lambda_k} \tag{5.25}$$

As a consequence, an error or a deviation in the real claim charge $C_{n,1}$ will impact sharply the ultimate charge for this quarter $\hat{C}_{n,n}$. The evolutions can come from the increase in the reported claims at this quarter or a delay in the administrative report -for instance if too many claims are reported, and the company faces difficulties to register them. Thus, the provision for this occurrence quarter $\hat{R}_n$ will be impacted and, because of its weight, the total IBNR also.

$$\hat{R}_n = \hat{C_{n,n}} - C_{n,1} \tag{5.26}$$

For instance, at 2018Q3, the provision for the occurrence quarter 2018Q2 $\hat{R}_n$ represents a 35% of the total amount to reserve. Therefore, $C_{n,1}$ plays an important role among the known claim costs. If this claim cost $C_{n,1}$ increases by 10%, the total IBNR provision increases by 3.28%. However, if $C_{n-1,1}$ increases by 10%, the total IBNR provision decreases only by 0.65%. A 40x40 development triangle has been chosen to capture the full development and the impact of a change in $C_{n,1}$. The changes in the first development quarter for the last occurrence quarter play a key role in the total IBNR estimation. This amount and its variations should be watched by the expert in charge of the provisioning process.

[6] $RSE = \frac{SE(\hat{R}_{total})}{\hat{R}_{total}} = \frac{\sqrt{M\hat{S}EP(\hat{R}_{total})}}{\hat{R}_{total}}$

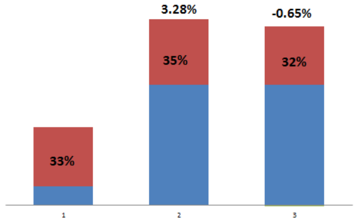


Figure 5.9: Impacts on the IBNR at 2018Q3 of: 1.The initial IBNR amount 2.An increase of 10% of $C_{n,1}$ and 3.An increase of 10% of $C_{n-1,1}$. In red: the % of $\hat{R}_n$ on the total provision with a 40x40 development triangle

The importance of the last occurrence quarter and its impact on the total amount to put aside can be measured through the boni/mali study. Thanks to the previous claim data bases used to compute the previous IBNR amounts at previous valuation date, the past information can be compared with the current information. For instance, the cost of the claims occurred at 2016Q3 and reported at 2016Q3 was 8.68x10^8 ie $C_{40,1}$=8.68x10^8 at the valuation date 2016Q4. The total amount to put aside to cover the claim cost at 2016Q4 was $\hat{R}_{40}$=6.31x10^8. At 2018Q3, this cost has become 2,68x10^9. If it is assumed that after 2 years all the occurred claims have been notified, the real claim cost is 2,68x10^9. As a consequence, the amount put aside in 2016Q4 -6.31x10^5 - does not cover the real charge -2,68x10^9: there is a mali on the occurrence quarter 2016Q3. If all the claims were reported "on time", the insurer would have been insolvent.

Valuation Date	$C_{n,1}$	$\hat{R}_{2016Q3}$	$\hat{R}_{2016Q3}$ in % of the total IBNR at valuation date
2016Q4	8.68x10^5	6.31x10^5	34.12
2017Q4	2,68x10^9	3.10x10^8	5.41
2018Q3	2,68x10^9	1.98x10^8	3.48

The previous chart underlines the weight of the last occurrence quarter: it stands for a 34.12 % of the total IBNR amount at 2016Q4. However, this occurrence quarter 2016Q3, only weights 5.41% one year late and finally, 3.48 % two years later.

The last quarter estimate is very risky. Not only it weights a lot in the total IBNR to cover the period, but it is very volatile. It is usually under estimated. In addition, the issue of the available data can be underlined.

The last month adjustment

Not only the last occurrence quarter is very sensitive but all the data to perform its estimation is not available. Because of the delay to get the claim data from the partners and the Claim Department, the IBNR is "adjusted" with one-month roll out. .

$$\mathrm{IBNR}_{\mathrm{Total}} = \mathrm{IBNR}_{\mathrm{without}} + \mathrm{IBNR}_{\mathrm{Adj}} \tag{5.27}$$

With $\mathrm{IBNR}_{\mathrm{without}}$ the amount of IBNR estimated without considering the last month of the last occurrence quarter and $\mathrm{IBNR}_{\mathrm{Adj}}$ the last month roll out. For instance, at 2017Q4, the last month taken into account is 10/2017.
The last month estimation is based on the evolution of the previous IBNR and the moving average of the historical loss ratios by risks. It stands for around 5% of the $\mathrm{IBNR}_{\mathrm{without}}$.

The impacts of late claims

Late claims are very dangerous for the development pattern. They are claims which are reported a long time after their occurrence. Late claims impact the top right part of the triangle and so, affect the whole "snowballing" development. As a consequence, the ultimate charge estimates are also impacted -especially the last occurrence quarters n. They are very sensitive to the change in the LDF as they rely on their estimation [7]. Its risk has been shown in the previous part . The highest is the claim development, the highest is the increase in the IBNR to cover the whole period.

[7] especially for the last occurrence quarter - $\hat{C_{n,n}} = C_{n,1} \times \prod_{k=1}^{n-1} \hat{\lambda_k}$

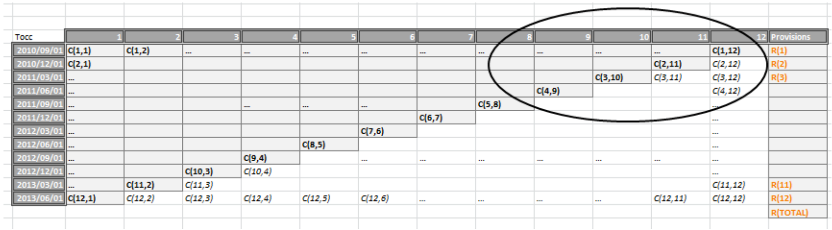


Figure 5.10: The impact of late claims

We will try to quantify the late claim impact on the IBNR for the CI coverage. This coverage is more volatile and has a longer development pattern than the death coverage. From the 2018Q3 claim data basis, a claim -or several claims- occurred in 2010Q3 were reported in 2016Q4. Their cost was 9.26x10^6 JPY. We will consider the aggregated amount. This claim appears in the upper right part of the development triangle. Its reporting delay is 32 quarters -more than 10 years after the occurrence. To include this late claim in the development triangle, a 40x40 development triangle is used. When it is taken into account, the total IBNR at 2018Q3 is 6.35x10^9 JPY vs 5.71x10^9 JPY (+11%) with a 40x32 development triangle. The 40x32 does not include the late claim. Hence, the impact of late claim is important on the provision amount.

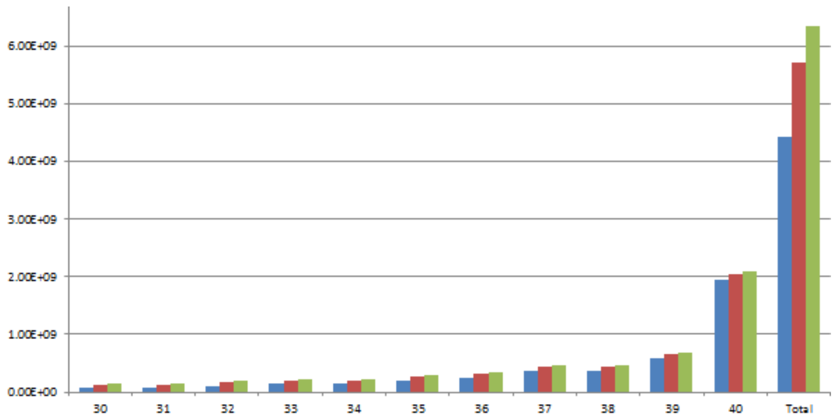


Figure 5.11: The impact of late claims on the $\hat{R}_i$ and the $\hat{R}_{\text{total}}$ - in red: the 40x32 development triangle, in blue: the 40x18 development triangle and in green: the 40x40 development triangle

From the previous chart, several development triangles are compared:

- The 40x40 development triangle includes all the late claims.
- The 40x32 development triangle is a tradeoff. It includes some late claims: between 18 and 32 development quarters.
- The 40x18 development triangle includes no late claims [8]

As previously, the development triangle shows that the last occurrence quarter gives a higher amount of provision -$\hat{R}_n = 2 \times \hat{R}_{n-1}$. The late claim increases the weight of this occurrence quarter n.
Considering all the late claims with the 40x40 triangle increases by 48% the IBNR in comparison with the 40x18 triangle. However, this triangle can be riskier: all the claims developed after 18 are not taken into account. If the insurer does not include these claims in its reserves, it may become insolvent when they are notified and have to be paid. Henceforth, the 40x32 development triangle is a good compromise: it captures some late claims but "only" increases the IBNR amount by 30% in comparison with the 40x18 triangle.

[8]For the CI coverage, late claims are defined as claims with 18 or more development quarters

Regarding the RSE, there is no clear effect of the late claims. Indeed, this indicator is almost the same for the 3 run-off triangles: 9.94% for the 40x18, 9.30% for the 40x32, and 10.14% the 40x40. All the sizes show a "good" quality.

Finally, late claims can have a tough impact on the total IBNR to put aside. They appear in the development triangle on a random basis. Their appearance increases the amount to reserve. However, if they are not considered, the insurer could become insolvent. It could not have enough money to face the cost of these late claims. Therefore, to monitor the risks due to this phenomenon, the issue is to define:

1. What is a late claim? After how many development periods is a claim late?
2. Are they only random or do they appear on a regular basis?
3. How much buffer does the insurer want to put aside to cover these unexpected costs? That is to say by how much it wants to rise its IBNR provisions to cover these late claims?

The answers belong to the actuarial function and the management. In the study, the 40x32 development triangle seems to be a good compromise between softening the impact of late claims and being able to cover the late claim charge.

5.2.4 Alternatives to the Mack Chain Ladder method

Methods with other factors

As reveal in the boni/mali study, the Mack Chain Ladder method with the traditional LDF does not seem appropriate when its hypothesis do not hold. Indeed, large amounts of boni/mali have been highlighted. Mainly, they arise from the "bad" adjustments of the LDF estimates when compared with the reality. It seems that there is a trend in the LDF.
Some alternatives to compute the LDF have been implemented.

The traditional LDF are estimated as:

$$\forall j \in [1, n-1], \hat{\lambda_j} = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}} \tag{5.28}$$

This estimation gives the same weight to all the occurrence quarters i. $\hat{\lambda_j}$ is an average based on the claim development. However, with the 2018Q3 data, it seems that some occurrence quarters tend to be different or show a different pattern in their development.

To test alternative to the usual LDF, different LDF have been tested to estimate the ultimate charge. Using $\lambda_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}$, we have:

- The median: $\hat{\lambda}_j = \text{mediane}(\lambda_{i,j})_i$ with $1 \leq$ i $\leq$ n-j-1
- The average: $\hat{\lambda}_j = \frac{\sum_{i=1}^{n-j-1} \lambda_{i,j}}{n-j}$
- The past x periods: $\hat{\lambda}_j = \frac{\sum_{i=n-j-x-1}^{n-j-1} C_{i,j+1}}{\sum_{i=n-j-x-1}^{n-j-1} C_{i,j}}$

All these methods give different amounts of provisions at 2018Q3:

LDF	$\hat{R}_{2016Q3}$
Traditional	6.35x10^9
Median	3.07x10^9
Average	6.82x10^9
Past 10	4.49x10^9
Past 20	5.33x10^9

where "past 20" is based on the past 20 periods and "past 10" on the past 10 periods. The traditional method refers to the classic LDF.
If we only take into account the last 10 or 20 occurrence quarters, the IBNR decreases. It can be explained by the decreasing trend observed in $\hat{\lambda}_{i,1}$ for the last occurrence periods.
Using the median LDF also results in lowering the total amount of previsions: it is almost divided by 2 in comparison with the traditional LDF. With the median LDF, after 11 development quarters, the LDF $\hat{\lambda}_j$ are equal to 1. This method smooths the development but does not capture late claims. The chart below only shows the difference in the IBNR amount at 2018Q3 for the CI risk.

To complete the comparison, we perform a back-test or a boni/mali study to assess the differences when using the different LDF. The amount of provisions computed with the distinct methods and the previous claim data -at 2017Q4 and 2016Q4- are compared with the reality that is to the real claim costs known at 2018Q3. The RBNS are set constant for all the back tested years.

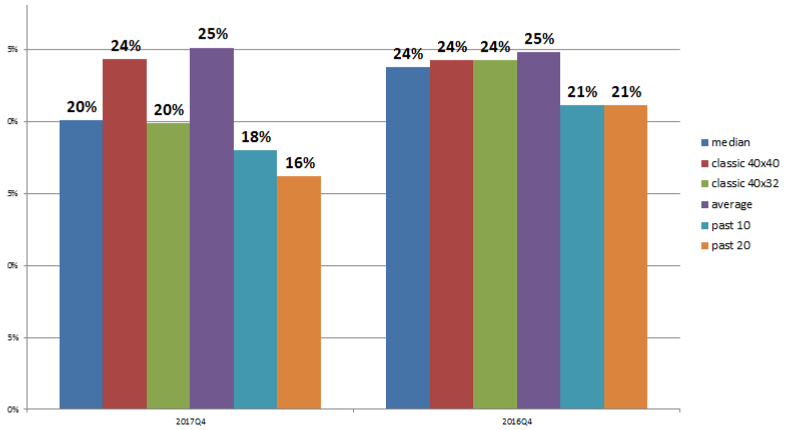


Figure 5.12: Back testing the 2017Q4 and 2016Q4 claim data with the different LDF

At 2016Q4, there are no late claims after the development quarter 32. At 2017Q4, the last late claims occur at the development quarter 33. The late claims impact the average LDF of the different methods.

$\hat{\lambda}_{33}$	Value
$\hat{\lambda}_{33}^{median}$	1.023
$\hat{\lambda}_{33}^{average}$	1.003
$\hat{\lambda}_{33}^{40x40}$	1.061
$\hat{\lambda}_{33}^{40x32}$	1.000
$\hat{\lambda}_{33}^{past10}$	1.000
$\hat{\lambda}_{33}^{past20}$	1.000

The impact of the late claims increases sharply the IBNR amount with the 40x40 classic development triangle: indeed, for 2016Q4, the boni is in the same range as the 40x32 triangle which does not capture claims developed after 32 quarters. But, for 2017Q4, the late claims at quarter 33 result in increasing the IBNR amount with the 40x40 triangle. From the latest, the boni is 29.71% vs 23.25% with the 40x32 triangle.
The average LDF method seems to over-estimate the claim development. It shows the highest boni at 2017Q4 -25%- and at 2016Q4 -25%.
From all the LDF tested, the best choice seems to be the LDF based on the last 20 occurrence quarters. It has the "smaller" boni at 2017Q4 and 2016Q4. This factor captures the last trends without considering the old past claim development which may not be accurate at the valuation date.

These alternatives can help to find the "best" LDF. They should fit the future claim development pattern. Moreover, the issue of late claims has to be added in the balance between a prudential vision with "too much" provisions and a risky vision with "just enough" provisions.
The main drawback of these methods is the lack of indicators to assess the quality of the estimates. Only back tests can help to measure the pertinence of the methods proposed.

5.2.5 The Bornhuetter-Ferguson method

The Bornhuetter-Ferguson method was developed by Bornhuetter and Ferguson in *The actuary and the IBNR* in 1972. It uses an a priori information to stabilize the claim development.

Theory

The reserve forecast can be unstable. The Bornhuetter Ferguson method helps to stabilize the IBNR estimates. It uses an a priori on the ultimate charge $\hat{C}_{i,n}$. As a result, the last development patterns are less dependent on the first developments. This method is a way to circumvent unstable development pattern and to monitor the impact of the last development.

Hypothesis: The model relies on the following hypothesis.

1. The occurrence periods are independent.
2. $\forall i \in [1, n]$, $\exists$ $(\mu_1,..,\mu_n)$ and $(\beta_1, ..., \beta_n)$ with $\beta_n = 1$ so that:
$$E(C_{i,1}) = \beta_1 \mu_i \tag{5.29}$$
$$E(C_{i,1+j}|C_{i,1}, ..., C_{i,j}) = C_{i,j+1} + (\beta_{j+1} - \beta_j)\mu_i \tag{5.30}$$
where β_j captures the claim development pattern, μ_i is the a priori on the ultimate charge and $C_{i,j}$ the aggregated claim costs for the occurrence quarter i and the development quarter j.

The model: To compute the claim reserve based on the Bornhuetter-Ferguson model, an a-priori about the ultimate charge is needed -$\forall i \in [1, n]$, $\hat{\mu}_i$. In addition, β_j can be estimated thanks to the estimates of the LDF provided by the Mack Chain Ladder method. We have:

$$\forall j \in [1, n-1], \hat{\beta}_j = \prod_{k=j+1}^{n} \frac{1}{\hat{\lambda_k}} \tag{5.31}$$

with $\hat{\lambda_k}$ the estimates of the LDF of the Mack Chain Ladder method. They are also called the cadence. The triangle can be filled:

$$\hat{C_{i,j}} = C_{i,n-i+1} + (\beta_j - \beta_{n+1-i})\hat{\mu}_i \tag{5.32}$$

For the estimate of the ultimate charge, it becomes:

$$\forall i \in [2, n], \hat{C_{i,n}} = C_{i,n-i+1} + (1 - \hat{\beta}_{n-i+1})\hat{\mu}_i \tag{5.33}$$

with $\beta_n = 1$ and $\hat{\mu}_i$ the a priori information on the expected ultimate charge for the occurrence quarter i. If i=1, $\hat{C_{1,n}} = C_{1,n}$, where $C_{1,n}$ is known.
From the estimation of the ultimate charge, the IBNR amounts are computed as previously for each occurrence quarter i :

$$\begin{aligned}
\hat{R}_1 &= 0 && (5.34)\\
\hat{R}_i &= \hat{C_{i,n}} - C_{i,n+1-i} && (5.35)\\
&= C_{i,n-i+1} + (1 - \hat{\beta}_{n-i+1})\hat{\mu}_i - C_{i,n+1-i} && (5.36)\\
&= (1 - \hat{\beta}_{n-i+1})\hat{\mu}_i && (5.37)\\
& && (5.38)
\end{aligned}$$

where $\hat{\beta_{n-i+1}} = \prod_{k=n-i+1}^{n} \frac{1}{\hat{\lambda_k}}$.

The Bornhuetter-Ferguson method is based on an a-priori information: the expected ultimate charge $\hat{\mu}_i$. On the one hand, it enables to control the claim development which is very sensitive -especially, in the case of late claims. But, on the other hand, this expected ultimate charge has to be selected carefully. A deep analysis of the claim development and the claim amounts is required, as well as the validation by the expert in charge of the reserving process.

Application

The first step in the estimation is to find the expected ultimate charge $\hat{\mu}_i$. Several values will be tested to assess their impacts on the total IBNR provision. The expected claim cost is based on the value of the premiums and of the loss ratios:

$$\hat{\mu}_i = \text{Expected ultimate charge} = \text{Premium x Loss ratio}$$

Indeed, the expected ultimate charge $\hat{\mu}_i$ is the expected claim cost that the insurer will pay once all the occurred claims at quarter i will be notified to the latest. The claim cost and the premiums are linked: the premiums represent the risk covered by the insurer, and the amount it will have to pay in case a claim occurs. Henceforth, the premium is the theoretical claim cost which is multiplied by the probability that claims occur. This probability is measured by the loss ratio [9].
Finally, choosing the expected ultimate charge goes back to selecting the loss ratio. To measure their impacts on the estimation of the ultimate charge and on the total provision, several values will be tested. Because it represents the highest ultimate charge, only the last occurrence quarter n will be displayed. It is the same for all the loss ratios.

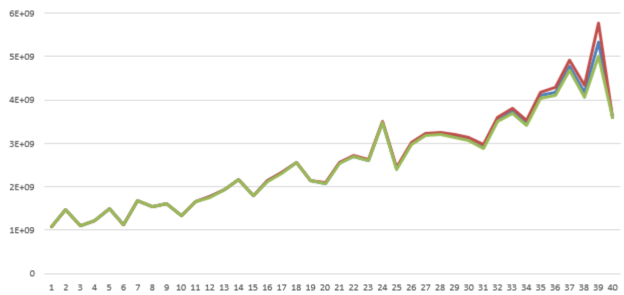


Figure 5.13: The estimated ultimate loss $\hat{C}_{n,n}$for the different loss ratios

Indeed, the estimated ultimate charge shows little difference in function of the loss ratio. A small difference appears for the last occurrence quarters - but not for the last one.
The highest is the loss ratio, the highest is the IBNR amount to put aside. Indeed, when the loss ratio is more than 1, it means that the premiums do not cover the risk anymore: $\frac{\text{claims}}{\text{premiums}} \geq 1$ ie claims $>$ premiums. Henceforth, the claim reserve amount has to be bigger to cover this case.

Loss ratio	$\hat{C}_{n,n}$	IBNR
102%	3.59x10^9	4.86x10^9
84%=min(LR)	3.59x10^9	3.81x10^9
132%=max(LR)	3.59x10^9	6.29x10^9

With min(LR) the minimum of the loss ratio based on the historical data.

The impact of late claims can also trigger the provision amount, but its impact is less important than for the Mack Chain Ladder method.
The impact of late claims is higher when the loss ratio is lower.

Finally, when we compare the Bornhuetter-Ferguson IBNR amount at 2017Q4 with the reality. There is a boni of 8.94% with a loss ratio of 102%. It seems that this method enables the insurer to monitor its provisions especially when late claims are reported. Indeed, thanks to the expected ultimate charge, the IBNR amount is limited. It depends less on the claim development and on events which can shift the claim development. However, the main drawback of the Bornhuetter-Ferguson is the lack of indicators to measure the quality of its estimates.

[9]Loss ratio= $\frac{\text{paid claims}}{\text{sum of premiums}}$

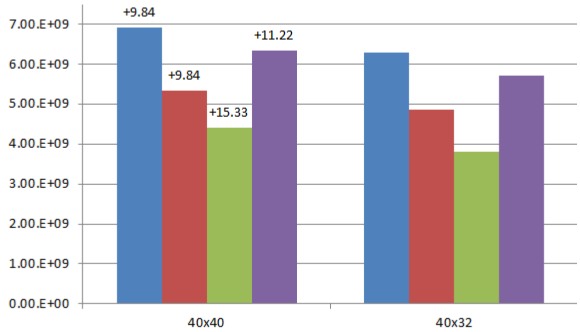


Figure 5.14: The impact of the late claims with the Bornhuetter-Ferguson estimates. In purple: the Mack Chain Ladder estimate. In red: the loss ratio =102%. In blue: the loss ratio =132%. In green: the loss ratio =84%.

5.3 Stochastic methods

Stochastic methods to estimate the claim reserves -the IBNR- have been risen. Although the models are more complicated than deterministic models, their main attractive feature is that they provide a measure of the errors made in the estimations. This measure is a warning for the insurers: it helps them to be aware of the risks contained in the provision estimates. Hence, they can monitor the risk. For instance, they can opt to reserve more to be sure not to become insolvent because of the bad quality of the claim reserve estimations.

The stochastic methods have already been developed with the Mack Chain Ladder method and the RSE. The latest quantifies the errors in the estimates. From the Mack Chain Ladder method, we will compare other stochastic methods: the GLM and the bootstrap. The change in the IBNR amounts as well as in the RSE will be highlighted.

5.3.1 The main advantages of the stochastic methods

Stochastic methods help to study the volatility of the IBNR reserves and the uncertainty in their estimations. Volatility can be due to:

1. Late claims: these claims are reported late in the development pattern, until the ultimate charge, and will impact sharply the amount to reserve. Their frequency is random.
2. Change in the claim amounts: the amounts of the occurred claims can change from notification to payment.
3. A too small number of data: in the case of the last occurrence quarter for instance.

All these parameters have a key role in the estimates and, as a consequence, on the total amount to reserve. Before booking the claim reserves, their impacts have to be taken into account. Indeed, the claim reserves have to take into account all the scenarios which may occur.

5.3.2 The GLM regression

The Generalized Linear Models -GLM- help to choose the best model in function of the available claim data. They also give indicators to quantify the estimation errors.

The GLM reserving method have been developed by Renshaw et Verrall -1994. It relies on the idea that the occurrence and the development periods explain the amount to reserve. The model uses the incremental claim costs $(Y_{i,j})_{i,j}$ at each quarter of occurrence i and notification j, to forecast the total amount of IBNR.

Theory

The GLM are based on the linear models and regressions. They are used to understand the dependent variable thanks to the information brought by several explicative variables.

Hypothesis: it is assumed that the dependent variable -the incremental claim cost $(Y_{i,j})_{i,j}$ are independent.

The GLM model[10]: The purpose of the GLM is to model a variable $(Y_{i,j})_{i,j}$ - thanks to several explicative variables $(X_{i,j})$. In the case of reserving, $(Y_{i,j})_{i,j}$ is the incremental claim cost for the occurrence quarter i. The development or reporting quarter j, and $X_{i,j}$ are the development and occurrence quarters. Their main feature is that the relation between the variable to explain $Y_{i,j}$ and the observations -$X_{i,j} = (a_i, b_j)$- may be non-linear. Thus, the residuals $\epsilon_{i,j}$ may not followed a $N(0,1)$ - whereas it is a key assumption for the linear regression models.
The GLM includes three main elements:

1. The random element : it is assumed that the probability distribution of the $(Y_{i,j})_{i,j}$ is one of the exponential distributions. That is to say that their densities have the following form:

$$f(y_{i,j}|\theta_{i,j}, \phi) = exp(\frac{y_{i,j} \times \theta_{i,j} - b(\theta_{i,j})}{a(\phi)} + c(y_{i,j}, \phi)) \tag{5.39}$$

   where $a(.)$, $b(.)$ and $c(.)$ are functions chosen in function of the distribution wanted, $\theta_{i,j}$ the natural parameter and ϕ the scale or dispersion parameter -$\phi \leq 0$.
   The first moments of the variables $(Y_{i,j})_{i,j}$ becomes:

$$E(Y_{i,j}) = b'(\theta_{i,j}) = \mu_{i,j} \tag{5.40}$$

   and :

$$\begin{aligned} V(Y_{i,j}) &= b''(\theta_{i,j}) \times \phi & (5.41)\\ &= b''(b'^{-1}(\mu_{i,j})) \times \phi & (5.42) \end{aligned}$$

   Where b' and b'' are the first and second derivatives of the function b, b' is invertible. The variance function $b''(b'^{-1}(.))$ depends only on the parameter $\theta_{i,j}$. It features the distribution of the exponential family.

2. The link function: we introduce the link function g defined as:

$$g(\mu_{i,j}) = \theta_{i,j} \tag{5.43}$$

   Therefore, we have:

$$\mu_{i,j} = g^{-1}(\theta_{i,j}) \Longleftrightarrow \mu_{i,j} = b'(\theta_{i,j}) \tag{5.44}$$

   With $g^{-1} = b'$. g gives the link between the mean $\mu_{i,j}$ and the natural parameter $\theta_{i,j}$. It is assumed that g is strictly monotonous, invertible and differentiable.

3. The determinist element or systemic element: we introduce $\nu_{i,j}$ as:

$$\nu_{i,j} = \mu + \alpha_i + \beta_j \tag{5.45}$$

   where $\alpha_0 = \beta_0 = 0$, α_i the occurrence quarter, β_j the development quarter and μ the calendar quarter which are assumed to be constant. It implies that the systematic element is a linear combination of the explicative variables α_i and β_j.

It results that the link function creates a link between the random element and the systemical element.

$$\begin{aligned} g(E(Y_{i,j})) &= g(\mu_{i,j}) & (5.46)\\ &= \nu_{i,j} & (5.47) \end{aligned}$$

[10]Denuit and Charpentier 2005, *Mathematiques de l'assurance non vie*, chapitre 11

The GLM model with a Poisson distribution is the main model used to explain the incremental claim costs $Y_{i,j}$ -and then the IBNR amount to reserve. Under this model: it is assumed that: $\forall i \in [1,n], j \in [1,n]$

- The incremental claim costs follow a Poisson distribution ie $Y_{i,j} \sim P(\lambda_i)$.
  It implies that the incremental costs have to be positive due to the definition of the Poisson distribution [11]. The model does not work if there is a negative incremental payment.
- The link function is g=ln. As a result, the density f becomes:

$$f(y_{i,j}|\lambda_i) = exp(-\lambda_i)\frac{\lambda_i^{y_{i,j}}}{y_{i,j}!} \tag{5.48}$$

  with: $a(\phi) = 1$, $b(\theta_{i,j}) = exp(\theta_{i,j}) = \lambda_i$, $c(_{i,j}, \phi) = -log(y_{i,j}!)$, $\theta_{i,j} = log(\lambda_i)$ and $\phi = 1$.
- The systemic element is:

$$\nu_{i,j} = \mu + \alpha_i + \beta_j \tag{5.49}$$

$$\nu_{i,j} = g(E(Y_{i,j})) = ln(\mu) \Longleftrightarrow \mu_{i,j} = exp(\mu + \alpha_i + \beta_j) \tag{5.50}$$

The coefficients $\hat{\mu}$, $\hat{\alpha_i}$, $\hat{\beta_j}$ are estimated by Maximum Likelihood Estimation -MLE. We get:

$$\hat{\mu_{i,j}} = exp(\hat{\mu} + \hat{\alpha_i} + \hat{\beta_j}) \tag{5.51}$$

and, finally:

$$\begin{aligned} E(\hat{Y_{i,k}}) &= \hat{\mu_{i,k}} && (5.52)\\ &= exp(\hat{\mu} + \hat{\alpha_i} + \hat{\beta_k}) && (5.53) \end{aligned}$$

The previous equation helps to compute the total amount of provisions $\hat{R_i}$ for the occurrence quarter i:

$$\begin{aligned} E(\hat{R_i}) &= (E(\hat{C_{i,n}} - C_{i,n+1-i}) && (5.54)\\ &= E(\hat{C_{i,n}}) - C_{i,n+1-i} \text{because } C_{i,n+1-i} \text{ is knwon -the anti-diagonal of the development triangle} && (5.55)\\ &= \sum_{k=1}^{n} E(\hat{Y_{i,k}}) - \sum_{k=1}^{n+1-i} Y_{i,k} && (5.56)\\ &= \sum_{k=1}^{n+1-i} E(\hat{Y_{i,k}}) + \sum_{k=n-i}^{n} E(\hat{Y_{i,k}}) - \sum_{k=1}^{n+1-i} Y_{i,k} && (5.57)\\ &= \sum_{k=1}^{n+1-i} Y_{i,k} + \sum_{k=n-i}^{n} E(\hat{Y_{i,k}}) - \sum_{k=1}^{n+1-i} Y_{i,k} && (5.58)\\ &= \sum_{k=n-i}^{n} E(\hat{Y_{i,k}}) && (5.59)\\ &= \sum_{k=n-i}^{n} exp(\hat{\mu} + \hat{\alpha_i} + \hat{\beta_k}) && (5.60)\\ & && (5.61) \end{aligned}$$

with $C_{i,n} = \sum_{k=1}^{n} Y_{i,k}$.
As a consequence, the total amount of IBNR provisions is:

$$\begin{aligned} E(\hat{R}_{\text{Total}}) &= \sum_{i=1}^{n} E(\hat{R_i}) && (5.62)\\ &= \sum_{i=1}^{n}(\sum_{k=n-i+2}^{n} exp(\hat{\mu} + \hat{\alpha_i} + \hat{\beta_k})) && (5.63)\\ &= \text{IBNR} && (5.64) \end{aligned}$$

[11] The Poisson distribution is defined on N^+

From the GLM Poisson model, the total IBNR estimation at quarter t IBNR_t is the same as the one calculated with Mack Chain Ladder method.

Finally, the Poisson distribution is featured by:

Distribution	$E(Y)$	$Var(Y)$
Poisson	$exp(\theta)$	$exp(\theta)$

with $\theta \in R^+$.

The indicators to validate the model

The scaled deviance:
The quality of the adjustment of the GLM model is assessed by the scaled deviance. It measures the distance between the likelihood of the model and the likelihood of the saturated model -the perfect model.

$$\Delta = -2 \times ln(\frac{L(Y|Y)}{L(\mu|Y)}) = -2 \times (ln(L(Y|Y)) - ln(L(\mu|Y))) \tag{5.65}$$

with $L(Y|Y)$ the likelihood of the model with all the coefficients equal to 0 -the maximized log-likelihood of the saturated model. The statured model is featured by a parameter for every observation which explains Y_i totally. $L(\mu|Y)$ is the likelihood of a model with less parameters than variables to explain -the maximized log-likelihood of the fitted model.
The scaled deviance captures the difference between the model's fit and the fit of a perfect model featured by $E(Y_{i,j}) = Y_{i,j}$ -with no bias in the estimates.

Validation of the model:
If the scaled deviance is closed to 1 -that is to say when the values forecast by the fitted model and the values forecast by "the most complete model we could fit" are close-, the model is well adjusted to the real data. Indeed, the likelihood of the model is close to the saturated model which is the "perfect model". On the other hand, if Δ is big, there is evidence for a model lack-of-fit.

The standardized Pearson residuals:
The standardized Pearson residuals are used instead of the "classic" residuals because the model shows heteroscedasticity. Hence, the residuals are not random. The standardized Pearson residuals are defined as :

$$\forall i \in [1,n], \forall j \in [1,n] r^P_{i,j} = \frac{Y_{i,j} - \hat{\mu_{i,j}}}{\sqrt{V(\hat{\mu}_{i,j})}} \tag{5.66}$$

with $\hat{\mu_{i,k}} = E(\hat{Y_{i,k}}) = exp(\hat{\mu} + \hat{\alpha}_i + \hat{\beta}_k)$. Their mean is 0 and their variance tends toward 0.

Validation of the model: The GLM model works if the standardized Pearson residuals are randomly spread around 0 when plotted against $Y_{i,j}$.

Over dispersion and the over-dispersed Poisson distribution:
When the model does not fit the data, a phenomenon of over-dispersion can appear. It can be one of the reasons for a bad adjustment. Over dispersion is featured by:

Loi	$E(Y)$	$Var(Y)$
Poisson	$exp(\mu + \alpha_i + \beta_k)$	$exp(\mu + \alpha_i + \beta_k)$
Over-dispersed Poisson	$exp(\theta)$	$\phi \times exp(\theta)$

The variance is estimated and the dispersion parameter ϕ is fixed to the value of its estimation. In case of over dispersion, an over-dispersed Poisson distribution is better to estimate the incremental claim cost $Y_{i,j}$.

Application

The 2018Q3 claim data basis is used.
The first step consists in building a new development triangle based on the GLM Poisson. For all $(i, j) \in [1, 40]^2$, the incremental claim payments $Y_{i,j}$ are assumed to be independent and to follow a Poisson distribution ie $Y_{i,j} \sim P(exp(\mu + \alpha_i + \beta_j)$ with $\alpha_i = i$ -the occurrence quarter- and $\beta_j = j$ -the development quarter. The parameters $(\alpha_i)_i$ and $(\beta_j)_j$ are estimated by MLE. Thus, we get: $\hat{\mu_{i,j}} = E(\hat{Y_{i,j}}) = exp(\hat{\mu} + \hat{\alpha_i} + \hat{\beta_j})$.
The outputs of the model are shown in appendixes for the CI coverage (appendix 6.14). The model gives a new development triangle. Most of the parameters $(\alpha_i)_i$ and $(\beta_j)_j$ are significant -except for $j \in [29, 40]$. Hence, the results of the model have to be considered carefully: the model does not capture all the development.
The total provision IBNR_{2018Q3} is the same as with the Mack Chain Ladder method.

Risk	IBNR_{2018Q3}
CI	$5.71\text{x}10^9$
D	$1.09\text{x}10^9$

Hypothesis:
The hypothesis of the GLM Poisson are checked with the CI claim data basis.
The standardized Pearson residuals are plotted in function of the observed values should show a random distribution around the mean 0.

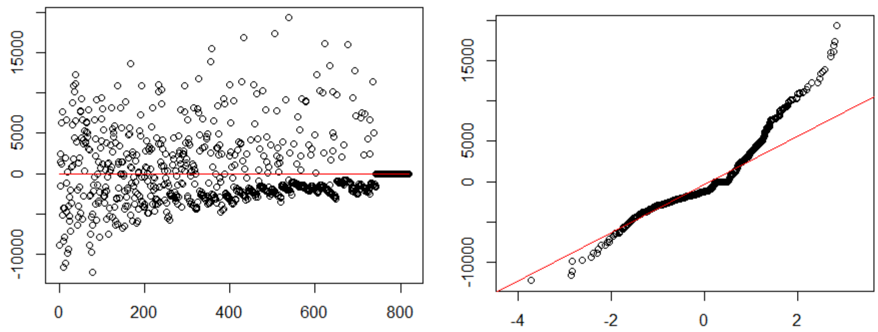


Figure 5.15: On the left: The standardized Pearson residuals with a 40x32 development triangle for the CI coverage- in red : the line with slope 0 -On the right: the qq plot of the standardized Pearson residuals

From the figure above of the standardized Pearson residuals, we can conclude that they are not randomly spread. It seems that there is a trend for the last residuals which are aligned with 0. The following assumption can be checked with their distribution: it does not fit the distribution of a Normal (0,1) as shown with the qqplot -the points should be aligned with the red straight line.

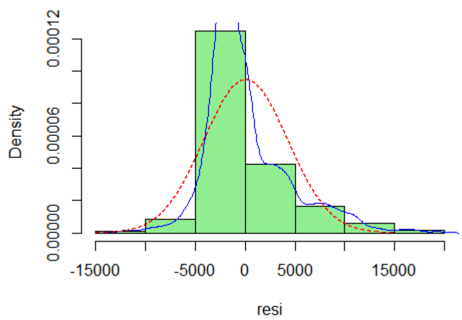


Figure 5.16: The histogram and the empirical density of the standardized Pearson residuals issued from the Poisson GLM with the CI coverage at 2018Q3 -in blue- the theoretical density of a normal (0,1) distribution -in red

To go deeper, we plot the histogram and the density of these residuals -the figure above. The blue curve is the empirical distribution density of the standardized Pearson residuals. It does not match with the theoretical normal distribution. As a result, the assumptions of the GLM model can be questioned.

In addition, the Shapiro test is used to check if the standardized Pearson residuals $r^P_{i,j} = r^P_k$ follow a normal distribution.

H_0: the standardized Pearson residuals follow a Normal distribution vs H_1

The test statistic is:

$$W = \frac{(\sum_{k=1}^{n} a_k \times r^P_{(k)})^2}{\sum_{k=1}^{n} (r^P_k - \bar{r})^2} \tag{5.67}$$

with n the size of the vector with all the r^P_k, $r^P_{(k)}$ the values of the ordered standardized Pearson residual, a_k the constants generated from the means, variances and covariances of the order statistics of a sample of size n from a normal distribution, $(r^P_k)_k$ the values of the standardized Pearson residuals and $\bar{r} = \frac{\sum_{k=1}^{n} r^P_k}{n}$ the mean of the residuals. We have: W = 0.90145.
From the latest, the p value is computed. It is lower than the level chosen -here, 0.05% to accept or reject the null hypothesis. We have p-value inferior to 2.2x10^{-16}: the null hypothesis is rejected. Therefore, the standardized Pearson residuals do not follow a normal distribution. Therefore, the validity of the results has to be considered carefully. The data does not check the hypothesis of the model.

The indicators of the quality of the model:

The scaled deviance ratio Δ gives the adjustment of the model with the data -here, the incremental claim payments for the CI coverage at 2018Q3.

Risk	Δ
CI	7.22
Death	8.29

When there is a "good" adjustment, this ratio is around 1. With the 2018Q3 data, the ratio is very high. It shows that the model is not well adjusted.

Regarding the errors in the estimates, the RSE is lower in comparison with the other models. It is 8.26% for the CI and 16.44% for the death coverage.

Re-estimation of the model:

Based on the results of the scaled deviance and the high dispersion, a phenomenon of over-dispersion can be assumed. The mean does not equal the variance. In this case, using a Poisson distribution to model the incremental claim costs would lead to an under-estimation of the variance. As a result, a lot of estimations of the $\hat{\alpha}_i$ and $\hat{\beta}_j$ may be significant. That is to say that they are different from 0 at a risk of 5 % only because of the under estimation of the variance. To test over dispersion, we use the R function "dispersiontest". It performs the following test:

H_0: There is equi-dispersion ie $Var(X) = \mu$ vs H_1: $Var(X) = \mu + \nu + \alpha \times f(\mu)$ where f is a transformation of μ

If $\alpha > 0$ there is over dispersion -resp. if $\alpha < 0$ there is under dispersion. The coefficient α can be estimated by an auxiliary ordinary least square -OLS- regression and tested with the corresponding t statistic. This statistic follows asymptotically a standard normal under the null hypothesis.
From the 2018Q3 claim data basis, t=13.22 and the p-value is inferior to 2.2x10^{-16}. Henceforth, H_0 is rejected: there is no equi-dispersion. But, H_1 cannot be accepted neither. The test conclusion is that there is no equi-dispersion, but the data might be over dispersed. In this case, the estimated dispersion is 1.15x10^7.

As a consequence, an over dispersed Poisson distribution is used. Under this model, the dispersion parameter is fixed. With an over-dispersed Poisson distribution, it becomes $\hat{\phi} = 1.68\text{x}10^7$. Both models lead to the same value of IBNR provisions. The over dispersed Poisson GLM has a smaller scale deviance: the fit is better than with a Poisson distribution.

Law	Poisson	Over dispersed Poisson
Δ	7.95	7.22
Dispersion	1	$1.68\text{x}10^7$
IBNR amount	$5.71\text{x}10^9$	$5.71\text{x}10^9$

5.3.3 Bootstrap

The bootstrap method provides simulations which enables to create several future feasible scenarios for the claim reserve amount. From the previous, the IBNR amount is estimated. Thus, the variability of parameters -using inference- can also be assessed. Knowing these parameters is tremendous to control the risks around the estimates as well as their reliability.

The main advantage of the bootstrap is the lack of assumptions on the claim provision distribution. It also provides a random distribution of the IBNR amount which can help to compute the PM -the quantile at X% of the provision distribution.

Theory

The bootstrap theory was presented by B.Efron in *Bootstrap Methods: Another Look at the Jackknife* in 1980. The bootstrap enables to estimate the features of a sample. It is a non-parametric re-sampling method. From a sample of size n, the bootstrap method creates a new development triangle of the same size performing N random re-samples. In the reserving process, the sample is the initial incurred development triangle. After the N re-sampling, the last development triangle has less volatility. The volatility in the development triangle may be due to various reasons: late claims, exceptional claims... Nonetheless, the re-sampled triangle is the result of the hazard. The volatility has been "erased" through the random re-samplings. But, according to the bootstrap theory, this new triangle still contains some of the original information on the claim development.

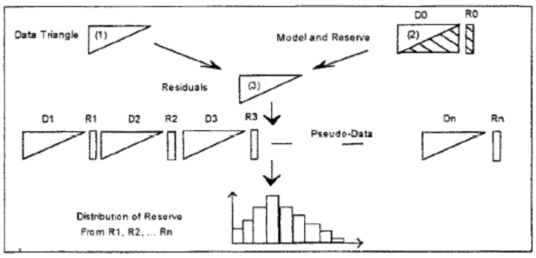


Figure 5.17: The main steps of the bootstrap

Hypothesis:
There are hypothesis to check before bootstrapping the data. The variables of the first sample have to be independent and identically distributed -iid. However, be they incremental or cumulated, the claim costs by occurrence and development quarters are not iid. It results that the bootstrap uses the Pearson residuals issued from the GLM approach. They are defined as:

$$r_{i,j} = \frac{C_{i,j} - \hat{C_{i,j}}}{\sqrt{Var(\hat{C_{i,j}})}} \tag{5.68}$$

Where $\hat{C_{i,j}}$ is the claim cost for the occurrence quarter i and the development quarter j, estimated thanks to the GLM method.

The main steps of the bootstrap can be summed up as follows:

1. We consider one run off triangle with the development in columns and the occurrence in rows:

$$\begin{array}{lll} C_{1,1} & C_{1,2} & C_{1,3} \\ C_{2,1} & C_{2,2} & \\ C_{1,2} & & \end{array}$$

   with $C_{i,j}$ the aggregated claim costs for the claims occurred in i and notified in j. The Mack Chain Ladder assumptions are assumed to be checked.

2. A GLM model with an over-dispersed Poisson distribution is performed -as in the previous model- and a "new" triangle is obtained, with the expected claim costs: $\hat{C_{i,j}}$

$$\begin{array}{lll} \hat{C_{1,1}} & \hat{C_{1,2}} & \hat{C_{1,3}} \\ \hat{C_{2,1}} & \hat{C_{2,2}} & \\ \hat{C_{3,1}} & & \end{array}$$

3. From the last triangle, the Pearson residuals -the residuals from the GLM model- are computed thanks to the following formula: $r_{i,j} = \frac{C_{i,j} - \hat{C_{i,j}}}{\sqrt{Var(\hat{C_{i,j}})}}$

$$\begin{array}{lll} r_{1,1} & r_{1,2} & r_{1,3} \\ r_{2,1} & r_{2,2} & \\ r_{1,2} & & \end{array}$$

4. From these residuals which are iid, a re-sampling is done thanks to a uniform distribution:

$$\begin{array}{lll} r_{1,3} & r_{2,1} & r_{2,2} \\ r_{2,1} & r_{1,3} & \\ r_{1,2} & & \end{array}$$

5. From the previous triangle, a new triangle of claim costs is computed with the following formula: $C_{i,j}^{\text{boot}} = r_{i,j} \times \sqrt{Var(\hat{C_{i,j}})} + \hat{C_{i,j}}$

$$\begin{array}{lll} C_{1,3}^{\text{boot}} & C_{2,1}^{\text{boot}} & C_{1,3}^{\text{boot}} \\ C_{2,1}^{\text{boot}} & C_{2,2}^{\text{boot}} & \\ C_{1,2}^{\text{boot}} & & \end{array}$$

6. Mack Chain Ladder is applied with the computation of the LDF and the ultimate charge to estimate the IBNR amount.

7. For each occurrence quarter i -$i \in [1, n]$, the total reserve is computed $\hat{R}_i^{\text{boot,k}} = \hat{C}_{i,n}^{\text{boot}} - C_{i,n+1-i}$ as well as the total provisions for the period $\hat{R}_{total}^{\text{boot,k}} = \sum_{i=2}^{n} \hat{R}_i^{\text{boot}}$ with k the kth re-sample -$k \in [1, N]$- and $R_1 = 0$.

8. From the last triangle, the steps 4 to 7 are performed N times - with N a large number.

9. At the end of the process, only the last development triangle is kept. It is "fully random". Moreover, a new random distribution of $(\hat{R}_{total}^{\text{boot,k}})_k = \text{IBNR}_{2018Q3}^{k}$ is obtained : the IBNR amount estimated at the end of each re-sample k are kept.

From the $(\hat{R}_{total}^{\text{boot,k}})_k$ distribution, the mean and the variance can be estimated:

$$\hat{R}_{total}^{\text{boot}} = mean(\hat{R}_{total}^{\text{boot,N}}) = \frac{1}{N} \times \sum_{k=1}^{N} (\hat{R}_{total}^{\text{boot,k}}) \tag{5.69}$$

with N the number of simulations. Regarding the variance of the total provision amount:

$$\hat{Var}(\hat{R}^{\text{boot}}_{total}) = \frac{1}{N} \times \sum_{k=1}^{N}((\hat{R}^{\text{boot,k}}_{total})^2) - mean(\hat{R}^{\text{boot,N}}_{total})^2 \tag{5.70}$$

with N the number of simulation.
In addition, the total forecast error on $\hat{R}^{\text{boot}}_{total}$ is estimated:

$$M\hat{S}EP(\hat{R}^{\text{boot}}_{total}) = \sum_{i=2}^{n}(M\hat{S}EP(\hat{R}^{\text{boot}}_{i})^2 + \hat{C}^{\text{boot}}_{i,n} \times (\sum_{j=i+1}^{n} \hat{C}^{\text{boot}}_{j,n}) \times \sum_{k=n+1-i}^{n-1} \frac{2 \times (\hat{\sigma}_k)^2}{(\hat{\lambda}_k)^2} \times \frac{1}{\sum_{m=1}^{n-k} C_{m,k}} \tag{5.71}$$

with $\hat{\lambda}_k$, $\hat{\sigma}_k$ obtained from the Mack Chain Ladder method at each step k.

Application

From the GLM approach, the over-dispersed Poisson distribution is used rather than the Poisson distribution. The latest has shown a bad adjustment with the data as well as over dispersion.
The plot of the standardized Pearson residuals is centered around 0. They seem to be randomly spread.

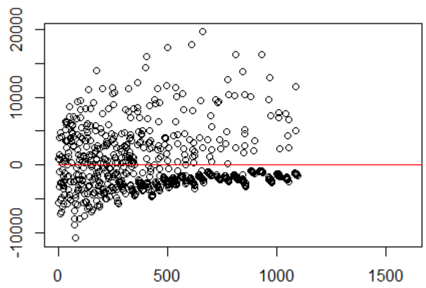


Figure 5.18: The standardized Pearson residuals from the last 40x32 bootstrapped development triangle for the CI at 2018Q3

The input of the bootstrap algorithm - performed with the software R- is the aggregated claim cost triangle - the triangle of size 40x32 also used in the Mack Chain Ladder method. Several numbers of computation will be chosen to measure the quality of the estimates as well as the IBNR amounts estimated. The higher is N, the longer is the algorithm and, hence, the computing time to obtain the estimates. For the CI coverage at 2018Q3, we have:

N	$\hat{R}^{\text{boot}}_{total}$	Lower	Upper	RSE
1 000	5.82x10^9	5.80x10^9	5.86x10^9	8.63
5 000	5.82x10^9	5.81x10^9	5.84x10^9	8.36
10 000	5.82x10^9	5.82x10^9	5.84x10^9	8.33
20 000	5.82x10^9	5.82x10^9	5.83x10^9	8.39

Lower and Upper stand for the confidence interval with a normal law -it is the parametrical distribution which better fits the data as shown in the following part. thus, the bounds are not negative which allow us to use this distribution. Lower is the inferior bound of the confidence interval at 95% and Upper is the superior bound. The details can be found in the appendixes -6.3.2 with n=N. $\hat{R}^{\text{boot}}_{total}$ = IBNR$_{2018Q3}$ is the amount to reserve provided by the bootstrap method. When N increases, the estimation of the IBNR rises a little. Between, 1 000 and 5 000 estimations, the IBNR amount increases by 0.004%. Regarding the confidence interval at 95%, it is larger when N decreases. The number of simulations increases the exactness of the estimates -except for N=20 000. As a result, it reduces the variance in the estimates and the confidence interval. With a risk of 5%, it can be assumed that the amount to reserve for the CI coverage at 2018Q3 is included in the interval 5.82x10^9 and 5.83x10^9.
The increase in the RSE between N= 20 000 and N=10 000 is suspicious. Indeed, as seen with the other values of N, the RSE should decrease when the number of re-samples rises. Re-sampling gives a better quality of the estimates. It should reduce the RSE.

Risk	$\hat{R}^{\text{boot}}_{total}$	$\hat{R}^{\text{MC}}_{total}$	RSE^{boot}	RSE^{MC}
CI	5.82x10^9	5.71x10^9	8.33	9.30
D	1.10x10^9	1.09x10^9	18.16	21.20

with $\hat{R}^{\text{boot}}_{total} = \text{IBNR}^{boot}_{2018Q3}$ the IBNR estimate from the bootstrap method, $\hat{R}^{\text{MC}}_{total}$ from the Mack Chain Ladder method and N= 10000.

The total reserve $\hat{R}^{\text{boot}}_{total}$ issued from the bootstrap is close to the Mack Chain Ladder reserve amount. It is a little higher. Nonetheless, the bootstrap method has a lower RSE -8.33% vs 9.30% for the CI coverage. The estimation is better because more scenarios are considered in the bootstrap method. Regarding the death coverage, at 2018Q3, the provision issued from the bootstrap is 0.74% higher but it reduces by 14% the RSE. The death IBNR estimates show a better quality with the bootstrap than with the Mack Chain Ladder method.

The bootstrap method leads to a small increase of the IBNR amount, but it shows a better quality in terms of estimates. The RSE is smaller for both coverage than with the Mack Chain Ladder method.

The adjustment of the PM with the Bootstrap

From the bootstrap method, a new distribution of $\hat{R}_{total}$ is obtained. Indeed, at each re-sample k, the amount of the total provisions $\hat{R}^{\text{boot,k}}_{total}$ is kept. A priori, no distribution has been given to the bootstrap $(\hat{R}^{\text{boot,k}}_{total})_k$ distribution. This new distribution is assumed to be random thanks to the re sampling. The following chart shows the IBNR distribution.

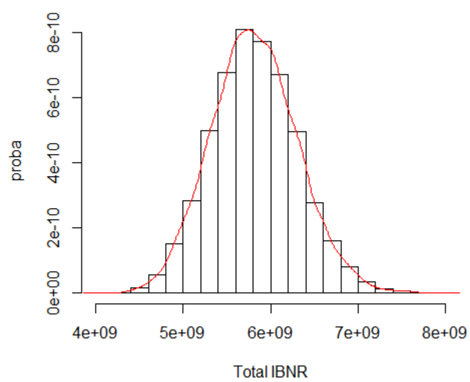


Figure 5.19: The empirical distribution of the IBNR_{2018Q3} for the CI risk with a 40x32 development triangle size. In red: its density

<u>Testing several adjustments for the PM:</u>
In the following, the focus is led on the total amount to reserve R_{total} but the bootstrap also provides estimates by quarter of occurrence. From this new distribution of $(\hat{R}^{\text{boot,k}}_{total})_k$, we will try to find the best distribution which fits the IBNR provision distribution. Thanks to the latest, a new a-priori distribution will be proposed to compute the PM. With the current methodology, the PM is the quantile at X% of the log-normal distribution. Its parameters are:

$$(\hat{\mu},\hat{\sigma}^2)=(ln(\hat{R}^{\text{boot}}_{total}) - \frac{\hat{\sigma}^2}{2}, ln(1 + (\frac{\hat{SE}(\hat{R}^{\text{boot}}_{total})}{\hat{R}^{\text{boot}}_{total}})^2))$$

Where SE is the standard error issued from the Mack Chain Ladder method.

Various probability distributions are candidates :

- Normal distribution
- Lognormal distribution
- Poisson distribution

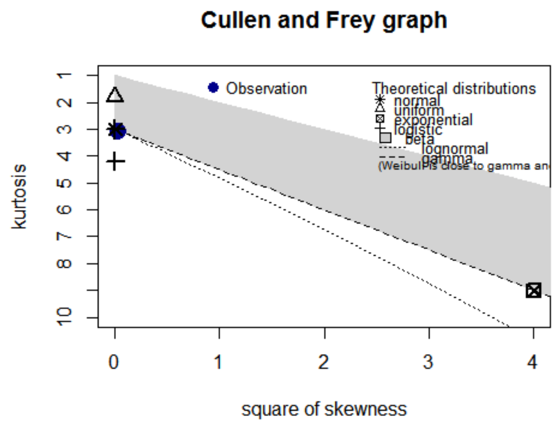


Figure 5.20: Several theoretical distributions are tested from the bootstrapped IBNR amount- the distribution which are closer to the point- are the best candidates -the CI coverage at 2018Q3

Their main features can be found in appendix (appendix 6.1). The parameters of these a priori distributions are estimated using the Maximum-Likelihood Estimation -MLE.
Once the parameters are estimated, a theoretical distribution can be simulated. From the following, the best fitting law will be selected. To choose the best law, the RSE is minimized. It measures "the distance" between the estimates issued from the theoretical distribution and the values issued from the bootstrap process. The less spread is the data, the best is the estimation. Therefore, minimizing the RSE results in finding the best approximation for the IBNR distribution. Once the best distribution is chosen the quantiles at X% can be calculated.
In particular, the choice of the a priori lognormal can be challenged. The a priori lognormal distribution is the one used to compute the PM -part 4.1.4 Indeed, this "a priori" distribution may be "far" from the real IBNR distribution. It will be compared with the other distributions. Their parameters are approximated by MLE.

Application:
At 2018Q3, with the log-normal used as a reference -part 4.1.4, we have:

X%	$PM_{X\%}$ CI	$PM_{X\%}$ Death
90%	13.76	23.27
95%	19.76	30.28
99%	23.40	35.76

To choose the best theoretical probability distribution, the R function "fitdist" from the MASS package is used. It estimates the parameters which best fit the data -using the MLE. From the $(\hat{R}_{total}^{\text{boot,k}})_k$ outputs of the bootstrap, this function gives back the features of the wanted distributions.
For instance, for the CI at 2018Q3, with N=10 000, we have:

Distribution	$\widehat{Parameter}1$	$\widehat{Parameter}2$
Normal	Mean= 5.81x10^9	Variance = 4.9x10^8
Lognormal	Shape= 22.5	Location= 0.084
Poisson	Mean = 5.82x10^9	
Lognormal Ref	Shape= 22.5	Location= 0.0086

The last row of the chart gives the values of the parameters for the a priori log-normal used as a reference. Once the parameters have been estimated, their distributions are simulated using the software R.

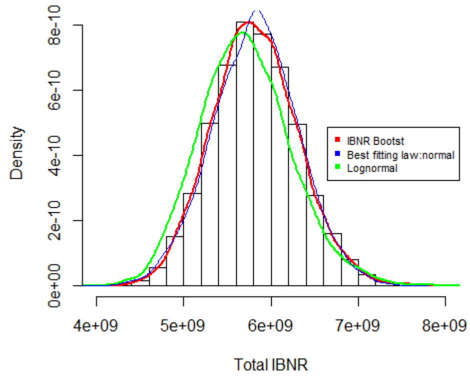


Figure 5.21: Comparison of the IBNR distributions. In red: the density of the $(\hat{R}^{\text{boot,k}}_{total})_k$, in blue: the normal distribution and -in green- the log normal distribution N=20 000

From the previous figure, the distribution in blue -a normal of parameters (Mean= 5.81x10^9, Variance= 4.9x10^8)- seems to better fit the IBNR distribution -in red. The IBNR distribution is issued from the bootstrap performed previously. Once the distribution and its parameters estimated, we look at the RSE: with N=20 000, for the CI coverage at 2018Q3,

Law	RSE
Normal	11.90%
Lognormal	12.08 %
Poisson	100.35%

The Poisson distribution is "far" from the bootstrapped distribution: this distribution is put aside. The difference in the RSE between the lognormal and normal distributions is very small. We can conclude that both distributions have a good fit with the distribution of the $(\hat{R}^{\text{boot,k}}_{total})_k$. Even though the normal shows a smaller RSE, the lognormal is preferred because it simulates positive provision amounts and bounds for the confidence interval.
The RSE for the a priori lognormal provided can also be estimated.

Law	RSE
Lognormal Ref	1354.26 %

It is high: this distribution does not fit the distribution of the $(\hat{R}^{\text{boot,k}}_{total})_k$. The lognormal $(\hat{\mu},\hat{\sigma}^2)$ is badly adjusted to the IBNR distribution provided by the bootstrap method.

The PM can be computed from the previous laws - in % of the $\hat{R}^{\text{boot}}_{total}$ for the CI coverage at 2018Q3:

Law	PM 90%	PM 95%	PM 99%
Lognormal	11.07	14.47	21.74
Normal	11.02	14.11	19.71
Lognormal Ref	13.76	19.76	23.40

The PM issued from the best fitting normal and lognormal are almost the same -except for the risk level of 99%. The lognormal Ref -from -part 4.1.4- requires a higher PM -until 23.40% of the IBNR_{2018Q3} to be covered at 99%.

As a consequence, the PM can be estimated using the quantile at X% of a normal or log normal distribution. However, it seems that the lognormal used as a reference does not fit well the random IBNR distribution issued from the bootstrap. Henceforth, this distribution may not capture well the real provision distribution and, so, it may not protect the insurer against deviations which can impact the reserves.

5.4 Summary

Various methods to estimate the claim reserves have been reviewed. A back test of the methods based on the previous claim data bases will be develop in the following chapter. This comparison in the real claim cost with the past reserve estimations is also called boni-mali study.

5.4.1 RBNS

To sum up the findings on the RBNS with have: at 2018Q3,

Acceptance rate	CI	death
100%	3.89×10^9	1.09×10^9
3-year average	3.46×10^9	8.20×10^8

The 3-year average acceptance rate reduces the RBNS amount. As a consequence, there is a smaller boni: 8% instead of 19% for 2017Q4 at 2018Q3.

5.4.2 IBNR

Finally, based on the RBNS estimated through the 3-year average acceptance rate, the following amounts of IBNR are computed at 2018Q3, with the methods reviewed.

Method	$IBNR_{2018Q3}$ CI	$IBNR_{2018Q3}$ death
Mack Chain Ladder	5.71×10^9	1.09×10^9
GLM Poisson	5.71×10^9	1.09×10^9
Bootstrap	5.82×10^9	1.10×10^9

The Mack Chain Ladder and the Poisson methods give the same IBNR amount for the period 2018Q3. However, they do not estimate the same development by occurrence quarters. There is a small difference in the ultimate charge.

Regarding the quality of the IBNR estimations, the model of the GLM Poisson has the lowest RSE, for all the periods. Nonetheless, it has been shown that the data -at least at 2018Q3- does not check the hypothesis of the model. Hence, the estimated IBNR amount may not be accurate to estimate the future claim cost development. The bootstrap also has a low RSE -after the Poisson model. It only requires the variables of the first development triangle -the standardized Pearson residuals- to be iid. This assumption is also questioned.
The following chart displays the RSE for the CI coverage at different valuation dates:

Method	2018Q3	2017Q4	2016Q4	2015Q4
Mack Chain Ladder	9.30	9.02	10.84	10.65
GLM Poisson	8.26	8.19	9.75	10.08
Bootstrap	8.33	8.40	10.17	10.62

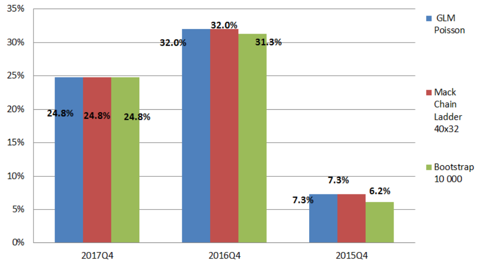
And, for the death coverage, the RSE is:

Method	2018Q3	2017Q4	2016Q4	2015Q4
Mack 40x32	21.20	19.47	23.99	20.84
GLM Poisson	16.44	15.07	16.98	14.15
Bootstrap	18.16	16.91	19.02	15.96

The RSE is higher for the death coverage as it is more volatile than the CI coverage. It results that the quality of the estimations is smaller. The IBNR estimates for the death coverage are riskier than for the CI coverage. It applies for all the models developed.

5.4.3 Global boni/mali on the total claim reserves

From all the methods developed in the study, we perform a boni/mali study. We compare the estimates at the valuation date based on the claim data available at valuation date with the estimates calculated from the current claim data basis 2018Q3.



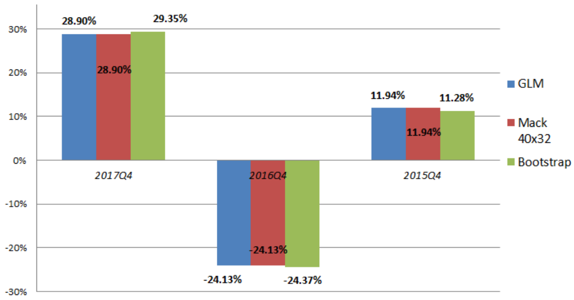


Figure 5.22: On the left: the back test of the claim reserves with different processes at 2017Q4, 2016Q4 and 2015Q4 for the CI coverage; On the right: the back test of the claim reserves with different processes at 2017Q4, 2016Q4 and 2015Q4 for the death coverage

The chart gives the view on the previous claim reserve estimates from the valuation date 2018Q3. As of 2018Q3, "too much" money had been reserved in 2016Q4 in comparison with the real claim development at 2018Q3. Less claim reported than excepted can be one reason for the boni. The method used to compute the IBNR and RBNS can also be part of the high boni. The opposite applies for the death coverage at 2016Q4. It can be explained by several reasons: more late claims reported during 2018Q3, a bad estimation at 2016Q4,...
The boni or mali are in the same range for all the methods. They are equal for the Mack Chain Ladder and the GLM Poisson estimates due to the same IBNR estimates.

To back test the different methods, we are interested in making a comparison between the real claim development and the expected claim development computed in the past years. This expected development is estimated with our methods. To assess, such a change, the ultimate claim cost is used$(\hat{C}_{i,n})_i$. It captures several effects:

- The real information of the occurred and reported claims
- The RBNS estimates
- The expected claim development

The following chart compares the initial claim development estimated at 2016Q4, with its update at 2017Q4 and at 2018Q3.

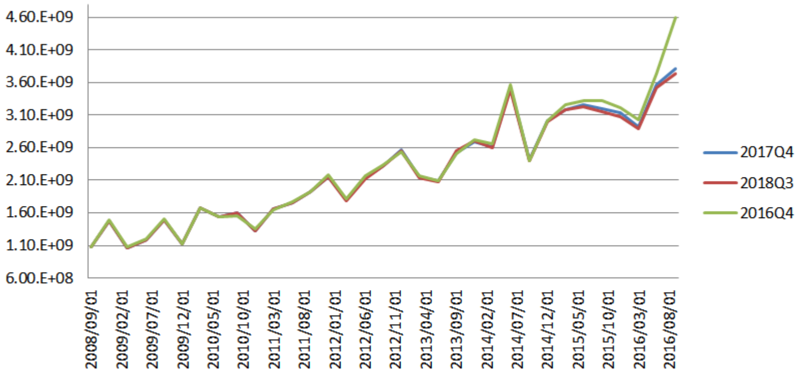


Figure 5.23: Back test of the ultimate charge -2016Q4- for several valuation periods: 2016Q4, 2017Q4 and 2018Q3

The ultimate charge does not change a lot from 2018Q3 to 2017Q3. However, for the last occurrence quarters, the amounts vary a lot between the estimations at 2016Q3 and 2017Q3. A 49% of the boni -the over estimation of the claim development between 2016Q4 and 2018Q3- is due to the last occurrence quarter n. This quarter is 2016Q3=n. Indeed, at 2016Q4, the IBNR amount for this quarter was 4.60×10^9. At 2017Q4, it became 3.74×10^9 and, at 2018Q3, it is only 3.81×10^9. Between its valuation at 2016Q4, it has decreased from a -19%: -17% between 2016Q4 and 2017Q4, and -2% between 2017Q4 and 2018Q3. This over estimation is due to the Chain Ladder estimations. Indeed, the development pattern is not constant for our claim data.

It results that the insurer should be careful and aware of the risks when it estimates the claim provisions. The Mack Chain Ladder method is the most famous but there are several issues affecting its estimations. It is easy to use, and it does not require a lot of data. In addition, it is also approved by the regulators as it leads to over estimations of the claim reserves. But, its 3 hypotheses need to be checked by the data.

Chapter 6

The provisioning risks under Solvency II: the ultimate vision versus the one-year vision

Several methods have been reviewed to estimate the claim reserves. They all give different estimations of the future expected claim development that the insurer will have to face. The value of the claim reserves is key because it reflects the balance between profits and prudence. If too much is reserved, the insurance company loses money. But, on the other hand, if too less money is reserved, the latest may become insolvent and not be able to face its commitments. Over estimation is preferred rather than under estimation of the reserves.

Thanks to the stochastic reserving methods, the insurer can assess the quality of the estimations. It helps him to know how risky the estimates are and how it can rely on the amounts issued of the reserving processes. For instance, if the RSE is very high, the estimates show a bad quality: to be more prudent, it may reserve more. Hence, quantifying the processing risk at a different time scope is also useful for the insurer.
If the stochastic methods give a risk vision until the complete development of the claims through the MSEP, the Claim Development Result -CDR- provides a one-year vision. This time scope is used by Solvency II. It is based on the accounting year which may be useful for the insurer when it wants to consider the risks in the reserve estimates at one year.

6.1 The reserving risk

6.1.1 The Claim Development Result : CDR

The CDR measures the claim development at one year: the difference between the initial estimation of the claim development and the estimation of the claim development done one year later. It assesses how much the insurer would need in case of liquidation. If the CDR is negative, it means that the insurer has under-estimated the future claim development. It becomes insolvent one year later. Hence, the CDR is the difference between two estimations of the ultimate charges at different periods. Indeed, it measures the total claim development over all the periods, for all the occurrence periods $\hat{C}_{i,\cdot}$. Measuring the difference in the ultimate charge is the same as measuring the difference between the IBNR provisions with the adjustment of the incremental claim notified between the two periods.

The CDR: notations

Notations: The following notations will be used to describe the information on the claim amounts:

- $I \in N$ and $J \in N$ defines the time period -for instance 2018Q3
- $D_I = \{C_{i,j} | i + j \leq n + 1 \text{ and } i \leq n\}$ ie the available information on the claim costs at time I

- $D_{I+1} = \{C_{i,j} | i + j \leq n + 2 \text{ and } i \leq n + 1\}$ ie the available information on the claim costs at the next period

For $i \in [2, n]$, the notations from the previous parts will be used except for:

- The provisions estimated at time I with the information D_I: $\hat{R}_i^I = \hat{C}_{i,n} - C_{i,n-i}$ and $\hat{R}_1^I = 0$.
- The provisions estimated at time I+1 with the information D_{I+1}: $\hat{R}_i^{I+1} = \hat{C}_{i,n} - C_{i,n-i+1}$ and $\hat{R}_1^{I+1} = 0$.

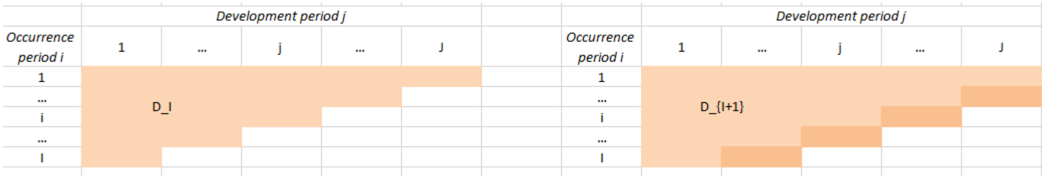


Figure 6.1: The information available at time I and I+1 - a new anti-diagonal is known

The CDR: definition

The CDR captures the difference in the estimations between the period I and I+1.

$$CDR_i(I + 1) = E(R_i^I | D_I) - E(R_i^{I+1} | D_{I+1}) - Y_{i,n-i+2} \tag{6.1}$$

where $Y_{i,n-i+2}$ is the incremental claim cost for the occurrence period i and the development period n-i+2 and R_i^I the IBNR amounts with the information D_I for the occurrence period i. $Y_{i,n-i+2}$ is known at time I+1 but estimated at time I. It stands for the claim amounts reported between I and I+1. It is the new anti-diagonal of the development triangle at I+1.

6.1.2 The reserving risk at one-year horizon

The CDR helps to assess the reserving risk at a one-year horizon. If the Mack Chain Ladder gives the reserving risk with an ultimate vision thanks to the RSE -which is the MSEP in terms of IBNR-, the equivalent can be estimated at one-year horizon.
This new measure of the quality of the estimations is different from the MSEP previously calculated. It only compares one period with another. Regarding the Mack Chain Ladder method, the MSEP is based on an ultimate vision of the risks in the estimations for the complete claim development. Indeed, the MSEP gives the errors in the forecast for the whole development triangle. With the CDR, we have a one period vision, and, so, n estimations of the errors in the forecast for only one period.
The one-year scope is used by the norm Solvency II: it captures the uncertainty over an accounting year. It is used by the insurer to quantify the technical risk -the risk linked to the method used to compute the reserves- in its balance sheet.

The Merz and Wuthrich method -2007- gives the indicators to measure the uncertainty in the CDR. To get a distribution and -hence- a quantile at X%, the re-reserving uses the bootstrap.

6.2 The model of Merz and Wuthrich

Based on the Mack Chain Ladder model, Merz and Wuthrich have introduced a new method to estimate the uncertainty in the provision estimates at one period. Their idea is based on the boni/mali study: they asses the change in the total provision estimate -or the ultimate charge- thanks to the new information on the claim cost development.

6.2.1 Theory

Hypothesis

As in the Mack Chain Ladder method, the model relies on various tremendous hypothesis:

1. The occurrence periods are independent ie $\forall i \neq j$, $(C_{i,1}, C_{i,2}, ..., C_{i,n})$ is independent with $(C_{j,1}, C_{j,2}, ..., C_{j,n})$.
2. The claim cost $C_{i,j}$ are a Markov chain $C_{i,j+1}$ only depends on $C_{i,j}$ and :

$$\forall j \in [1, n-1], \exists \lambda_j \text{ and}, \exists \sigma_j > 0 \mid \forall i \in [1, n], E(C_{i,j+1}|C_{i,j}) = \lambda_j \times C_{i,j}$$

and

$$V(C_{i,j+1}|C_{i,j}) = \sigma_j^2 \times C_{i,j} \tag{6.2}$$

Here, $C_{i,j+1}$ depends on all the claim cost $(C_{i,j})_j$ not only the previous development period $(C_{i,j})$ as in the Mack Chain Ladder process.

Estimating the CDR

If the insurer is aware of the real CDR, it would help him to solve all the risks contained in the provisioning. However, the real CDR is unknown but it can be estimated. Henceforth, for $i \in [1, n]$, we have:

$$CDR_i(I+1) = E(R_i^I|D_I) - (C_{i,I-i+1} - C_{i,I-i} + E(R_i^{I+1}|D_{I+1})) \tag{6.3}$$
$$= E(C_{i,J}|D_I) - E(C_{i,J}|D_{I+1}) \tag{6.4}$$

where i is the occurrence period, R_i^{I+1} the provision for the occurrence quarter i at the period I+1, $C_{i,I-i+1} - C_{i,I-i}$ the incremental claim cost developed between time I and time I+1, D_I the information available at time I and CDR_i the real claim development for the occurrence period i. For the complete period to cover, it becomes:

$$CDR = \sum_{i=1}^{n} CDR_i(I+1) \tag{6.5}$$

Because $CDR_i(I+1)$ is unknown, Merz and Wuthrich use an estimate to approach the real value of $CDR_i(I+1)$.

$$C\hat{D}R_i(I+1) = \hat{R}_i^I - (C_{i,I-i+1} - C_{i,I-i} + \hat{R}_i^{I+1}) \tag{6.6}$$
$$= \hat{C}_{i,n}^I - \hat{C}_{i,n}^{I+1} \tag{6.7}$$

And, finally, for the total period:

$$C\hat{D}R = \sum_{k=1}^{n} C\hat{D}R_k(I+1) \tag{6.8}$$

$C\hat{D}R$ gives the estimation of the full claim development for the period.

Not only the value of the estimate is important, but indicators to measure its quality are useful. As in the previous methods, for $i \in [1, n]$, the main indicator to assess the quality of the estimation is the MSEP:

$$MSEP(C\hat{D}R_i(I+1)) = E((CDR_i(I+1) - C\hat{D}R_i(I+1))^2|D_I) \tag{6.9}$$

It measures the distance between the estimate and its real value. The MSEP can be divided between the process variance and the estimation error:

$$MSEP(C\hat{D}R_i(I+1)) = Var(CDR_i(I+1)|D_i) + (E(CDR_i(I+1)|D_i) - C\hat{D}R_i(I+1))^2 \tag{6.10}$$

In particular, we are interested in the distance between the real CDR and 0 $MSEP(0)$. This distance represents the "prospective solvency point of view". Indeed, under the scope of Solvency

II, the insurer needs to know how far is its CDR from 0. 0 is when the insurer becomes insolvent once all the claims developed. It measures the quality of the forecast of $C\hat{D}R_i(I+1)$ by 0. This distance will be noted:

$$MSEP(0) = E((C\hat{D}R_i(I+1) - 0)^2|D_I) \tag{6.11}$$

Thus, the insurer is interested in knowing the distance between the CDR estimated and the real CDR. This measures the "retrospective" point of view.

$$MSEP(CDR_i(I+1)) = E((CDR_i(I+1) - C\hat{D}R_i(I+1))^2|D_I) \tag{6.12}$$

with $CDR_i(I+1)$ the real CDR and $C\hat{D}R_i(I+1)$ the estimated CDR. From Merz and Wuthrich, the following formulas provide estimates for $MSEP(C\hat{D}R_i(I+1))$ and $MSEP(0)$:

$$MS\hat{E}P(0) = (\hat{C}^I_{i,n})^2 \times (\hat{\Gamma}^I_{i,n} + \hat{\Delta}^I_{i,n}) \tag{6.13}$$

where the details of $\hat{\Gamma}^I_{i,n}$ and $\hat{\Delta}^I_{i,n}$ can be found in the appendixes and J=n the ultimate development. For the retrospective measure, we have:

$$M\hat{S}EP(CDR_i(I+1)) = (\hat{C}^I_{i,n})^2 \times (\hat{\phi}^I_{i,n} + \hat{\Delta}^I_{i,n}) \tag{6.14}$$

The forecast error between the estimated CDR and the null CDR is higher than the forecast error between the real CDR and the CDR estimated.

Because of the correlations between the occurrence periods i, the total error on all the CDR becomes:

$$M\hat{S}EP_A(\sum_{k=1}^{n} C\hat{D}R_i(I+1)) = (\sum_{k=1}^{n} M\hat{S}EP(C\hat{D}R_k(I+1))) + 2\sum_{k\geq i+1\geq 1} \hat{C}^I_{i,n} \times \hat{C}^I_{k,n}(\hat{\phi}^I_{i,n} + \hat{\Lambda}^I_{i,n}) \tag{6.15}$$

where $M\hat{S}EP_A$ is the estimate of the MSEP for the aggregated occurrence periods and $A = \sum_{k=1}^{n} CDR_i(I+1)$. The details of $\hat{\Lambda}^I_{i,n}$ can be found in the appendixes (appendix 6.3.2). And for the estimate of the MSEP(0):

$$M\hat{S}EP_A(0) = (\sum_{k=1}^{I} M\hat{S}EP(0) + 2\sum_{k\geq i+1\geq 1} \hat{C}^I_{i,n} \times \hat{C}^I_{k,n}(\hat{\phi}^I_{i,n} + \hat{\Xi}^I_{i,n}) \tag{6.16}$$

where the details of $\hat{\Xi}^I_{i,n}$ can be found in the appendixes (appendix 6.3.2).

Comparison with the MSEP in the Mack Chain Ladder model

The MSEP of the model of Merz and Wuthrich only considers one anti-diagonal: the new information arriving at the time I+1. However, in the Mack Chain Ladder process, all the anti-diagonals of the development triangle are used to measure the variance process. This distinction explains why, the first process is said to be a one period vision, whereas the latest is an ultimate vision -all the periods are compared.

As a result, the Merz and Wuthrich MSEP is lower than the MSEP estimated with the Mack Chain Ladder method. The one-year volatility is lower than the total volatility -for all occurrence and development periods-.

6.2.2 Application

Checking the hypothesis

The same claim data basis is used: 2018Q3. The results will be presented for the CI coverage. We choose n=11 and compute the triangle with a yearly development pattern. Even though one quarter is missing for the last occurrence year 2018, it is assumed to be full. As in the Mack Chain Ladder method, the assumption regarding the independence of the occurrence periods is questioned and rejected by the test proposed by T.Mack.

To assess the validity of the model, we check that: $\forall j \in [1, n-1]$,

$$(\frac{\hat{\sigma}_j}{\hat{\lambda}^I_j})^2 \times (\frac{1}{C_{n-j,j}}) \leq 1 \tag{6.17}$$

	1	2	3	4	5	6	7	8	9	10	11	12
Values	0,00E+00	0,00E+00	6,47E-04	1,08E-03	1,83E-03	5,11E-03	6,26E-03	1,02E-02	1,54E-02	2,41E-02	3,87E-02	8,06E-02

Figure 6.2: The values of $(\frac{\hat{\sigma}_j}{\hat{\lambda}_j^I})^2 \times (\frac{1}{C_{n-j,j}})$ for $j \in [1, 11]$ and n=11

with j the development period,$\hat{\sigma}_j$ is estimated in the Mack Chain Ladder model. The following chart shows the values of this ratio for the main portfolio. The development patterns will be compared. The assumption is checked for the 11x11 development triangle for the CI and death coverage at 2018Q3.

Comparison between the Mack volatility and the Merz and Wuthrich volatility

The estimate of the ultimate standard error -error of the Mack Chain Ladder model,$(M\hat{S}EP(CDR^{MW}(i))^{\frac{1}{2}}$ - is higher than the estimate of the one-year standard error $(M\hat{S}EPCDR^{\infty}(i))^{\frac{1}{2}}$. For the CI coverage, the first one is 25% higher. Indeed, in Mack Chain Ladder, the MSEP has to capture all the developments of the triangle.

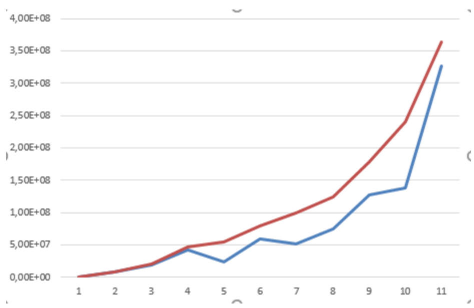


Figure 6.3: The $(M\hat{S}EP)^{\frac{1}{2}}$ for $CDR^{MW}(1)_i$ in function of the occurrence years $i \in [1, n]$ -in blue- and for CDR_i^{∞} -in red- for the CI coverage at 2018

From the estimation of the MSEP at different time scopes, we can conclude that 75% of the total volatility comes from the first year of development, 45% from the second development quarter, 33% from the third.... Once the triangle is full developed, there is no more uncertainty in the CDR $(M\hat{S}EP(CDR^{MW}(11))^{\frac{1}{2}} = 0$. Hence, the first period has the biggest weight in the total volatility of the development triangle: indeed, after the first development, there are still a lot of uncertainties in the total claim charge.

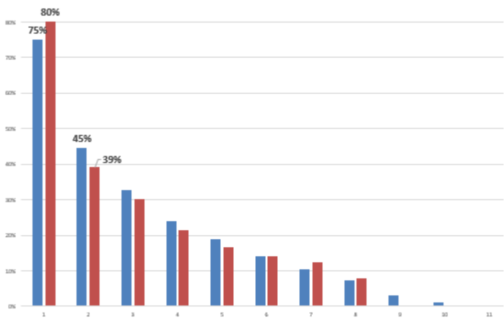


Figure 6.4: The repartition of the reserving risk with the ultimate vision by years of development for the death -in red- and CI coverage -in blue

The first development year show more volatility -and, hence, a low quality of estimations. Nonetheless, the volatility decreases faster - -51%- for the death coverage than in the case of the CI coverage - -40% for the second year. It is due to a shorter claim development: the claims are reported faster for the death coverage.

6.3 Extension: the distribution of the CDR with the bootstrap method

The main drawback of the Merz and Wuthrich method is that it does not give a distribution for the CDR. The distribution helps to provide the quantile at 99.5% to measure the risk in the reserving process under the frame of Solvency II.
The new vision conveys by the European norm is more prudential. As a consequence, it is key to consider all the possible scenarios of the CDR. Nonetheless, the Merz and Wuthrich method only gives one amount. To solve this issue and to estimate the reserving risk, the re-reserving method provides the wanted distribution based on the bootstrap. From the latest, the quantile at 99.5% is used to answer the Solvency II requirements.

6.3.1 Theory

The re-reserving method uses bootstrap to simulate a distribution of the CDR and then, compute a quantile at X%.

The main steps of the re-reserving method are:

1. With the Mack Chain Ladder process, the total provisions are estimated from the initial development triangle $\hat{R}_i^I$ with the available information at time I. The Mack Chain Ladder hypothesis are assumed to be checked by the data.
2. The residuals of Mack Chain Ladder are computed: $\forall i \in [1,n], \forall j \in [1,n]$ with $i+j \leq n+1$,

$$r_{i,j} = \frac{\sqrt{C_{i,j}}(\lambda_{i,j} - \hat{\lambda_j})}{\hat{\sigma_j^2}} \tag{6.18}$$

They are adjusted to check the hypothesis of iid ie $(r_{i,j}^A)_{i,j}$ are iid.

$$r_{i,j}^A = \sqrt{\frac{n-j}{n-j-1}} \times r_{i,j} \tag{6.19}$$

$$\begin{array}{lll} r_{1,1}^A & r_{1,2}^A & r_{1,3}^A \\ r_{2,1}^A & r_{2,2}^A & \\ r_{1,2}^A & & \end{array}$$

3. The $(r_{i,j}^A)$ are re-sampled with a uniform distribution:

$$\begin{array}{lll} r_{1,3}^A & r_{2,1}^A & r_{2,2}^A \\ r_{2,1}^A & r_{1,3}^A & \\ r_{1,2}^A & & \end{array}$$

4. From the previous triangle , new individual LDF are computed: $\hat{\lambda}_{i,j}^I = \hat{\lambda}_j + r_{i,j}^A \times \sqrt{\frac{\hat{\sigma}^2}{C_{i,j}}}$

$$\begin{array}{lll} \hat{\lambda}_{1,1}^I & \hat{\lambda}_{1,2}^I & \hat{\lambda}_{1,3}^I \\ \hat{\lambda}_{2,1}^I & \hat{\lambda}_{2,2}^I & \\ \hat{\lambda}_{1,2}^I & & \end{array}$$

5. The individual LDF are used to get new LDF and to complete the anti-diagonal of the triangle. The new LDF are: $\hat{\lambda}_j^I = \frac{\sum_{i=1}^{n-j} C_{i,j} \times \hat{\lambda}_{i,j}^I}{\sum_{i=1}^{n-j} C_{i,j}}$
6. Hence, a new anti-diagonal is simulated: $Y_{i,n-i+1} = C_{i,n\hat{-}i+1} - C_{i,n-i}$. $C_{i,n\hat{-}i+1}$ is the new simulation of $C_{i,n-i+1}$. It includes the process error $\hat{\sigma}_j^2$ and the information available at time I. The new claim cost $C_{i,n\hat{-}i+1}$ is simulated thanks to an a-priori distribution: a log-normal, normal or gamma. In our case, the log-normal is kept:

$$C_{i,n\hat{-}i+1} = LN(ln(C_{i,n-i+1} \times \hat{\lambda}_{n-i}^I) - \frac{1}{2} \times ln(1+\frac{\hat{\sigma}_{n-i}^2}{C_{i,n-i+1} \times (\hat{\lambda}_{n-i}^I)^2}), ln(1+\frac{\hat{\sigma}_{n-i}^2}{C_{i,n-i+1} \times (\hat{\lambda}_{n-i}^I)^2})) \tag{6.20}$$

7. From this new information at time I+1, a new estimation of the LDF is made:

$$\hat{\lambda}_j^{I+1} = \frac{\sum_{i=1}^{n-j} C_{i,j} \times \lambda_{i,j} + C_{n-j,j} \times \hat{\lambda}_{n-j,j}^{I+1}}{\sum_{i=1}^{n-j} C_{i,j}} \tag{6.21}$$

8. The low part of the triangle is filled :

$$\forall i \in [2, n], \hat{C_{i,n}} == \hat{C}_{i,n-i+1} \times \prod_{k=n-i+1}^{n-1} \hat{\lambda_k}^{I+1} \tag{6.22}$$

9. The new provisions can be estimated for each occurrence period i and in total:

$$\hat{R}_i^{I+1} = \hat{C_{i,n}} - \hat{C}_{i,n+1-i} \tag{6.23}$$

And:

$$\hat{R}_{total}^{I+1} = \sum_{i=1}^{n} \hat{R}_i^{I+1} \tag{6.24}$$

10. Finally, the CDR is: $CDR(I+1)^k = \hat{R}^I - \hat{R}^{I+1} - \hat{Y}^I$ with $\hat{Y}^I$ the re estimation of the incremental claim costs between time I and I+1

The steps 2 to 10 are performed N times -$N \in N$. Finally, a distribution of the $(CDR(I+1)^k)_k$ is obtained as well as the estimations of the MSEP. The latest will be used to compute the SCR and the Risk Margin under the norm Solvency II.

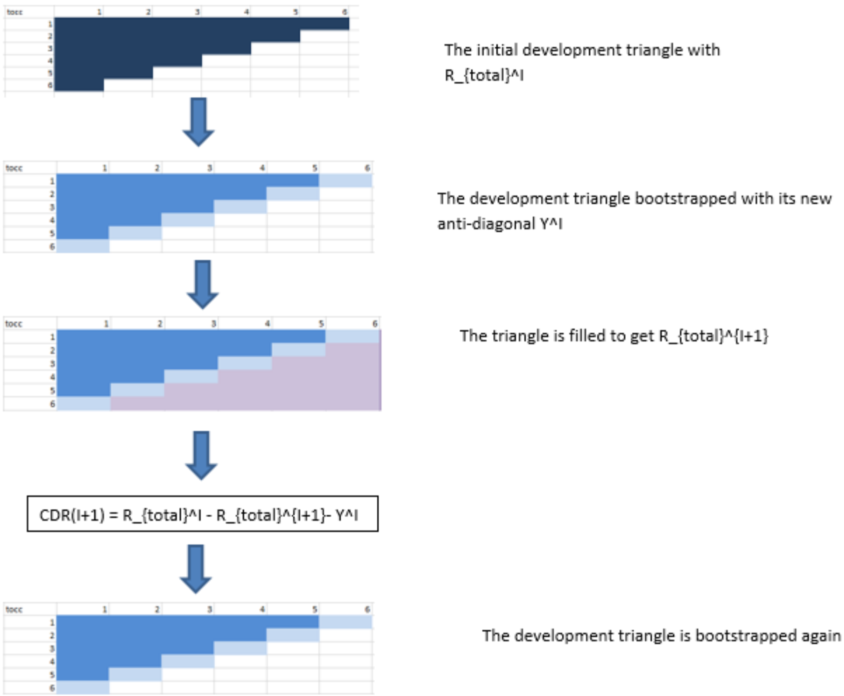


Figure 6.5: The main steps of the re-reserving process

6.3.2 Application

The same 11x11 yearly development triangle is used for the CI coverage. N is set to be 20 000. The IBNR amount as well as the RSE issued from the re-reserving method are in the same range as the IBNR amount issued from Mack Chain Ladder. Indeed, the IBNR increases by 3% and the RSE is 7% higher for the re-reserving method.
Once the distribution of the CDR simulated, the total amount to cover the provisioning risk at

one year is a 14% higher if the risk taken is 1% instead of 5%. The lower is the risk, the higher is the amount : 5.04x10^9 with a risk of 5% and 5.75x10^9 with a risk of 1% for the CI coverage. Hence, under the Solvency II scope, we have:

Risk	$q_{99.5\%}$
CI	5.75x10^9
Death	1.17x10^9

with $q_{99.5\%}$ the quantile at 99.5% of the $C\hat{D}R(1)$ amount issued from the bootstrap.

Both the Merz and Wuthrich and the bootstrap methods give MSEP which are in the similar range. However, the one-year provisioning risk issued from the bootstrap is almost always higher than the provisioning risk issued from the Merz and Wuthrich method. In addition, the bootstrap smooths the distribution by years of occurrence and provides the quantiles.

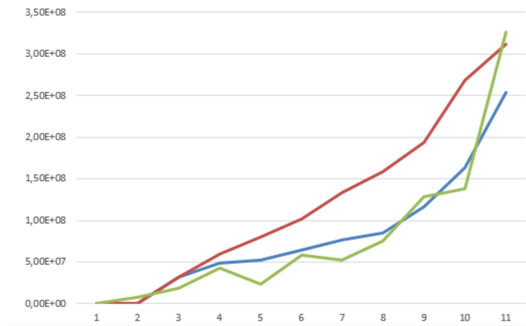


Figure 6.6: The comparison between the $\sqrt{MSEP}$ for the CDR at one year with the Merz and Wuthrich method -in green-, with the bootstrap -in blue- and the $\sqrt{MSEP}$ for the CDR at(∞) issued from the bootstrap -in red -for the CI coverage

As with the previous method, the ultimate MSEP is higher than the MSEP of the CDR at one year.

Risk	$RSE(\infty)$	$RSE(CDR(1))$
CI	17.45%	12.34%
Death	23.29%	18.74%

The previous chart shows the total RSE [1] for the ultimate charge and one year horizon issued from the bootstrap.
At last, we note that the RSE is higher for the death coverage for both the ultimate and the one-year scope. There are more risks around the estimates than for the CI coverage. It is in line with the previous findings : the RSE of the death coverage in comparison with the CI coverage is always higher due to less data point and more volatility.

[1] $RSE(\infty) = \frac{(M\hat{S}EP(CDR(\infty)_{bootst})^{\frac{1}{2}}}{IBNR^{BE}}$ and $RSE(CDR(1)) = \frac{(M\hat{S}EP(CDR(1)_{bootst})^{\frac{1}{2}}}{IBNR^{BE}}$

Conclusion

Based on the claim experience of BNP Paribas Cardif Japan for the CPI coverage, the reserving process is analyzed and assessed. Two main guarantees of the insurer are considered: the death and CI coverage. Under the coverage of the CI, if the insured faces one of the identified CI or in case of death of the latest, the insurer pays back the sum insured to the beneficiary -mainly banks and financial institutions.
The reserves or technical provisions under the scope of Solvency II, are a tool for insurers to face their commitments. Indeed, they have various constraints from the insured, the shareholders and the regulators. Especially, towards their insured, they have committed to cover the claim costs. In case the insured dies or is affected by one critical illnesses, the insurance company pays back the remaining amount of the loan -also called, the outstanding balance. Because all the claims are not reported, stated and paid on the spot, it builds reserves to cover the future claim development.

The claim reserves gather: the estimation for the RBNS, the IBNR and a prudence margin. The latest is an add on to face a deviation in the insurer sinistrality.
Including an acceptance rate based on the 3-year average of the historical data reduces the RBNS amount. It smooths the estimates and better capture the real status of the reported claims. When the reported claims are back tested thanks to the previous claim data bases, their expected "acceptance" fits the real acceptance. As a result, there is a small boni: the past estimations match with the current reality and the reserve amount is higher than the real claim development. However, as the boni decreases in comparison with an acceptance rate of 100%, a deviation in the claim status may impact the RBNS estimation. The amount put aside may not be sufficient to cover the claims.
Regarding the IBNR, various methods have been reviewed to compute their amount. The amount to reserve as well as the errors in the reserving process are considered. The errors quantify how much the insurer can rely on the estimates. The GLM Poisson seems to be the best compromise between all the methods: it gives the same amount to put aside as the Mack Chain Ladder method, but it has a lower RSE. The RSE is the main indicator to quantify the errors done in the estimations. Although, the claim data does not check the underlying assumptions of the GLM. As a result, the risk of mis-adequation between the estimates and the reality may be higher.
To complete the analysis of the reserving process, the estimates are compared based on the past data. The expected claim developments calculated in the past are compared with the real claim developments. All the methods show an over-estimation of the IBNR reserve. But, they are in the same range of boni. It is a positive difference between the expected and the real claim development. As a result, we cannot conclude that there is an obvious and safe method to estimate the claim reserves -especially the IBNR. The GLM, bootstrap and Mack Chain Ladder methods for the same development triangle are in the same range in terms of RSE, claim reserve amounts and checks of the model assumptions with the data.
Some adjustment can be done: for instance, the parameters of the theoretical IBNR distribution to compute the prudence margin could be estimated instead of being settled a priori, the LDF of the Mack Chain Ladder model can be based only the last 10 or 20 periods... But there are still some risks in the estimates.

In addition, the risk in the reserving process in its self is estimated at one-year horizon. It measures the volatility and the risks over one year for the insurer. It is more concrete for the latest than the reserving risk with an ultimate vision -ultimate as it goes until the full development of the run off triangle. It helps to compute the risk SCR and the RM required by the norm Solvency II. This indicator helps the insurer to measure the risks around their estimations for an accounting year.

At last, there are various options to estimate the claim development and the amount to reserve which will cover it. The lowest is the amount, the highest is the amount that insurers can invest to make profits. Nonetheless, this sum has to cover the claim costs so that the latest avoid insolvency. From the amounts estimated, it has to measure how far the latest are from the real values and how much risk there is in the estimates. As a result, there is no obvious method which fits the real claim development of the CPI coverage : the insurer has to balance between profits and prudence, once the risks assessed.

Note de synthèse

Les compagnies d'assurance sont caractérisées par des revenus réguliers et des dépenses aléatoires : chaque mois, elles reçoivent les primes d'assurance versées par les assurés et elles payent les sinistres qui arrivent. Elles ont un business model contre-cyclique. Par conséquent, et en raison des contraintes qui émanent principalement des assurés, des actionnaires et des régulateurs, les assureurs constituent des "provisions". Les provisions les aident à répondre à tous ces engagements, à rester solvable et à mettre en place un business soutenable. Elles sont un outil clef pour faire face à toutes les incertitudes liées au fonctionnement des assureurs.

Il y a plusieurs catégories de provisions pour couvrir les futurs cash flows des assureurs. L'estimation de ces derniers apparait au passif de leurs bilans : en effet, cette somme est due à un tiers. Dans le cadre de la norme européenne Solvabilité II, les provisions -appelées "provisions techniques" - sont estimées en Best Estimate. A ces dernières, une Marge de Risque est ajoutée. D'un côté, les provisions techniques aident les assureurs à rester solvables et à avoir une vision "juste" de leur bilan. En effet, les estimations incluent les couts futurs liés à l'année en cours. Mais, d'un autre côté, cette somme est "mise de côté" et ne peut pas être investie. Ainsi, le montant des provisions techniques est un compromis entre profits et prudence.

Parmi toutes les provisions, la provision pour les sinistres à payer -PSAP- est l'une des principales. Elle couvre tous les futurs cash-flows qui émanent des sinistres. L'incertitude envers les sinistres arrivés réside dans : le délai de report -la période entre l'arrivée du sinistre et son report à l'assureur- et le statut -tous les accidents reportés ne sont pas acceptés. Ces incertitudes sont capturées par les estimations des sinistres reportés mais non statués -RBNS- et les estimations des sinistres arrivés mais non reportés -IBNR. A ces montants et pour pouvoir faire face à la volatilité des sinistres, l'assureur ajoute une marge de prudence -PM. Cette dernière permet à l'assureur d'être protégé avec un risque de X% choisi par celui-ci. Plus X est petit, plus le montant de la PM est grand.

Avant de comparer les différentes méthodes de provisonnement, la qualité et la pertinence des données utilisées dans l'étude est évaluée. A partir de l'expérience de BNP Paribas Cardif Japan, des ajustements sont réalisés dans les données. En outre, une étude descriptive de l'historique des sinistres est faite afin d'en souligner les grandes tendances.

A partir de l'expérience du portefeuille de BNP Paribas Cardif Japan pour le produit d'assurance emprunteur, l'étude présente différentes méthodes pour estimer les PSAP.

$$\text{PSAP} = \text{IBNR} + \text{RBNS} + \text{PM}$$

L'estimation des RBNS repose sur l'inclusion ou non du taux acceptation des sinistres par l'assureur. En effet, la seule incertitude concernant le montant à mettre de coté pour couvrir les sinistres reportés est décision d'accepter ou de rejeter le sinistre. Inclure un taux d'acceptation permet d'avoir une estimation plus fine des RBNS: si l'estimation de ce taux est juste, l'assureur met de coté le montant exact de sinistres qui seront acceptés et payés par ce dernier. Afin de vérifier si cette estimation est juste, un back test est réalisé par rapport aux estimations faites au cours des années précédentes. Pour les risques identifiés, l'inclusion du taux d'acceptation basé sur les 3 dernières années améliore la qualité de l'estimation.

Une fois les RBNS estimés, pour chaque méthode, les sommes à provisionner pour couvrir les IBNR sont estimées ainsi que les indicateurs qui mesurent la qualité de l'estimation. En effet, pour contrôler ses risques, la compagnie d'assurance doit avoir à l'esprit la fiabilité du montant estimé : s'il n'est pas fiable, elle peut décider d'augmenter ses provisions pour s'assurer de rester solvable et de pouvoir respecter ses engagements. L'erreur standard relative -RSE- est le principal indicateur qui quantifie les erreurs d'estimations.

$$RSE_i = \frac{\sqrt{M\hat{S}EP(\hat{R}_i)}}{\hat{R}_i} \tag{6.25}$$

Avec $M\hat{S}EP(\hat{R}_i)$ l'estimation de l'erreur quadratique moyenne lors de l'estimation de du montant des IBNR -$\hat{R}_i$- pour une période d'occurrence i ou pour le total à mettre de coté pour couvrir la période donnée: dans ce cas, $i = total$. L'erreur quadratique moyenne mesure la "distance" entre l'estimateur et la vraie valeur. Un RSE faible montre une bonne qualité d'estimation. Elle contient peu d'incertitude.

De tous les modèles abordés et pour les risques décès et maladies critiques -CI, le modèle GLM Poisson présente le plus petit RSE, bien que la différence avec les autres modèles soit faible. Ce dernier donne le même montant de provisions -IBNR- que la méthode de Mack Chain Ladder qui est la plus répandue et facile à mettre en oeuvre. Cependant, les estimations restent risquées car les données des sinistres ne vérifient pas les hypothèses du modèle GLM. Il ne semble pas y avoir de modèles qui soit meilleur : les RSE sont semblables pour les différents risques et à différentes dates d'estimation. On remarque que l'estimation du montant des réserves est de meilleure qualité pour le produit CI que pour le produit décès, notamment en raison du plus petit nombre de sinistres dans le cas de celui-ci. Par conséquent, la marge de prudence est supérieure pour couvrir le risque d'estimation du montant des réserves.
De plus, en utilisant les bases de données des sinistres des années précédentes, nous comparons le montant des provisions estimés à cette époque avec leur développement réel. Ce dernier est ainsi confronté au développement attendu estimé au cours des années précédentes. Si la méthode de provisionnement est adaptée, le développement attendu doit correspondre au développement réel. Les données soulignent un sur-provisionnement de la PSAP estimée à travers les différentes méthodes.

A partir de la comparaison avec le développement réel et l'estimation du RSE, il ne semble pas y avoir de méthode pour estimer la PSAP qui capture parfaitement le développement des sinistres et présente un faible RSE. Le GLM, le bootstrap et la méthode de Mack Chain Ladder pour le même triangle de développement conduisent à des estimations du même ordre pour le RSE et le montant des IBNR. De plus, les hypothèses de ces différentes méthodes ne sont pas toujours vérifiées.
Des ajustements peuvent être faits : par exemple, les paramètres de la distribution théorique des IBNR pour calculer la PM peuvent etre estimés plutôt que fixés a priori. Les facteurs de développement de la méthode de Mack Chain Ladder peuvent aussi être ajustés : au lieu de prendre la moyenne sur toutes les périodes d'occurrence, les 10 ou 20 dernières pourraient être gardées. Le choix de la taille du triangle de développement ainsi que l'inclusion des sinistres reportés tardivement conduisent à différents montants d'estimation de la PSAP ainsi que différents RSE. La sélection de ces paramètres est un arbitrage entre prudence et profits fait par l'assureur.

Finalement, dans le cadre de la norme Solvabilité II, le risque de provisionnement est estimé à un horizon d'un an. Il mesure la volatilité du développement des sinistres pour une année, qui est plus concret que la vision ultime du risque de provisionnement donné par les méthodes précédentes. Cette dernière est dite ultime car elle inclut le développement complet du triangle.
La comparaison entre le risque de provisionnement à un an et à l'ultime montre que les premières années de développement des sinistres contiennent le plus d'incertitude: les sinistres associés aux garanties décès et maladies critiques sont reportées rapidement après la survenance de ces derniers. En outre, en utilisant la méthode du re-serving bootstrap, la qualité de l'estimation du risque de provisionnement à un an est estimée: la garantie CI montre une meilleur estimation.
De plus, le risque de provisionnement sert à estimer le Solvency Capital Requirement -SCR- et la Marge de Risque -RM- sous Solvabilité II.

Executive summary

Insurance companies are featured by regular earnings and random spending: each month, they receive the premiums from their insured and they pay the claims. Their business model is a counter cycle model. As a consequence, and due to the constraints from the insured, the shareholders and the regulators, insurers build reserves. The amount of reserves helps them to answer all sides, to remain solvent and to develop a sustainable business. They are a key tool to face all the uncertainties which emerge from the workings of the insurance companies.
There are various reserves to cover the future cash flows of the insurers. The estimations of the reserves appear in the liabilities of the balance sheet: indeed, their amount is owed to a third party. Under the European norm Solvency II, the reserves are gathered under the name "Technical provisions" and estimated in "Best Estimate". To the latest, a Risk Margin is added.
On the one hand, the technical provisions help insurers to remain solvent and to have an accurate vision of their balance sheet. Especially, because the estimations include the future costs linked to the current year. But, on the other hand, this amount is "put aside" and cannot be invested. It results that the amount of the technical provisions is a trade-off between profits and prudence.

Among all the reserves, the claim reserve is one of the main. It covers all the futur cash flows which arise from the claims. The uncertainty towards the occurred claims lies in: the reporting date -there is a delay between the occurrence and the notification to the insurance company- and the status -all the claims reported are not accepted. These uncertainties are captured by the estimations of the Reported But Not Settled claims -RBNS- and the Incurred But Not Reported claims -IBNR. To the latest -and to face claim deviations- a prudence margin -PM- is added by the insurance company before booking the amount in the balance sheet. The latest enables the insurer to be covered against a claim deviation with a risk of X% -where X is chosen by the insurer. The smaller X is, the higher the amount to add to the provisions is.

Before reviewing several ways to estimate the claim reserves, the quality and accurateness of the claim data from BNP Paribas Cardif Japan is assessed. Some adjustments of the claim data are done, and a statistical description of the claim data basis is made. It gives an idea of the specificity of the insurance products and sinistrality.

Based on the experience of BNP Paribas Cardif Japan for the CPI insurance product, the study reviews several methods used to compute the claim reserves.

$$\text{Claim reserves} = \text{IBNR} + \text{RBNS} + \text{PM}$$

The estimations of the RBNS rely on the inclusion of the acceptance rate. Indeed, the only uncertainty in the amount to reserve to cover the costs for the reported claims is the status: whether or not the insurer will accept the claim. When an acceptance rate is included, the estimation of the RBNS is thinner: if this rate is well estimated, the insurer puts aside the exact amount to cover the claims which are to be accepted. In order to assess the pertinence of the acceptance rate estimated based on the 3-year acceptance rate, a back test is done. It compares the estimations done in the past with the present. For the CI and death coverage, including an acceptance rate improves the RBNS estimations.

Once the RBNS estimated, the differences in terms of IBNR amounts as well as in the quality of estimations are computed with various methods. Indeed, to monitor its risks, the insurer has to be aware of the reliability of the amount to reserve: if there is a bad quality in the estimates, it may reserve more to be sure to remain solvent and face its commitments. The Relative Standard Error -RSE- is the main indicator to quantify the errors done in the estimations.

$$RSE_i = \frac{\sqrt{M\hat{S}EP(\hat{R}_i)}}{\hat{R}_i} \tag{6.26}$$

With $M\hat{S}EP(\hat{R}_i)$ the Mean Square Error of Prediction, estimated when the IBNR amount $\hat{R}_i$ is computed for one period of occurrence i or for the total amount to reserve to cover the full development triangle: in this case, i=total. The MSEP measures the "distance" between the

estimates and the real value. A low RSE shows a good quality of estimation. There is a low uncertainty and the insurer can rely on the amounts estimated.
Among all the models, the Poisson GLM has the lowest RSE -the difference is small with the other methods. It provides the same amount of IBNR reserves as the Mack Chain Ladder method -the easiest and most famous reserving method. However, there is still a risk in the estimates as the data does not check the assumptions of the model -for both the GLM Poisson and Mack Chain Ladder. We cannot conclude that one model is better than the others : the RSE are in the same range for the different risks and valuation dates.
The estimations of the IBNR for the CI risk are better than for the death risk. For all the methods, the latest has a lower RSE. It results that the PM to cover the risk in the IBNR estimations of the death coverage is higher than for the CI coverage. Less claims are reported for the death than for the CI coverage.
Thus, thanks to the access to the previous claim data bases, a comparison with the past claim reserve estimations is done. It compares the real claim development -as of now- with the expected claim development estimated in the previous years. If the method fits the claim development, the expected and the real development should match. Our data shows high amounts of boni -the expected claim development is over-estimated when compared with the current development. It underlines an un-adapted fit between the models and the real claim development.

From the comparison with the real claim development and the assessment of the RSE, there is no obvious method which leads to a perfect match between the estimates and the reality, and which shows a low error in the estimations. The GLM, bootstrap and Mack Chain Ladder methods for the same development triangle are in the same range in terms of RSE, claim reserve amount and check of the assumptions with the data.
Some adjustments can be done: for instance, the parameters of the theoretical IBNR distribution to compute the PM could be estimated instead of being settled a priori. The loss development factors -LDF- of the Mack Chain Ladder model can also be adjusted: instead of taking an average on all the occurrence periods, only the last 10 or 20 could be selected. The choice of the development triangle as well as the inclusion of late claims also shows differences in the amount to reserve and the quality of the estimates. Hence, it is an arbitration between prudence and profits done by the insurer.

Finally, under the scope of Solvency, the risk in the reserving process in itself is estimated at one-year horizon. It measures the volatility in the reserving estimation over one year. For the insurer, it is more concrete than the reserving risk with an ultimate vision provided by the other methods. It is ultimate as it goes until the full development of the run off triangle.
The comparison between the reserving risk at one year and at an ultimate development shows that the first development years capture most of the uncertainty. The death and CI claims are reported quickly after their occurrences. From the re-reserving method, the quality of the reserving risk is estimated: the CI guarantee shows a better estimation than the death coverage.
This one-year reserving risk is used to compute the SCR, the Risk Margin -RM- and hence, to comply with the Solvency II regulation.

Key words

Key words: technical provisions, reserving risk, Mack Chain Ladder, MSEP, claim reserves, IBNR, RBNS, boni/mali, CDR, GLM, boostsrap

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Appendixes

Acquisition and partnership

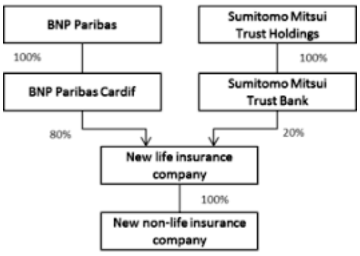

Figure 6.7: Completion of Share Acquisition of BNP Paribas Cardif Japan Assurance Vie

Evolution of the real estate prices in Japan

Figure 6.8: The evolution of the real estate prices in Japan

Claim evolution for death coverage

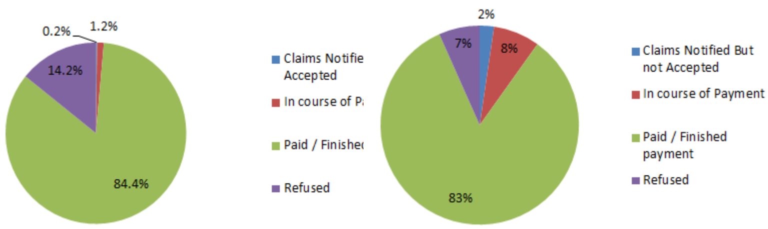

Figure 6.9: On the left: The evolution of the claims notified in 2017 at 2018Q3 and on the right: The distribution of the claim status at 2018Q3

Distributions and their parameters

Distribution	First moment	Second moment
Poisson(λ)	λ	λ
Normal(m,σ^2)	m	σ^2
Gamma(α,β)	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Lognormal(μ,σ^2)	exp($\mu + \frac{\sigma^2}{2}$)	(exp($2\mu + \sigma^2$)(exp(σ^2)-1)

Table 6.1: The different distributions and their features

RSE by risks and reserving methods

Values	Identification	Moments
death	1.92x10^{-5}%	1.76x10^{-5}%
CI	4.45x10^{-6}%	4.76x10^{-6}%

Table 6.2: The RSE in function of the risks and reserving methods

The LDF estimates are not biased

$$\begin{aligned} E(\hat{\lambda_j}) &= E(E(\hat{\lambda_j}|F_n)) && (6.27)\\ &= E(E(\frac{\sum_{i=1} n-jC_{i,j+1}}{\sum_{i=1} n-jC_{i,j}}|F_n)) && (6.28)\\ &= E(\frac{\sum_{i=1} n-jE(C_{i,j+1}|F_n)}{\sum_{i=1} n-jC_{i,j}}) = E(\frac{\sum_{i=1} n-j\lambda_j * C_{i,j})}{\sum_{i=1} n-jC_{i,j}}) && (6.29)\\ &= \lambda_j && (6.30) \end{aligned}$$

The LDF estimates are not correlated

$\forall k \leq j$

$$\begin{aligned} E(\hat{\lambda_j} \times \hat{\lambda_k}) &= E(E(\hat{\lambda_j} \times \hat{\lambda_k}|F_n)) && (6.31)\\ &= E(\hat{\lambda_k} \times E(\hat{\lambda_j}|F_n)) && (6.32)\\ &= E(\hat{\lambda_k} \times \lambda_j) && (6.33)\\ &= \lambda_j \times E(\hat{\lambda_k}) && (6.34)\\ &= E(\hat{\lambda_j}) \times E(\hat{\lambda_k}) && (6.35) \end{aligned}$$

Testing Mack Chain Ladder assumptions for other development quarters: H1

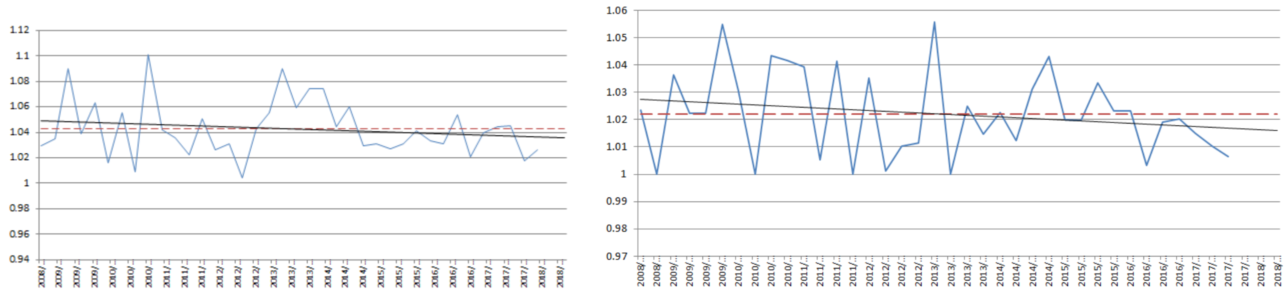

Table 6.3: Testing H1 for the development quarter 2 -on the right- and 3 -on the left- for the CI coverage

Testing Mack Chain Ladder hypothesis for the death coverage: H1

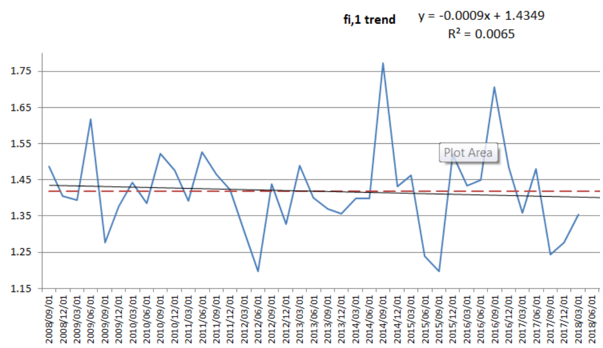

Figure 6.10: Testing H1 for the death coverage

Testing Mack Chain Ladder assumptions for the death coverage: H2

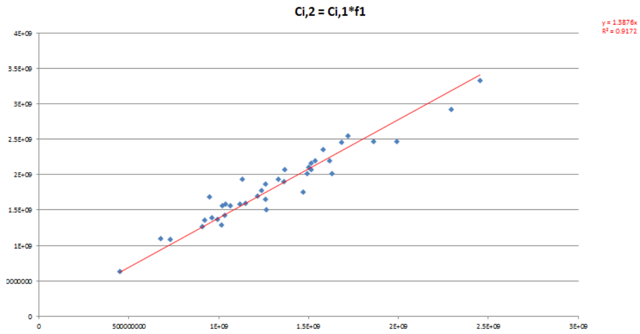

Figure 6.11: H1 for the death coverage

Testing Mack Chain Ladder assumptions for the death coverage: H3

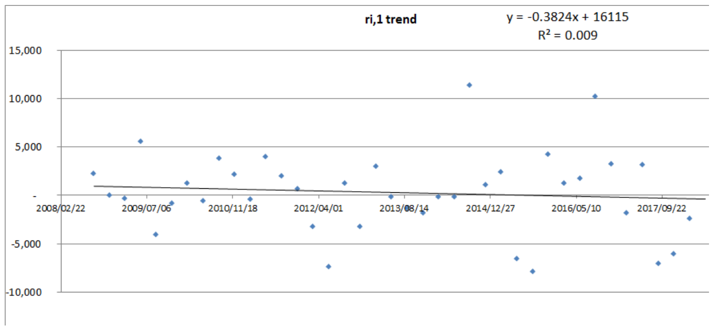

Figure 6.12: H3 for the death coverage

The results of the IBNR estimations with a 11x11 development triangle for the death coverage

Size	IBNR_{2018Q3}	RSE	PM at 95% in % of the IBNR_{2018Q3}
40x32	$1.09\text{x}10^9$	21.20%	30.28%
11x11	$7.36\text{x}10^8$	21.49%	30.71%

Table 6.4: Results of the IBNR estimations with a 11x11 triangle for the death coverage

Confidence intervals for the normal and log normal

If X_i is a positive variable and follows a normal (μ, σ^2). $\bar{X}_n$ is defined as: $\bar{X}_n = \frac{1}{n} \times \sum_{i=1}^{n} X_i$ the mean.

$$\sqrt{n}(\bar{X}_n - \mu) \approx N(0, \sigma^2) \tag{6.36}$$

1. For the normal: the confidence interval is straight forward:

$$IC_{1-\alpha} = [\bar{X}_n \pm q_{1-\frac{\alpha}{2}} \times \frac{\sigma^2}{\sqrt{n}}] \tag{6.37}$$

In our case: the estimation of the confidence interval is:

$$\bar{IC}_{1-\alpha} = [\hat{\bar{R}}_n \pm q_{1-\frac{\alpha}{2}} \times \frac{\hat{\sigma}^2}{\sqrt{n}}] \tag{6.38}$$

where $\hat{\bar{R}}_n = \frac{1}{n} \times \sum_{i=1}^{n} \hat{R}_i$ and $\hat{\sigma}^2$ is given by the Mack Chain Ladder method.

2. For the log normal: if we consider the function g which is C^1 in μ, and so that $g'(\mu) \neq 0$,it becomes:
$$\sqrt{n}(g(\bar{X}_n) - g(\mu)) \approx N(0, g'(\mu)^2 \times \sigma^2) \tag{6.39}$$
And the confidence interval for $g(\mu)$ at 1-α becomes:
$$IC_{1-\alpha} = [g(\bar{X}_n) \pm q_{1-\frac{\alpha}{2}} \times \frac{\sigma^2 \times g'(\mu)^2}{\sqrt{n}}] \tag{6.40}$$
Where $q_{1-\frac{\alpha}{2}}$ is the quantile at $1 - \frac{\alpha}{2}$ of a normal (0,1). If in addition, g is inversible and g^{-1} is strictly growing, then the confidence interval for μ is:
$$IC_{1-\alpha} = [g^{-1}(g(\bar{X}_n) \pm q_{1-\frac{\alpha}{2}} \times \frac{\sigma^2 \times g'(\mu)^2}{\sqrt{n}})] \tag{6.41}$$
We apply this with g=log and $g^{-1} = exp$ which checks the hyopthesis. We have $g'(\mu) = \frac{1}{\mu}$
$$IC_{1-\alpha} = [exp(ln(\bar{X}_n) \pm q_{1-\frac{\alpha}{2}} \times \frac{\sigma^2}{\sqrt{n} \times \mu^2})] \tag{6.42}$$
However, the following interval works only if n is very large. Not in the case of the confidence interval provided in the part -5.1 where n=1. It is used for the bootstrap in pat 5.3.3. In this case, we have: n=N and
$$\bar{X}_n = \bar{R}_n$$
σ^2 and μ^2 are estimated during the bootstrap process.

The formulas of the model of Merz and Wuthrich

With $S_j^I = \sum_{k=1}^{I-j} C_{i,j}$
$$\hat{\Xi}_{i,n}^I = \hat{\phi}_{i,n}^I + \frac{\hat{\sigma}_{I-i}}{\hat{\lambda}_{I-i}^I})^2 \times \frac{1}{S_{I-i}^{I+1}} \tag{6.43}$$
$$\hat{\Lambda}_{i,n}^I = \frac{\hat{\sigma}_{I-i}}{\hat{\lambda}_{I-i}^I})^2 \times \frac{1}{S_{I-i}^I} \times \frac{C_{i,I-i}}{S_{I-i}^{I+1}} + \sum_{j=I-i+1}^{n-1} (\frac{C_{I-j,j}}{S_j^{I+1}})^2 \times (\frac{\hat{\sigma}_j}{\hat{\lambda}_j^I})^2 \times \frac{1}{S_j^I} \tag{6.44}$$
$$\hat{\Delta}_{i,n}^I = (\frac{\hat{\sigma}_{I-i}}{\hat{\lambda}_{I-i}^I})^2 \times \frac{1}{S_{I-i}^I} + \sum_{j=I-i+1}^{n-1} (\frac{C_{I-j,j}}{S_j^{I+1}})^2 \times (\frac{\hat{\sigma}_j}{\hat{\lambda}_j^I})^2 \times \frac{1}{S_j^I} \tag{6.45}$$
$$\hat{\phi}_{i,n}^I = \sum_{j=I-i+1}^{n-1} (\frac{C_{I-j,j}}{S_j^{I+1}})^2 \times (\frac{\hat{\sigma}_j}{\hat{\lambda}_j^I})^2 \times \frac{1}{C_{I-j,j}^I} \tag{6.46}$$
$$\hat{\Psi}_i^I = (\frac{\hat{\sigma}_{I-i}}{\hat{\lambda}_{I-i}^I})^2 \times \frac{1}{C_{i,I-i}} \tag{6.47}$$
$$\hat{\Gamma}_{i,n}^I = \hat{\phi}_{i,n}^I + \hat{\Psi}_i^I \tag{6.48}$$

The GLM Poisson R outputs

```
Deviance Residuals:
   Min      1Q  Median      3Q     Max
-10992   -3073   -1493    1428   11082

Coefficients:
             Estimate Std. Error    z value Pr(>|z|)
(Intercept)  2.034e+01  3.063e-05  6.641e+05   <2e-16 ***
col2        -1.073e+00  7.902e-06 -1.357e+05   <2e-16 ***
col3        -2.904e+00  1.791e-05 -1.621e+05   <2e-16 ***
col4        -3.575e+00  2.528e-05 -1.414e+05   <2e-16 ***
col5        -3.806e+00  2.908e-05 -1.309e+05   <2e-16 ***
col6        -4.094e+00  3.434e-05 -1.192e+05   <2e-16 ***
col7        -4.259e+00  3.821e-05 -1.114e+05   <2e-16 ***
col8        -4.811e+00  5.138e-05 -9.364e+04   <2e-16 ***
col9        -4.712e+00  5.020e-05 -9.385e+04   <2e-16 ***
col10       -5.092e+00  6.223e-05 -8.183e+04   <2e-16 ***
col11       -4.678e+00  5.180e-05 -9.031e+04   <2e-16 ***
col12       -4.860e+00  5.811e-05 -8.362e+04   <2e-16 ***
col13       -5.382e+00  7.740e-05 -6.954e+04   <2e-16 ***
col14       -5.363e+00  7.893e-05 -6.795e+04   <2e-16 ***
col15       -5.733e+00  9.784e-05 -5.860e+04   <2e-16 ***
col16       -5.299e+00  8.122e-05 -6.524e+04   <2e-16 ***
col17       -5.883e+00  1.115e-04 -5.276e+04   <2e-16 ***
col18       -6.528e+00  1.601e-04 -4.076e+04   <2e-16 ***
col19       -5.898e+00  1.208e-04 -4.884e+04   <2e-16 ***
col20       -5.698e+00  1.133e-04 -5.031e+04   <2e-16 ***
col21       -5.941e+00  1.325e-04 -4.483e+04   <2e-16 ***
col22       -5.172e+00  9.320e-05 -5.549e+04   <2e-16 ***
col23       -6.792e+00  2.165e-04 -3.137e+04   <2e-16 ***
col24       -6.995e+00  2.502e-04 -2.795e+04   <2e-16 ***
col25       -5.422e+00  1.194e-04 -4.540e+04   <2e-16 ***
col26       -6.071e+00  1.726e-04 -3.517e+04   <2e-16 ***
col27       -5.446e+00  1.316e-04 -4.138e+04   <2e-16 ***
col28       -6.291e+00  2.118e-04 -2.971e+04   <2e-16 ***
col29       -2.991e+01  1.821e+01 -1.643e+00    0.100
col30       -2.989e+01  1.903e+01 -1.571e+00    0.116
col31       -2.987e+01  1.995e+01 -1.497e+00    0.134
col32       -2.987e+01  2.102e+01 -1.421e+00    0.155
col33       -2.985e+01  2.229e+01 -1.339e+00    0.181
col34       -2.982e+01  2.384e+01 -1.251e+00    0.211
col35       -2.978e+01  2.579e+01 -1.155e+00    0.248
col36       -2.980e+01  2.823e+01 -1.056e+00    0.291
col37       -2.975e+01  3.158e+01 -9.420e-01    0.346
col38       -2.975e+01  3.642e+01 -8.170e-01    0.414
col39       -2.981e+01  4.458e+01 -6.690e-01    0.504
col40       -2.965e+01  6.352e+01 -4.670e-01    0.641
lig2         3.229e-01  4.008e-05  8.056e+03   <2e-16 ***
```

Figure 6.13: The GLM with a Poisson distribution - R output - for the CI coverage at 2018Q3

```
factor(dev)2      -1.073e+00  3.239e-02 -33.110  < 2e-16 ***
factor(dev)3      -2.904e+00  7.342e-02 -39.549  < 2e-16 ***
factor(dev)4      -3.575e+00  1.037e-01 -34.490  < 2e-16 ***
factor(dev)5      -3.806e+00  1.192e-01 -31.926  < 2e-16 ***
factor(dev)6      -4.094e+00  1.408e-01 -29.081  < 2e-16 ***
factor(dev)7      -4.259e+00  1.567e-01 -27.185  < 2e-16 ***
factor(dev)8      -4.811e+00  2.106e-01 -22.841  < 2e-16 ***
factor(dev)9      -4.712e+00  2.058e-01 -22.893  < 2e-16 ***
factor(dev)10     -5.092e+00  2.551e-01 -19.959  < 2e-16 ***
factor(dev)11     -4.678e+00  2.124e-01 -22.028  < 2e-16 ***
factor(dev)12     -4.860e+00  2.382e-01 -20.398  < 2e-16 ***
factor(dev)13     -5.382e+00  3.173e-01 -16.961  < 2e-16 ***
factor(dev)14     -5.363e+00  3.236e-01 -16.573  < 2e-16 ***
factor(dev)15     -5.733e+00  4.011e-01 -14.293  < 2e-16 ***
factor(dev)16     -5.299e+00  3.330e-01 -15.913  < 2e-16 ***
factor(dev)17     -5.883e+00  4.571e-01 -12.870  < 2e-16 ***
factor(dev)18     -6.528e+00  6.566e-01  -9.943  < 2e-16 ***
factor(dev)19     -5.898e+00  4.951e-01 -11.914  < 2e-16 ***
factor(dev)20     -5.698e+00  4.644e-01 -12.272  < 2e-16 ***
factor(dev)21     -5.941e+00  5.433e-01 -10.936  < 2e-16 ***
factor(dev)22     -5.172e+00  3.821e-01 -13.536  < 2e-16 ***
factor(dev)23     -6.792e+00  8.876e-01  -7.652 6.17e-14 ***
factor(dev)24     -6.995e+00  1.026e+00  -6.818 1.91e-11 ***
factor(dev)25     -5.422e+00  4.896e-01 -11.074  < 2e-16 ***
factor(dev)26     -6.071e+00  7.078e-01  -8.578  < 2e-16 ***
factor(dev)27     -5.446e+00  5.395e-01 -10.093  < 2e-16 ***
factor(dev)28     -6.291e+00  8.682e-01  -7.246 1.08e-12 ***
factor(dev)29     -2.991e+01  7.464e+04   0.000  0.99968
factor(dev)30     -2.989e+01  7.800e+04   0.000  0.99969
factor(dev)31     -2.987e+01  8.180e+04   0.000  0.99971
factor(dev)32     -2.987e+01  8.616e+04   0.000  0.99972
factor(dev)33     -2.985e+01  9.137e+04   0.000  0.99974
factor(dev)34     -2.982e+01  9.776e+04   0.000  0.99976
factor(dev)35     -2.978e+01  1.057e+05   0.000  0.99978
factor(dev)36     -2.980e+01  1.157e+05   0.000  0.99979
factor(dev)37     -2.975e+01  1.295e+05   0.000  0.99982
factor(dev)38     -2.975e+01  1.493e+05   0.000  0.99984
factor(dev)39     -2.981e+01  1.828e+05   0.000  0.99987
factor(dev)40     -2.965e+01  2.604e+05   0.000  0.99991
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for Tweedie family taken to be 16807320)

    Null deviance: 4.1187e+11  on 819  degrees of freedom
Residual deviance: 1.1124e+10  on 741  degrees of freedom
AIC: NA
```

Figure 6.14: The GLM with a Poisson distribution - R output - for the CI coverage at 2018Q3