BUILDING OF A MORTALITY EXPERIENCE TABLE FOR A CREDIT LIFE PORTFOLIO IN GERMANY

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Greetings

First of all, I would like to thank my German colleagues who were always receptive for the different requests I had. I would like to specifically thank the actuaries working for the German branches (Sabatini Satkunananthan and Hata Coulibaly) who always made themselves available for any question I had.

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Résumé

Mots clés: Société Générale Insurance, Allemagne, assurance emprunteur, table de mortalité d'expérience, Hoem, Kaplan-Meier, Whittaker-Henderson, Makeham, Brass, Best Estimate, provisionnement, tarification.

L'objectif de ce mémoire est de présenter l'intérêt d'avoir recours à des tables de mortalité d'expérience pour la direction internationale de Sogecap. En effet, jusqu'à présent, en l'absence de suffisamment d'historique, les actuaires de la direction internationale de Sogecap (Société Générale Insurance) utilisaient les tables réglementaires pour les tarifications et le provisionnement. Avec une expérience de plus de 5 ans et l'entrée en vigueur de la directive Solvabilité 2, la direction a décidé de construire ses propres tables de mortalité.

Ce travail s'insère dans un large chantier démarré en 2016 afin d'industrialiser au maximum le traitement des données et l'automatisation de méthodes de construction de tables d'expérience pour le risque décès. Le plus gros produit d'assurance emprunteur (Bank Deutsches Kraftfahrzeuggewerbe GmbH – BDK) des succursales allemandes a été utilisé comme pilote : les programmes et les méthodes mis en place seront répliqués par la suite sur d'autres produits et d'autres pays (Italie, Pologne, Serbie, Roumanie, Maroc, etc.)

La première étape a été la collecte des données et le contrôle de leur fiabilité. Un certain nombre de contrôles inter et intra fichiers ont été mis en place pour s'assurer de la qualité des données.

Une fois que les données ont été contrôlées et nettoyées, le calcul des taux de mortalité bruts a pu être réalisé avec l'application de deux méthodes qu'on retrouve régulièrement dans les travaux sur les tables de mortalité d'expérience : Hoem, qui est le calcul du nombre de décès sur le nombre de personnes exposées à un âge x ; et Kaplan-Meier, qui prend en compte les sorties de portefeuille pour d'autres raisons que le décès (rachat anticipé de prêt, impayés, etc.) Les deux méthodes ont présenté des résultats très proches. Nous avons donc conservé les estimateurs Kaplan-Meier qui remplissaient de meilleures conditions pour l'application des méthodes de lissage.

Les taux bruts présentant certaines irrégularités, nous avons utilisé 3 méthodes de lissage (Whittaker-Henderson, Makeham et Brass) afin de lisser les taux de mortalité. Les 3 méthodes ont donné d'excellents résultats pour le cœur de la population étudiée [44 ans – 67 ans] : les méthodes ont été testées à travers plusieurs tests statistiques contrôlant la régularité de la courbe et la fiabilité par rapport aux données d'origine. Nous avons cependant décidé de conserver les taux provenant de la méthode de Brass pour faciliter l'extrapolation aux âges les plus jeunes et les plus vieux de notre portefeuille où nous manquons d'expérience et où les données sont assez volatiles. En effet, cette méthode faisant appel à une table de mortalité externe d'une population présentant des caractéristiques similaires à notre portefeuille, elle présente de vrais avantages en l'absence de données suffisantes.

Enfin, nous avons testé cette table d'expérience pour le calcul des réserves mathématiques et pour la tarification et nous avons comparé ces résultats avec ceux provenant des tables réglementaires

utilisées aujourd'hui. L'utilisation de cette table d'expérience permettra aux succursales allemandes de gagner en compétitivité dans les appels d'offre et de provisionner selon les règles *best estimate* pour être en ligne avec la directive Solvabilité 2.

Abstract

Keywords: Société Générale Insurance, Germany, credit life insurance, mortality experience table, Hoem, Kaplan-Meier, Whittaker-Henderson, Makeham, Brass, Best Estimate, reserving, pricing.

The objective of this project is to show the interest of using mortality experience tables for the international direction of Sogecap. Indeed, until now in the context of low experience, the actuaries of the international direction of Sogecap (Société Générale Insurance) were using the regulatory tables for the pricing and the reserving activities. In the framework of Solvency II legislation and thanks to the collection of data which took place over more than 5 years, the direction decided to build its own mortality experience tables.

This work takes part in a big project started in 2016 which objective is to automate as much as possible the reprocessing of data and the development of the methods for the building of the mortality experience tables. The biggest credit life insurance product (Bank Deutsches Kraftfahrzeuggewerbe GmbH – BDK) of the German subsidiaries was used as a pilot for the development of IT programs and methods: it is planned to replicate these methods on other products and countries (Italy, Poland, Serbia, Romania, Morocco, etc.)

The first step was to collect the data and check the consistency between the different files. Several programs were developed to automate the controls and check the accuracy of the data.

Once the data was controlled and corrected, the gross mortality rates were calculated with the application of two methods that are frequently used in the different work about the building of mortality experience tables: the first one is the Hoem method which is the calculation of the number of deaths over the number of years of exposure for each age. The second method is Kaplan-Meier which takes into account the exits of the insured that are not necessarily linked to the death (early reimbursement, unpaid, etc.) The results of both methods were very close to each other but we decided to keep the Kaplan-Meier estimators because the latter were meeting better conditions for the application of the selected graduation methods.

The gross mortality rates presented irregularities so we applied three graduation methods (Whittaker-Henderson, Makeham and Brass) to smooth the mortality rates. The 3 methods met very good results for the heart of the studied population [44 yo – 67 yo]: the methods were tested with different statistical tests about the fit to the crude mortality rates and about the smoothness of the graduated curves. Nevertheless, we decided to keep mortality rates coming out from Brass model for the extrapolation of the young and old ages where we lack data and where data is volatile. Indeed, as this method appeals to an external mortality table of a population with similar characteristics, it is particularly useful in our case.

Finally, we tested this experience table for the calculation of the mathematical reserves and for the pricing and we compared these results with the ones coming from the regulatory tables currently used. The use of the experience mortality table would enable the German subsidiaries to be more

competitive and to allocate the *best estimate* amount in the premium reserves to be in line with Solvency II norms.

Summary

Introduction

In the framework of Solvency II legislation, the insurers have the obligation to assess their risks as precise as possible because the provisions must correspond to the *best estimate* level and therefore externalize the safety margins (tables, rates…) that were often integrated in the calculation of the provisions under Solvency I. In order to reach the *best estimate* that corresponds to the probability-weighted average of future cash flows (as referred in Article 77(2) of Directive 2009/138/EC), the insurers must approach the risk as close as possible. Therefore, it is important to build experience tables which are consistent with the risk of the portfolio and check the reliability of the tables for the future projections.

Until now, in the context of low claims experience (the international direction of Sogecap (Société Générale Insurance / SGI) is relatively recent with most of the branches created between 2006-2007 and 2010), the actuaries were using the regulatory tables for the pricing and the reserving activities. Within the new Solvency II requirements and with the experiences acquired during the past years, it was important to build experience tables in order to calculate the provisions. Therefore, in 2016, a large project was launched by Société Générale Insurance to build the experience tables. This project covers two objectives:

- the building of *best estimate* experience tables for the calculation of reserves,
- the assessment of the safety margins required for the creation of the experience tables (used for pricing purposes): it was, indeed, an opportunity for Société Générale Insurance to review the pricing and be more competitive.

It has been decided to start with the biggest credit life insurance product insured by the subsidiary in Germany (Bank Deutsches Kraftfahrzeuggewerbe GmbH – BDK). This product has a sufficient number of good quality data that allowed the automation of some programs and the study of different methods of calculation of crude mortality rates and smoothing. Then, the methodology must be applied in other countries products (such as Italy, Poland, Serbia, Romania, Morocco, etc.)

The project was split into 7 steps:

- **Step 1: Data requirement and quality diagnostic**
- **Step 2: Corrective actions implementation and creation of unique Insured ID**
- **Step 3: Database in entry of Mortality Table Elaboration model**

  The objective of the step 3 is to aggregate the exposure and claims files at insured ID level in order to be able to calculate:
    - ✓ the real risk exposure duration per age
    - ✓ the number of death claims per age
- **Step 4: Observation period and products segmentation**

  This fourth step aims to define:
    - ✓ the relevant observation period to be retained for the study

Elaborate the relevant mortality table required to observe certain stability in term of insured characteristics and product management (underwriting process, medical selection, claim acceptance process...)

- ✓ the appropriate segmentation

  Indeed, in order to build a mortality table, volumes sufficiency criteria, in term of exposure and in term of claims, should be satisfied.

  At the end, the global analyses will outcome the relevant scope to be used for elaborating mortality table experience.

- **Step 5: Mortality table elaboration**

  This fifth step aims to define:
  - ✓ calculation of gross mortality rate per age
  - ✓ several smoothing methods
  - ✓ extrapolation of mortality rates for young and old ages

- **Step 6: Mortality table validation**

  This sixth step aims to define the different methods for the validation of the table and the definition of the final *best estimate* table.

- **Step 7: Building of experience table for pricing purpose**

  This seventh step aims to define the safety margin to be applied on the *best estimate* table for the creation of the tables used for the pricing.

The objective of this study is to build experience tables for a credit life portfolio insured by Société Générale Insurance Germany. So we will present the different methods applied on BDK portfolio and the results, and evaluate the impact on the pricing and the reserving when we use the experience tables.

This document is split into 3 chapters:

- **Chapter I: Study of credit life portfolio with BDK partner**

  This chapter presents the features of the product, the portfolio, the data processing and the tests performed to select the relevant observation period and the appropriate segmentation.

- **Chapter II: Building a mortality experience table**

  This chapter explains:
  - ✓ the different methods used for the calculation of the crude mortality rates (Hoem and Kaplan-Meier).
  - ✓ the different smoothing methods (Whittaker-Henderson, Makeham and Brass).
    For each method, the theory is reminded ahead and we see in a second step the application on BDK portfolio.
  - ✓ the extrapolation done for the tail distribution,
  - ✓ the tests performed for the validation of the *best estimate* table.

- **Chapter III: Impacts on the pricing and reserving with the experience table**

  This chapter evaluates the impact of using the experience tables on the tariff and the level of reserves. We will detail here the fine-tuning of the safety margin applied on the *best estimate* table for the definition of the mortality table used for the pricing.

Chapter 1 – Study of credit life portfolio with BDK partner

I. PRESENTATION OF THE PRODUCT

The Bank Deutsches Kraftfahrzeuggewerbe GmbH (BDK) is a German bank founded in 2000, subsidiary of the group Societe Generale since 2001, and specialist in financing solutions for car dealers and their clients.

BDK's products offer covers: car financing and leasing for private clients, financing solutions for car dealers, insurance and some other services. BDK works with more than 4.000 car dealers in all Germany and employs 685 people. Its outstanding loans and leasing are slightly less than 4,1 billion euros.

Societe Generale Insurance (SGI) that has concluded a partnership with BDK since 2008 is the insurer of the car loans distributed by BDK to car dealers and private. The insurance is limited to three guarantees (Death from any causes, Total Termporary Disability to work - TTD and Loss of employment - LE). All car loan subscribers who are aged between 18 and 74 years old at subscription are entitled to subscribe the credit life insurance. The contractual maturity age of this product is 75 years old.

Details of the offer:

- **Insured Person**
  - ✓ Car loan subscribers
  - ✓ Minimum Age at the inception of the loan: 18 years
  - ✓ Maximum Age at the inception of the loan:
    - Death: 74 years old
    - TTD: 64 years old
    - LE: 64 years old
  - ✓ Maximum Age at maturity:
    - Death: 75 years old
    - TTD: 65 years old
    - LE: 65 years old
  - ✓ Minimum period: none
- **Risks covered**
  - ✓ Death from any causes
  - ✓ Total Temporary Disability to work
  - ✓ Loss of employment

- **Benefit**
  - ✓ Death: The total outstanding amount of the loan at the date of death - maximum benefit per insured: 125 000 €
  - ✓ TTD: The total of monthly instalment – Max benefit: 2 000 € (monthly instalments covered up to 12 months in a row)
  - ✓ LE: The total of monthly instalment – Max benefit: 2 000 € (monthly instalments covered up to 12 months in a row)

- **Packages**
  - ✓ Package1 – Death (or Death Balloon) + TTD
  - ✓ Package2 – Death (or Death Balloon) + TTD + LE

  Death balloon corresponds to a loan where the instalments are not all equal so the amortization of the loan is not linear.

- **Medical formalities**
  - ✓ None

- **Exclusions in the event of death**

  We will focus on the exclusions in the event of death only as the purpose of this research paper is to build a mortality experience table.

  1) One particularity of the German market is the substitution of the standard exclusion on the preexisting illnesses that we have in France or other markets by the clause 12/24 defined below:

  <The insurer shall not be obliged to pay the benefit in the event of death, as a result of an existing illness of which the insured person was aware at the time at which he has submitted his Declaration of Enrolment, and for which the insured person has consulted or has been treated by a physician in the last 12 months prior to the submission of his Declaration of Enrolment.

  This exclusion shall only apply if the insured event occurs within the last 24 months following the commencement of the insurance cover and is causally connected with one of the illnesses indicated below:

  1. *Ovary- , testicle- , breast- , intestine-, prostata-, pancreas-, lymph node-, liver-, kidney-, gastric-, bones-, and respiratory system cancer as well as brain tumors, leukemia, malignant tumors, Hodgkin diseases and melanoma;*
  2. *Following illnesses of the cardiovascular system: high blood pressure, heart attack, thrombosis, cardiovascular diseases, strokes;*
  3. *Diabetes mellitus Type I or II as well as the resultant diseases diabetic retinopathy, neuropathy and nephropathy;*
  4. *Following chronic diseases of the respiratory system and the lungs: COPD (Chronic Obstructive Pulmonary Disease), asthma bronchiale, pathological changes to the pulmonary tissue (emphysema), pulmonary embolism, sleep apnea;*
  5. *Following liver and gall diseases: hepatitis (all kinds), cirrhosis, diseases of the gall bladder, fatty liver disease, chronic bile-duct inflammations, primary biliary cirrhosis, Crohn's disease;*

6. *All abrasions of joints (arthrosis) and defective position of joints if therefore a surgical intervention (also joint endoscopy) was done, artificial joints, state of stimulus or overload for hands and arms (tennis elbow), carpal tunnel syndrome, fibromyalgia, rheumatoid polyarthritis, sarcoidosis;*
7. *Renal failure, renal insufficiency, all deformities of kidney and the deducted urinary passages;*
8. *Epilepsy, Parkinson´s disease, dementia, congenital misfunctions of the spinal cord, multiple sclerosis;*
9. *HIV / Aids, immune system disorder as consequence of a HIV diagnosis;*
10. *Systemic and localized autoimmune diseases>*

2) Furthermore, the insurer shall not be obliged to pay the benefit in the event of death as a consequence of the results of an accident for which he has consulted or has been treated by a physician in the last 12 months prior to the submission of his declaration of enrolment.

This exclusion shall only apply if the insured event occurs within the last 24 months following the commencement of the insurance cover and is causally connected with the results of an accident. An accident is deemed to have occurred if the insured person suffers damage to health as a result of a sudden, involuntary, violent external impact (accident event).

3) The insurance coverage is also excluded in the event of death caused:

1. *by intentionally caused illnesses or accidents or owing to refusal to have treatment for illnesses or accidents;*
2. *by suicide in the course of the first three policy years. However, if it is proven that this act was committed in a condition of morbid disturbance of mental activity precluding the exercise of free will, the claim for benefits will continue;*
3. *by accidents when using aircraft with or without an engine, powered gliders, microlight planes, when parachuting, as pilot in command or as another crew member of an aircraft;*
4. *directly or indirectly by war or civil war, acts of terrorism or riots, but only of you were an active participant/ collaborator in such events;*
5. *by an addiction (e.g. misuse of drugs or medicaments), by alcoholism or by a disturbance of unconsciousness caused by drunkenness;*
6. *by exposure to nuclear energy, either directly or indirectly;*
7. *by surgical operations and medical treatments which were not carried out on medical grounds (e.g. cosmetic surgery, breast augmentation, piercings);*
8. *by accidents which you have as a result of taking part as a driver, co-driver or passenger of a motor vehicle in driving events, including the appurtenant practice drives, in which it is important to reach top speeds;*
9. *by mental illnesses, for example depressive illnesses (as for example depression, dysthymia, exhaustion syndrome), anxiety disorders, neuroses, schizophrenia, eating disorders, dementia, psychosomatic disorders (for example pain or feeling of illness without any apparent cause);*

10. *by accidents, which occurred in the participation of sports with a higher accidental/injury risk (extreme sport). These sports are for example fight sports, rock climbing, cave climbing, underwater diving with autonomous apparatus, bobsleigh, tobogganing, ski jumping, paragliding or bungee jumping. This enumeration is not final.*

II. PROCESSING OF DATA

II.1. *DATA DIAGNOSTIC*

Two files with the history of data since end of 2009 were requested to the German subsidiaries:

- ✓ The first file called "Exposure file"
- ✓ The second file called "Claims file"

Both files were extracted at contract level.

Name of file	Description	Nb of lines
Exposure file	Contain all contracts subscribed since end of 2009 to end of 2015 (extraction date - 31/12/2015)	274 419
Claims file	Contain all claims (death risk only) declared since the launch of the activity (extraction date – 31/08/2016)	1 275

To check the reliability of data, several controls on both files were automated:

- **File 1 – exposure**

Obs	Variable	NotMissing	Missing	Perc_Of_missing_	Min	Max	Minn	Maxn
1	CONTRACT_EFFECTIVE_DATE	274419	.	.	15/09/2009	15/12/2015	.	.
2	CONTRACT_ID	274419	.	.	.	.	.	.
3	DURATION	274419	.	.	.	.	9	96
4	INITIAL_SUM_INSURED	274419	.	.	.	.	0	92090
5	INSURED_BIRTHDATE	274414	5	0,00%	20/11/1929	23/11/1997	.	.
6	INSURED_FIRSTNAME	274273	146	0,05%	.	.	.	.
7	INSURED_GENDER_CODE	274419	197	0,07%	.	.	.	.
8	INSURED_ID	.	274419	100,00%	.	.	.	.
9	INSURED_SURNAME	274418	1	0,00%	.	.	.	.
10	OUTSTANDING_SUM_INSURED	274346	73	0,03%	.	.	0	72692
11	PRODUCT_ID	274419	.	.	.	.	.	.
12	PRODUCT_NAME	274419	.	.	.	.	.	.
13	QUOTITY_INSURED	274419	.	.	.	.	1	1
14	STATUS_AT_THE_CLOSING_DATE	274419	.	.	.	.	.	.
15	STATUS_CHANGE_DATE	274419	.	.	09/12/2009	14/01/2016	.	.
16	SUB_PRODUCT_ID	274419	.	.	.	.	.	.
17	SUB_PRODUCT_NAME	274419	.	.	.	.	.	.
18	THEORETICAL_ENDING_DATE	274418	1	0,00%	01/10/2010	15/12/2023	.	.

- ✓ ***Analysis of variables:***
  - The variables 1 to 4, 11 to 17 are fully completed.
  - The variable 5 "INSURED_BIRTHDATE" has 5 missing data that represent 0,002% of the portfolio. As we did not record any claim for these contracts, the latter were removed from the report.
  - The variable 6, 7, 9 and 10 have a few missing data but the gender only has an impact on the building of the experience tables by gender. As we start building a unisex experience table (for which the gender has no impact on the unisex experience table) and we build in a second step the experience tables by gender, we have considered that the 197 missing genders follow the same distribution

than the population with the same age. No specific treatment was executed for the other data.

- The variable 8 "INSURED_ID" do not exist in the database so a unique insured ID must be created.
- The variable "OUTSTANDING_SUM_INSURED" has 0,03% of missing lines. It is not significant but it was nevertheless taken into account for the control on the sum at risk.
- The variable 18 "THEORETICAL_ENDING_DATE" has one missing data but it has no impact on the calculation as we consider the status_change_date for the calculation of exposure.

➢ **File 2 – claims**

Obs	Variable	Notmissing	Missing	Perc_Of_missing_data	Min	Max	Minn	Maxn
1	CONTRACT_ID	1275	.	.	.	.	.	.
2	Claim_occurrence_date	1275	.	.	27/12/2009	05/08/2016	.	.
3	Decision	1275	.	.	.	.	.	.
4	INSURED_BIRTHDATE	1275	.	.	20/11/1929	27/07/1993	.	.
5	INSURED_FIRSTNAME	1275	.	.	.	.	.	.
6	INSURED_ID	.	1275	100,00%	.	.	.	.
7	INSURED_SURNAME	1275	.	.	.	.	.	.
8	PRODUCT_ID	1275	.	.	.	.	.	.
9	PRODUCT_NAME	1275	.	.	.	.	.	.
10	Payment_date	958	317	24,86%	01/04/2010	29/08/2016	.	.
11	RBNS_amount_local_cur	63	1212	95,06%	.	.	0	42225,2
12	Reporting_date	1275	.	.	02/02/2010	29/08/2016	.	.
13	Risk_involved	1275	.	.	.	.	.	.
14	SUB_PRODUCT_ID	1275	.	.	.	.	.	.
15	SUB_PRODUCT_NAME	1275	.	.	.	.	.	.
16	Total_Insured_Amount	1275	.	.	.	.	0	45024,3
17	paid_claims_local_cur	1274	1	0,08%	.	.	0	43044,9

✓ ***All variables are fully completed except:***

- The variable 6 "INSURED_ID" that must be created,
- The variables 11 "RBNS_amount_local_cur" and 12 "Reporting_date' as they represent the share of claims that are paid or pending. The rest of claims were refused,
- The variable "paid_claims_local_cur" has one missing data but it has no impact on the mortality study as the mortality rates are based on number.

In addition to this set of controls done on the missing data, several controls on the consistency between claims and exposure data file were performed to be sure that data between both files were matching:

➢ **Control n°1 – Consistency between claims and exposure data files**

All contracts present in the claims file are also present in the exposure file.

All contracts notified with a death in the exposure file are also present in the claims file.

- **Control n°2 – Consistency between date of birth of exposure file and claims file**

  The mistakes about the dates of birth were corrected. So the dates of birth of the exposure file and claims file match.

- **Control n°3 – Consistency between status_change_date from the exposure file and claim_occurence_date from the claims file**

  These dates were different in each file but the gap was a matter of a few days. All the status_change_date from the exposure file were replaced by the claim_occurence_date from the claims file that include the correct date. Indeed, the contract management tool being fulfilled later on, it often happens that the recording date is entered in the exposure file instead of the claim_occurrence_date.

- **Control n°4: Uniqueness of contract number**

  All contract numbers are unique. There are no doubloons in the file.

- **Control n°5: Uniqueness of insured number**

  This data is not fulfilled in the database so this control is not applicable.

To finalize the set of controls, it was important to check the completeness of the exposure and claims files. To do it, three accounting controls were done:

- **Control n°1: Check of the sum at risk**

Variable	N	Sum	Accounts	GAP	GAP %
INITIAL_SUM_INSURED	274419	3 419 737 543			
OUTSTANDING_SUM_INSURED	274346	1 694 033 488	1 681 239 020	12 794 468	0,76%

  The amount of the "outstanding_sum_insured" is 0,76% superior to the sum_at_risk booked in the account. As the gap is inferior to +1%, we considered that it is satisfactory. In any case, a prudential approach is followed in the building of the experience table because the operational risk is always present when big database is used.

- **Control n°2: Check of the amount paid for each accounting year**

Payment_year	Claims reporting	Accounts	GAP
2010	116 208,71	124 214,47	-8 005,76
2011	517 560,84	517 140,56	420,28
2012	760 320,03	760 320,03	-
2013	1 024 527,79	1 024 527,79	-
2014	1 155 400,82	1 155 400,82	-
2015	2 653 526,97	2 653 526,97	-
2016			-
Total	6 227 545,16	6 235 130,64	-7 585,48

  - The data of the claims reporting from the accounting years "2012 to 2015" are matching with the accounts.
  - For 2011, we observe a gap of 420,28 € that seems small and reasonable.
  - For 2010, we observe a gap of 8005,76 €.

  Because the claims management tool was launched in 2012, it is difficult to identify the reason of the gap for the accounting years 2010 and 2011. As we only take into account the effective dates of the contracts subscribed between 2011 and 2015 (that will be described in the document later on), we can consider that the control is successful (i.e. amounts are matching from 2012 to 2015 and the discrepancy observed in 2011 – 420,28 € – is marginal).

- **Control n°3: Check of the amount of RBNS with the closing at 31/08/2016**

Variable	Sum	Accounts	GAP
RBNS_amount_local_cur	597 042,64	616 396,08	- 19 353,44

The difference of 19 353.44 € corresponds to a claim that was counted twice in the accounts (in the paid claims and in the pending claims). The claims file was corrected to remove the doubloon.

Now that all the data were checked and corrected when necessary, we must check that the date of extraction of the claims file is relevant. Indeed, the timelag between the end of the observation period (exposure file reference date) and the reference date of the claims file must be sufficient to include all claims related to the contracts included in the exposure file.

The table represents the notification delay from the occurrence date of the claim.

Notification_Delay	Nb_claims	Sum_claims	Percent_of_claims_declared
nb of claims declared within 3 months	1172	1275	91,92%
nb of claims declared within 4 and 6 months	75	1275	5,88%
nb of claims declared within 7 and 9 months	14	1275	1,10%
nb of claims declared within 10 and 12 months	3	1275	0,24%
nb of claims declared within a period superior to 12 months	6	1275	0,47%
other	5	1275	0,39%

Notification_delay	Figures
Mean	1,07
Max	23

Almost 92% is declared within the 3 first months. Almost 98% is declared within the 6 first months. So we can consider that a delay of 6 months between the end of the observation period (exposure file reference date) and the reference date for the claims report is sufficient.

Now that we are confident that our data are reliable and consistent, we will work on the files to have a database at insured level instead of having it at contract level.

II.2. *PROCESSING OF DATABASE: FROM CONTRACT TO INSURED LEVEL*

The first step is to have a key in the database to be able to identify the insured. As the latter does not exist in the database, the first step was to create an insured_ID to fulfil this role. For simplicity reasons, our key is the concatenation of the insured_birthdate, the insured_firstname and the insured_surname.

Before proceeding to the concatenation, it is important to clean up the fields insured_firstname and insured_surname because some typing errors are present or different number of surnames are completed.

Four types of correction are done:

- Correction of the specificity of the language:
    - ✓ ü/ö/ä written ue/oe/ae or u/o/a in the administration tool
    - ✓ removal of titles like Dr. or Prof.
    - ✓ exclusion of some characters like dash
- Manual correction of surnames
- Implementation of a program under SAS to recognize the multiple surnames and limit the number of surnames to one
- Correction of typing errors on the surnames thanks to the function “Soundex” on SAS.

Once the insured_ID is created, it is important to update the exposure file with the most recent information. Indeed, as the claims file (extraction date: 31/08/2016) is posterior to the exposure file (extraction date: 31/12/2015), the claims and exposure files do not contain the same number of claims. A set of controls and processing are performed:

- A first control is to check that all claims recorded in the claims file have a correspondence in the exposure file (each contract ID in claims data file is present in the exposure data file). This control was performed during the data diagnostic.
- Then, the exposure file must be completed with the claims coming from the claims file which status is “closed”, “pending” or “refused” and which Claim_occurrence_date is anterior to 31/12/2015.
    - ✓ If the contracts are only present in the exposure file:
        - STATUS_AT_THE_CLOSING_DATE: Other
        - STATUS_CHANGE_DATE do not change
    - ✓ If the contracts are present in the claims file with a status “closed” or “pending” or “refused”:
        - STATUS_AT_THE_CLOSING_DATE: Death
        - STATUS_CHANGE_DATE: Claim_occurrence_date
- To aggregate the exposure file at insured ID level, it is important to define a starting date and a termination date for the calculation of the exposure.
    - ✓ When an insured has one single contract, the starting date and the termination date correspond respectively to the CONTRACT_EFFECTIVE_DATE and the STATUS_CHANGE_DATE.
    - ✓ When an insured has several contracts, we must identify the periods of interruption of exposure.

Example:

CONTRACTS	CONTRACT_EFFECTIVE_DATE	STATUS_CHANGE_DATE
Contract n°1	03/01/2010	24/07/2012
Contract n°2	16/06/2011	23/12/2013
Contract n°3	07/02/2015	31/12/2015

In this example, the insured is not exposed from 24/12/2013 to 06/02/2015.

To be able to calculate the exact exposure of each insured:

- ✓ A key must be created and incremented each time an interruption of exposure exists. In the above example, we will have:

CONTRACTS	CONTRACT_EFFECTIVE_DATE	STATUS_CHANGE_DATE	KEY
Contract n°1	03/01/2010	24/07/2012	1
Contract n°2	16/06/2011	23/12/2013	1
Contract n°3	07/02/2015	31/12/2015	2

- ✓ Then, a new INSURED_ID _New with the concatenation of two values (INSURED_ID and KEY) is created.
- ✓ For each INSURED_ID_New, the starting date corresponds to the oldest CONTRACT_EFFECTIVE_DATE and the termination date to the most recent STATUS_CHANGE_DATE. In the example, we would have two lines:

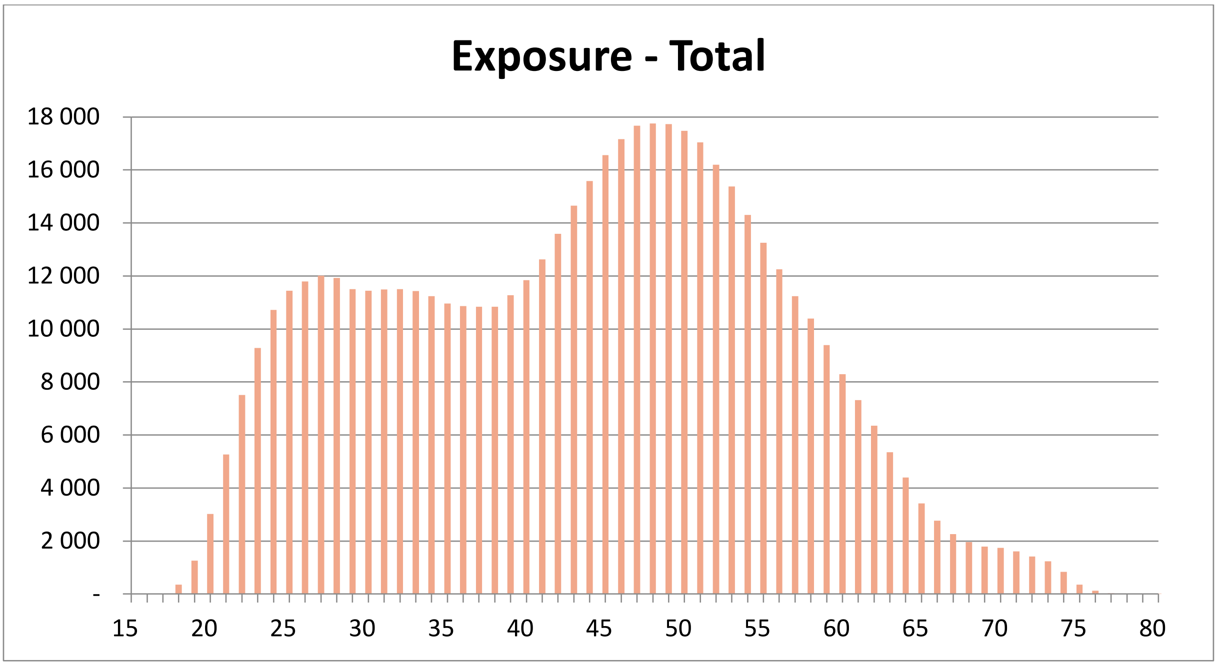
INSURED_ID_New	CONTRACT_EFFECTIVE_DATE	STATUS_CHANGE_DATE
1-1	03/01/2010	23/12/2013
1-2	07/02/2015	31/12/2015

- ➢ Once the CONTRACT_EFFECTIVE_DATE and the STATUS_CHANGE_DATE are defined, the final database is created with the different values:
  - ✓ INSURED_ID_New
  - ✓ INSURED_BIRTHDATE
  - ✓ INSURED_GENDER_CODE
  - ✓ CONTRACT_EFFECTIVE_DATE
  - ✓ STATUS_CHANGE_DATE
  - ✓ STATUS_AT_THE_CLOSING_DATE

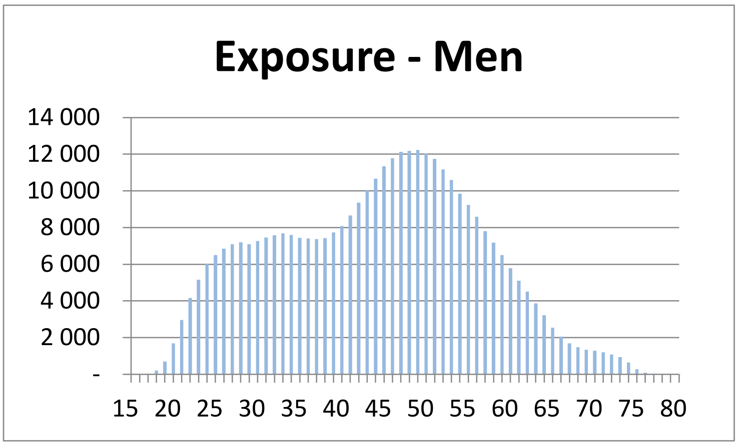
    → Hierarchisation of values
    - o If one of the contracts of the INSURED_ID_New was closed because of a death and the other contracts expired/lapse before the death had occurred, the STATUS_AT_THE_CLOSING_DATE must be completed with the value “death”.
    - o Otherwise the status is “other”.

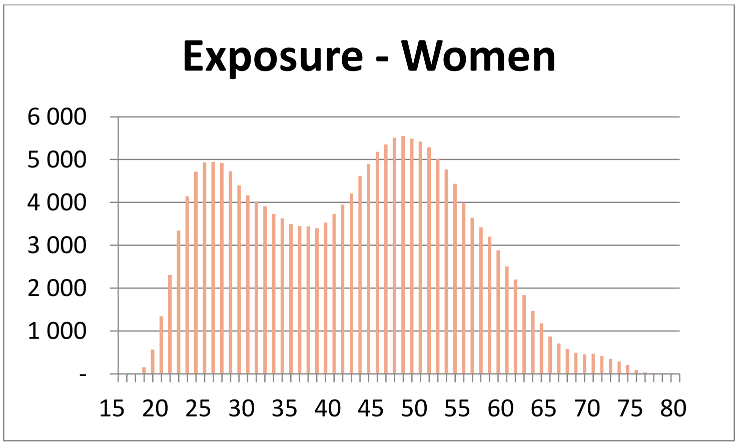
III. PRESENTATION OF THE PORTFOLIO

III.1. *ANALYSIS OF THE POPULATION - EXPOSURE*



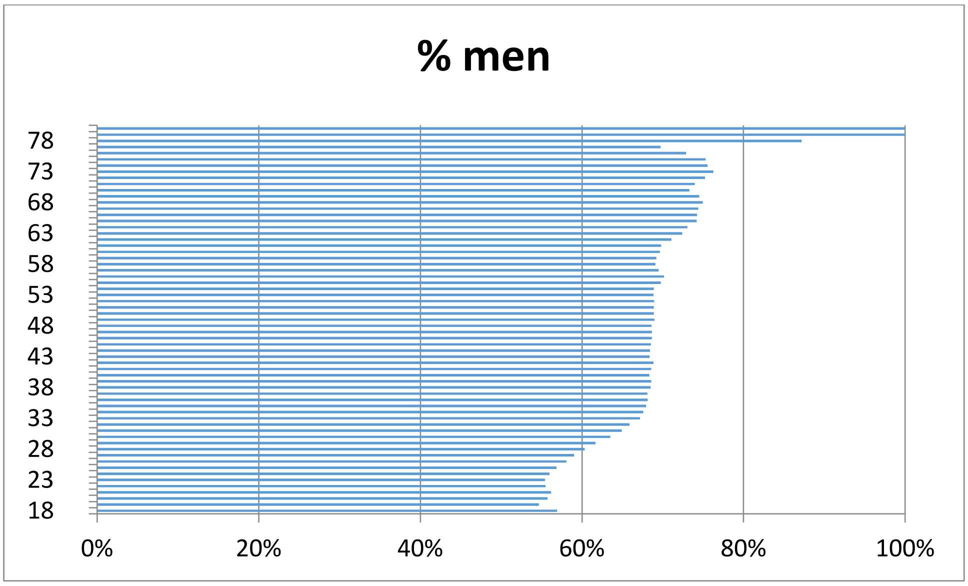
As the year 2009 was incomplete, we focused our study on the exposure from 2010. We see on this graphic that a major part of the exposure is included between **25 and 60** years old.





When we look at the men exposure, we see that the overall exposure has the same shape than the men exposure that is completely normal as the portfolio is constituted by around 2 third of men.

The shape for the women exposure is a little bit different as we see clearly a slowdown of exposure between 27 and 38 years old.



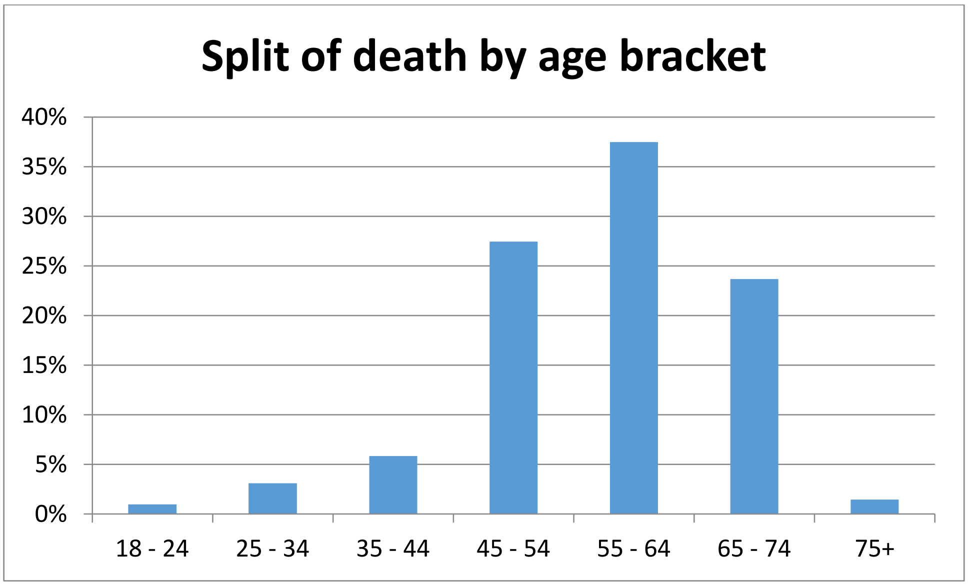
On this graphic, we see that the percentage of men having the insurance is globally higher with the age.

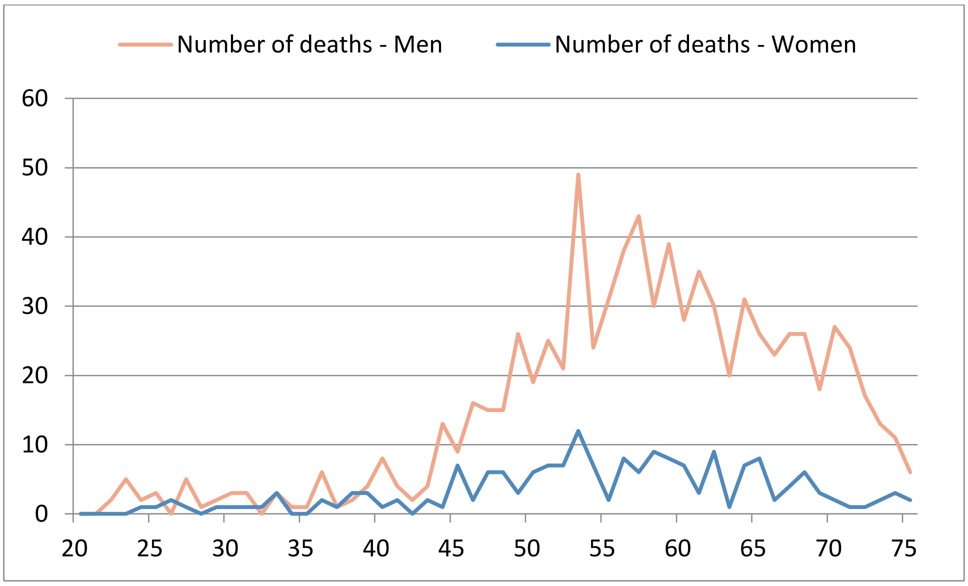
We will not compare BDK portfolio with the German national population as we clearly see that the interest for the final product bought (the car) is more important for men than women. It is therefore not interesting to compare this split between men and woman with the German population as the latter is balanced (49% of men in July 2016).

III.2. *DEATH*

III.2.1. Analysis of the deaths

We decided to include all declared death within the study (paid claims, reported but not settled claims and refused claims). A question was open for the refused claims but as the clause 12/24 was amended from mid-September 2015 to clarify the list of illnesses, we expect an increase of the acceptation rate. As it is too early to evaluate the impact, we preferred being prudent and integrate all claims including the refused ones.





Almost 89% of death is included between 45 and 74 years old. We can notice that some deaths are observed after 75 years old whereas the maturity age of the product is 75 years old. That is due to an operational failure: the control on the age at maturity is not performed at subscription by BDK. A study on the pricing is currently ongoing to extend this age at maturity at 78 years old.

The number of deaths is around 5 times more important for men than women. That is due to 2 factors: the nature of the good (car) financed by the loans and the lower mortality for women than men.

A peak of mortality (both for men and women) is observed at 53 yo that will need to be cautiously taken into account during the smoothing process.

III.2.2. Evolution of death by generation

Generations	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Total
2010	0,18%	0,15%	0,17%	0,20%	0,15%	0,16%	**1,02%**
2011	0,19%	0,22%	0,23%	0,16%	0,30%		**1,11%**
2012	0,15%	0,21%	0,18%	0,28%			**0,81%**
2013	0,15%	0,19%	0,20%				**0,54%**
2014	0,17%	0,19%					**0,35%**
2015	0,18%						**0,18%**

We will limit our study to the generations **2011 to 2015** as we observe that the mortality of the generation 2010 is inferior to the other generations. Indeed, after 6 years, the mortality rate of the generation 2010 is only equal to 1,02% whereas the mortality rate of the generation 2011 has already reached 1,11% after 5 years.

III.3. *Data sufficiency*

Now that we know we are limiting the period of observation to the years 2011 – 2015, we want to check the data sufficiency to build a experience table. To check it, we apply 2 criteria:

1. Cochran criteria:

Cochran criteria: $N_x \times \hat{q}_x \geq 5$ and $N_x \times (1 - \hat{q}_x) \geq 5$ with $\hat{q}_x = \frac{D_x}{E_x}$

$\hat{q}_x$ being the estimator of the mortality rate at age x, unbiased and convergent.

We note E_x the number of days (fraction of years) the person is observed at the age x and D_x the number of people who died at the age x.

Age	Nx * qx	Nx * (1-qx)	Age	Nx * qx	Nx * (1-qx)	Age	Nx * qx	Nx * (1-qx)
18	-	286	44	10	10 615	70	25	1 243
19	-	979	45	10	11 355	71	19	1 171
20	-	2 306	46	13	11 844	72	15	1 085
21	-	3 972	47	18	12 271	73	12	1 016
22	2	5 602	48	14	12 419	74	14	743
23	2	6 793	49	23	12 526	75	8	329
24	3	7 799	50	16	12 402	76	4	107
25	3	8 216	51	24	12 143	77	-	29
26	2	8 373	52	20	11 575	78	-	5
27	4	8 479	53	44	11 004	79	-	1
28	1	8 391	54	20	10 249	80	-	0
29	3	7 980	55	27	9 506	81	-	-
30	3	7 908	56	32	8 813	82	-	0
31	4	7 998	57	36	8 060	83	-	1
32	-	8 026	58	30	7 452	84	-	1
33	5	7 961	59	30	6 738	85	1	- 0
34	1	7 825	60	24	6 004	86	-	-
35	1	7 594	61	26	5 333	87	-	-
36	7	7 540	62	28	4 646	88	-	-
37	2	7 541	63	16	3 920	89	-	-
38	3	7 456	64	29	3 210	90	-	-
39	4	7 730	65	26	2 492	91	-	-
40	8	8 019	66	20	2 020	92	-	-
41	4	8 456	67	17	1 625	93	-	-
42	2	9 108	68	23	1 384	94	-	-
43	2	9 909	69	14	1 267	95	-	-

Results of Cochran criteria:

We observe that it exists some age brackets [from 18 yo to 43yo] and [from 76 yo] where the first criteria of Cochran $N_x \times \hat{q}_x \geq 5$ is not always fulfilled.

The period between **44 yo - 75 yo** is matching Cochran criteria.

2. Exposure criteria:

For the German portfolio, we consider that a minimum of 1 500 exposures by age is a minimum to integrate the age bracket in the period of observation. 1 500 was defined as a limit after we performed several simulations on the smoothing. Hence, the period of observation matching the threshold exposure is **20 yo – 67 yo.**

In order to meet the Cochran and the exposure criteria, the age range selection is **44 yo - 67 yo**.

Chapter 2 – Building of a mortality experience table

I. CALCULATION OF CRUDE MORTALITY RATES

I.1. *NOTATION*

q_x is the probability of dying for a person at age x.

T is the life expectancy of a person. T is a random variable $T \in \mathbb{N}$.

$S_x: t \to S_x(t)$ is the probability to survive at least until the age x.

I.2. *HOEM ESTIMATOR*

I.2.1. Reminder of Hoem method

The first method to build the mortality table is to calculate the number of deaths / number of years of exposure for each age. One big advantage of this method is its simplicity to implement.

The exposure of a person is the duration in years (not necessarily an integer) where the person is present in the portfolio.

For example, if a person subscribes a contract at 50,5 years old and if his/her contract is still active when he/she is 51 years old, the exposure of this person at 50 years old is half a year.

We note E_x the number of days (fraction of years) where we observe the person at age x and D_x the number of people who died at the age x. We can therefore calculate Hoem estimator of the mortality rate q_x:

$$\hat{q}_x = \frac{D_x}{E_x}$$

D_x is the sum of independent indicator functions $X_{i,x}$ (representing the death of the insured i before the age of $x+1$ whereas the insured i is alive at age x) with the same law "Bernouilli" so we can conclude that:

$$\forall_i \in [1, E_x] \;\; X_{i,x} \sim \mathcal{B}(q_x)$$

$$\forall_i \in [1, E_x] \;\; E(X_{i,x}) = q_x$$

$$\forall_i \in [1, E_x] \;\; Var(X_{i,x}) = q_x \cdot (1 - q_x)$$

$$\forall (i,j) \in [1, E_x]^2 \;\; i \neq j \;\; X_{i,x} \perp X_{j,x}$$

Then $\forall_i \in [1, E_x] \; E(\hat{q}_x) = \frac{E(D_x)}{E_x} = E_x * \frac{E(X_{i,x})}{E_x} = q_x$

Hoem estimator is therefore **unbiased.**

And $\forall_i \in [1, E_x]$ $V(\hat{q}_x) = \frac{V(D_x)}{{E_x}^2} = E_x * \frac{V(X_{i,x})}{{E_x}^2} = \frac{q_x \cdot (1-q_x)}{E_x}$

We know that $\frac{q_x \cdot (1-q_x)}{E_x} \xrightarrow[x \to +\infty]{} 0$ so we can conclude with the inequality of Bienaymé-Tchebychev that Hoem estimator is **convergent**.

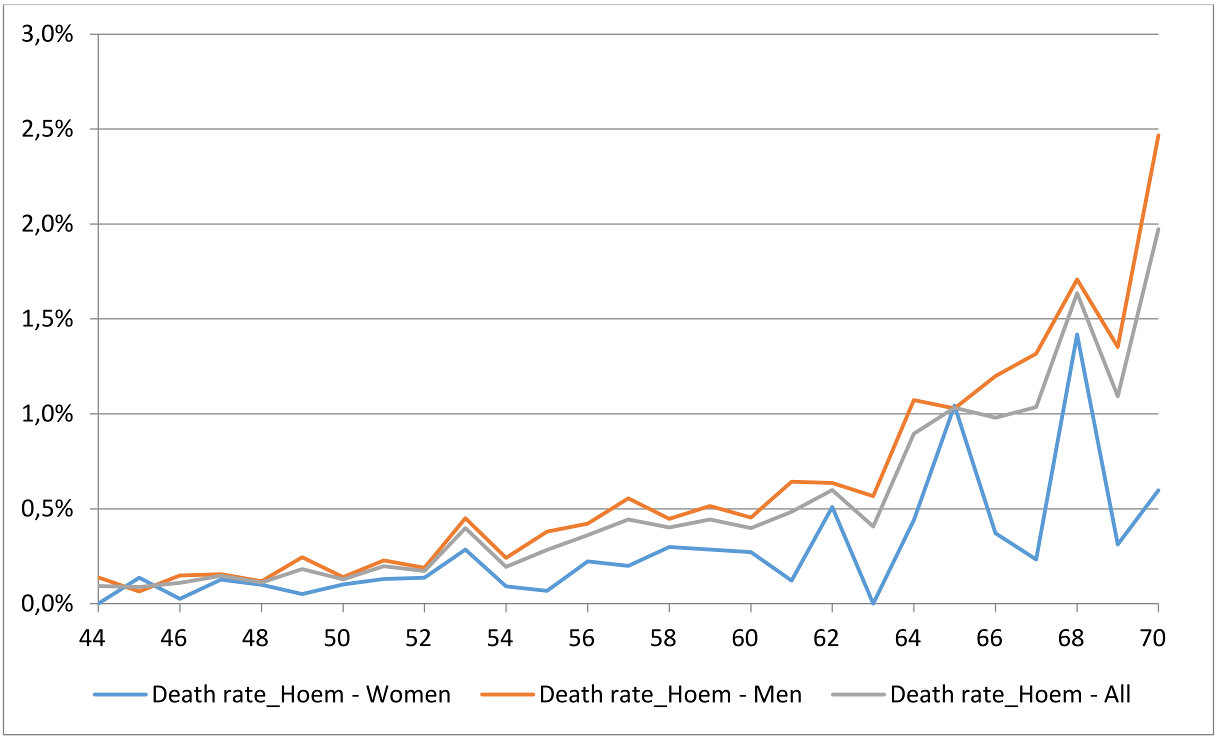
<u>Reminder of the inequality of Bienaymé-Tchebychev:</u>

$\forall \epsilon > 0 \qquad \mathbb{P}\left(\left|\hat{\theta}_n - \theta\right| > \epsilon\right) \leq \frac{Var(\hat{\theta}_n)}{\theta^2}$

As a consequence, we can say that $\hat{q}_x$ is a good estimator of the mortality q_x.

I.2.2. Application to credit life portfolio with BDK

Comparison of death rates by age for the period between 44 yo and 67 yo.



We observe that the unisex curve is between the curves of the women and the men population that is in line with our expectation. Moreover, we notice that the death rates of the global population are closer to the death rates of the male population that corresponds to the profile of the portfolio (69% men / 31% women for the age selection between 44 yo and 67 yo).

Asymptotic confidence intervals

We will now build a confidence interval of this estimator to see the precision.

We know that:

$\forall_i \in [1, E_x] \;\; X_{i,x} \sim \mathcal{B}(q_x)$

$\forall_i \in [1, E_x] \;\; E(X_{i,x}) = q_x$

$\forall_i \in [1, E_x] \;\; Var(X_{i,x}) = q_x \cdot (1 - q_x)$

$\forall (i,j) \in [1, E_x]^2 \;\; i \neq j \;\; X_{i,x} \perp X_{j,x}$

Then it is possible to apply the central-limit theorem defined below:

If $X_1, X_2, \dots, X_n$ is a random sample of size n that is a sequence of independent and identically distributed random variable with the same law then if we write down:

$$S_n = \sum_{k=1}^{n} X_k$$

And if we suppose that the variables have the same average μ and the same standard deviation (not null) σ then:

$Z_n = \frac{S_n - n\mu}{\sigma\sqrt{n}} \xrightarrow[n \to +\infty]{Law} \mathcal{N}(0,1)$ i.e.

$$\lim_{n \to +\infty} \mathbb{P}[Z_n \leq z] = \Phi(z)$$

Where Φ is the distribution function of a standard normal distribution.

In our case:

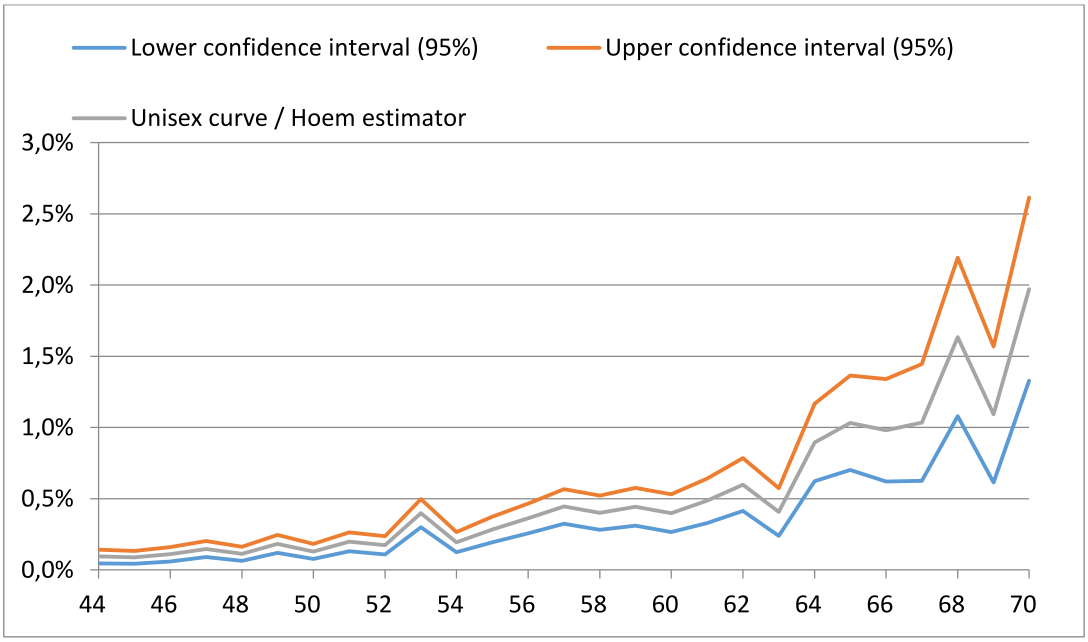
If $\quad S_{E_x} = D_x$

$$Z_{E_x} = \frac{S_{E_x} - E_x \cdot q_x}{\sqrt{q_x \cdot (1 - q_x) \cdot E_x}} = \sqrt{\frac{E_x}{q_x \cdot (1 - q_x)}} \cdot (\hat{q}_x - q_x) \xrightarrow[E_x \to +\infty]{Law} \mathcal{N}(0,1)$$

We can therefore build a 95% asymptotic confidence interval for q_x:

$$q_{\pm}(x) = \hat{q}(x) \pm u_{1-\frac{\alpha}{2}} \cdot \sqrt{\frac{q_x \cdot (1 - q_x)}{E_x}}$$

Where $u_{1-\frac{\alpha}{2}}$ represents the 95% fractile of the standard normal distribution. We will replace q_x by its estimator in the precedent equation.



We observe that the corridor is broader for the older ages (from 54 years old) and get wider with the age, that makes sense because the volatility is superior for the ages where the exposure is inferior.

I.3. *KAPLAN-MEIER ESTIMATOR*

I.3.1. Reminder of Kaplan-Meier method

Before we start explaining the Kaplan-Meier method, we will define what is censoring and truncation.

Censoring: Sources/events can be detected, but the values (measurements) are not known completely. We only know that the value is less than some number.

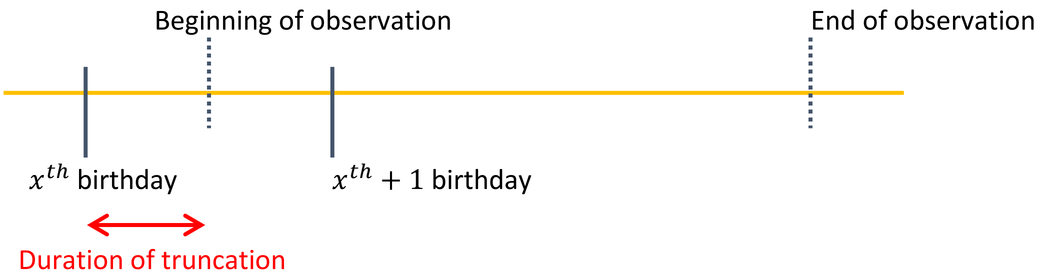
Truncation: An object can be detected only if its value is greater than some number and the value is completely known in the case of detection.

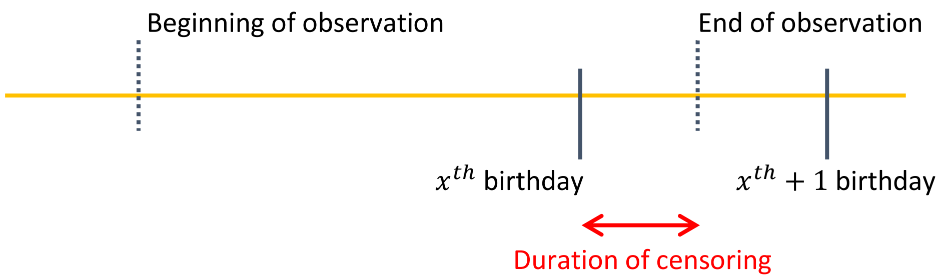
The main difference between censoring and truncation is that censored object is detectable while the object is not even detectable in the case of truncation.

To be more pragmatic, we will present an example of censoring and truncation:

A subject may not be observable during the totality of its $x^{th}+1$ year for 2 reasons mainly:

- It may have had its x^{th} birthday before the observation's starting date and it starts being observable between x and $x+1$: we call it **truncation**.
- Its $x^{th}+1$ birthday is posterior to the end of the observation period: we call it **censoring**.





In both cases, the ages at the beginning and the end of the observation period are known and not. The exit from the portfolio only is random.

The method of Kaplan-Meier was introduced in 1958 by Edward L. KAPLAN and Paul MEIER in an article [7] of the newspaper of the *American Statistical Association* called *Nonparametric estimation from incomplete observations.*

To estimate the proportion of subjects surviving at a given time point, and hence the survival probability to that time for the generic population from which the sample is extracted, the Kaplan-Meier method, also called product-limit estimator, is commonly used, which allows to deal with censored information. The simple idea underlying this method is that to survive to k intervals from the start of the study, it is necessary to survive to each one of the previous intervals and then also to the k^{th} one. This principle allows working with conditional and cumulative probabilities.

$$S(t) = \mathbb{P}(T > t) = \mathbb{P}(T > t|T > x) \times S(x) = S_x(t) \times S(x) \text{ for } t > x$$

Then if we note $p_i = \mathbb{P}(T > a_i|T > a_{i-1})$ with $a_i, i \in \{1, \dots, m\}$ the ages of people between x and $x + 1$, we have:

$$S(a_1) = \mathbb{P}(T > a_1|T > 0) \times S(0) = p_1$$

$$S(a_2) = \mathbb{P}(T > a_2|T > a_1) \times S(a_1) = p_2 \times p_1$$

$$S(a_i) = \mathbb{P}(T > a_i|T > a_{i-1}) \times S(a_{i-1}) = \prod_{k=1}^{i} p_k$$

A logical estimator of p_i is the number of survivors at the date a_{i+1} divided by the number of survivors at the date a_i:

$$\hat{p}_i = \frac{r_i - d_i}{r_i}$$

where r is the number alive and at risk at the beginning of the i^{th} interval ($r_i = r_{i-1} - d_{i-1} - c_{i-1} + t_{i-1}$ (c_{i-1} being the number of people censored on the period $]a_{i-1}, a_i]$, t_{i-1} the number of people truncated on the period $]a_{i-1}, a_i]$ and d_{i-1} the number of deaths during the same interval).

Survival at any time point is calculated as product of the conditional probabilities of surviving each previous time interval.

$$\forall t \in]0, a_m] \ \hat{S}(t) = \prod_{i\,£t} \frac{r_i - d_i}{r_i}$$

And:

$$\hat{q}_x = 1 - \frac{\hat{S}(x+1)}{\hat{S}(x)} = 1 - \frac{\prod_{i|a_i<x+1} \frac{r_i - d_i}{r_i}}{\prod_{i|a_i<x} \frac{r_i - d_i}{r_i}} = 1 - \prod_{i=a_1}^{a_m} \frac{r_i - d_i}{r_i}$$

This estimator is unbiased and the variance is:

$$Var\left(\hat{S}(t)\right) = -\hat{S}^2(t) \sum_{j|t_j \leq t} \frac{d_j}{r_j\left(r_j - d_j\right)}$$

As a miniature example of case, suppose that out of the example of 40 items, the following are observed:

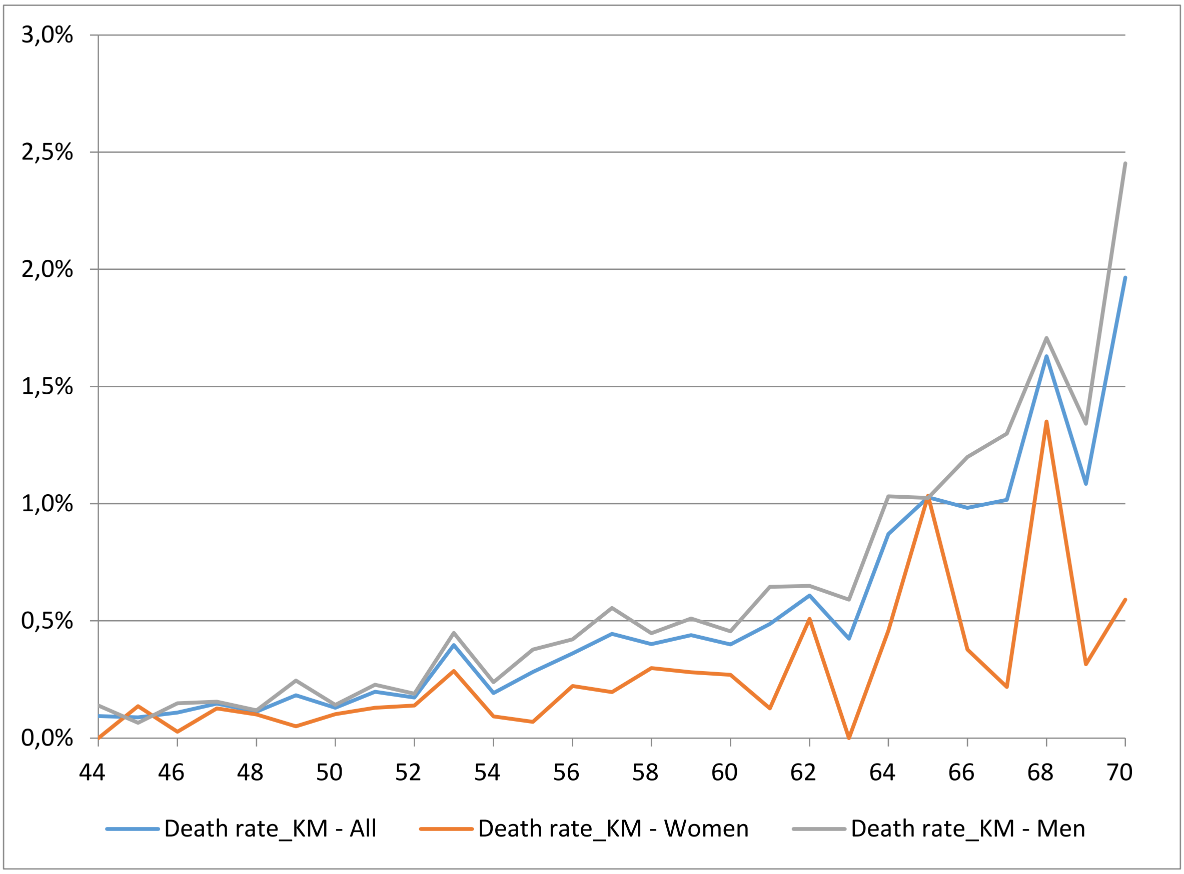
Weekdays	**Nb of leaving people**	**Nb of deaths**	**Nb of people present at the beginning of the period**	**Survival probability**
Monday	2	0	40	100%
Tuesday	1	1	38	97%
Wednesday	3	2	37	92%
Thursday	3	3	34	84%
Friday	4	3	31	76%
Saturday	5	3	27	67%
Sunday	5	4	22	55%

Each survival probability is obtained by multiplying the number of survivals / number of people present at the beginning of the period by the preceding survival probability.

A big advantage of the estimator of Kaplan-Meier is that the latter may be closed to the crude mortality rates without assuming any law a priori. This estimator is hard to implement because it is required to know the exact dates of subscription and exit from the portfolio.

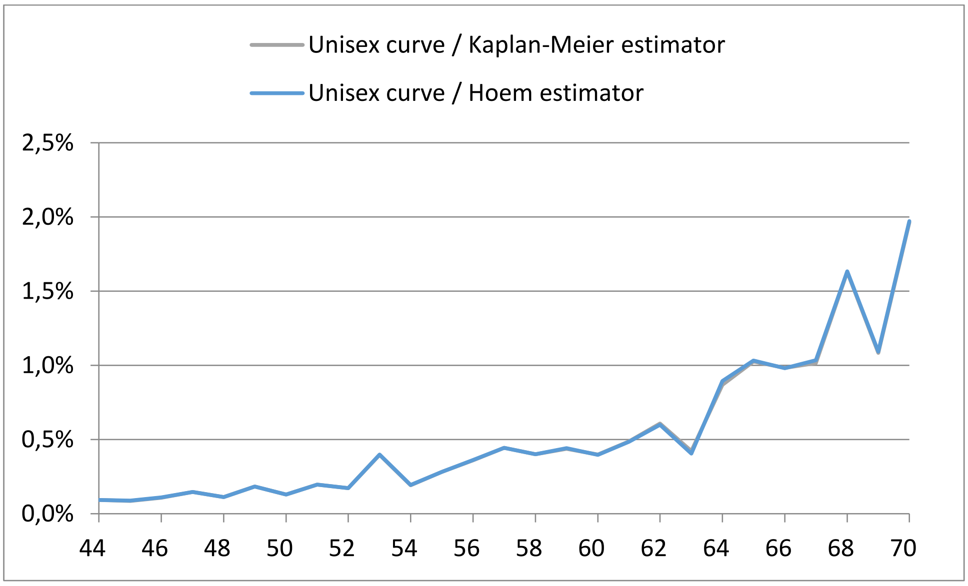
I.3.2. Application to credit life portfolio with BDK

Comparison of death rates by age for the period between 44 yo and 67 yo.



The Kaplan-Meier estimators are very close to Hoem ones that is in line with the expectations, the raw data being identical.

I.4. COMPARISON BETWEEN HOEM AND KAPLAN-MEIER METHOD



The crude mortality rate calculated with Kaplan-Meier method are very close to the ones calculated with Hoem method and are therefore not readable on the graphic. The difference for the class ages between 44 yo and 67 yo is inferior to 0,5% (on average).

II. SMOOTHING OF CRUDE MORTALITY RATES

The mortality curves of the crude mortality rates present some irregularities. It is therefore important to smooth them. A lot of methods exist: we tested 3 methods because the latter were already very efficient for the smoothing of the crude mortality rates of BDK portfolio:

- A non parametric model: Whittaker-Henderson where we allocated the same weight to all ages and Whittaker-Henderson where we chose different weighting depending on the exposure at age x
- A parametric method: Makeham and
- A semi-parametric / relational model: Brass

But other methods exist like the parametric method with Weibull law, the weighted-moving average model, the spline, etc. The curious reader can find information in the document *Lignes directrices mortalité*.

II.1. *WHITTAKER-HENDERSON GRADUATION METHOD*

II.1.1. Reminder of Whittaker-Henderson graduation method

This graduation method was introduced by E. T Whittaker (1923) and developed next by R. Henderson (1924) who showed how to apply the theory.

The method of Whittaker-Henderson consists of minimizing the value of $M = F + hS$ where:

- $F = \sum_{x=x\,\inf}^{x\,\sup} w_x (g_x - \hat{q}_x)^2$, a measure of fit;
- $S = \sum_{x=x\,\inf}^{x\,\sup-z} (\Delta^z g_x)^2$, a measure of smoothness;
- g_x, the graduated data;
- $\hat{q}_x$, the ungraduated /raw data;
- w_x, a nonnegative number representing the weight to be given to the difference between the graduated and ungraduated values associated with a particular value of x;
- h, a non-negative number representing the smoothness constant.

In other words, the graduation process dictates the best possible fit (smallest value of F) and the best possible smoothness (smallest value of S) with the proper balance defined by h. The shape of the underlying data is assumed to be close to a polynomial of degree $z - 1$. The magnitude of h determines how close the graduated data will be to the polynomial. The greater the magnitude of h, the more the graduated data resembles the polynomial; the smaller the magnitude of h, the more the graduated data resemble the original data.

II.1.2. Application to credit life portfolio with BDK

We launched the program with different levels of z and h to smooth the curve. We realized smoothing with $z = 2$ and 3, and $h = 1$, 100 and 1000. We also tested the smoothing with unweighted data $(\forall x, w_x = 1)$and weigthted data. The criteria we used for the weight is:

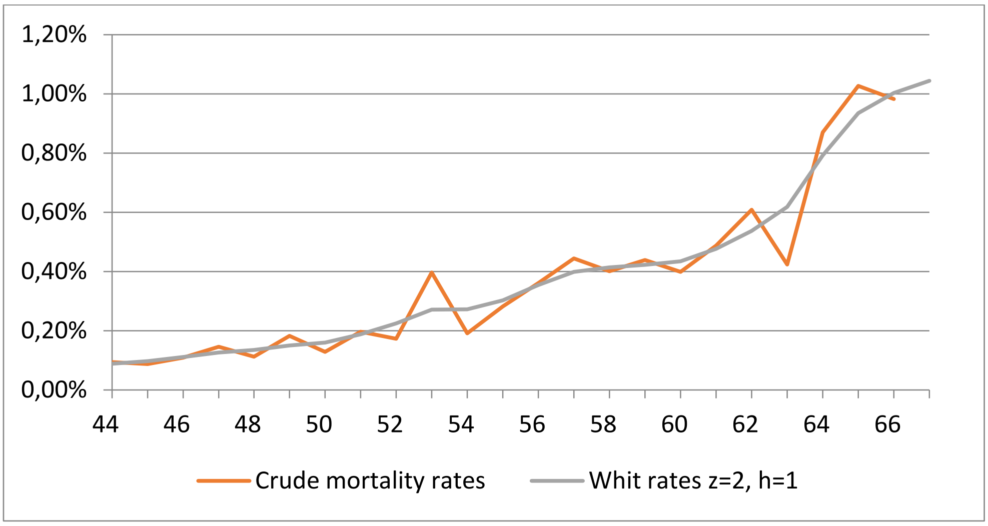
$$w_x = \frac{n_x}{\bar{n}} \text{ where } \bar{n} = \frac{\sum_{x=x\,\inf}^{x\,\sup} n_x}{x\,\sup - x\,\inf + 1}$$

This choice enables to limit the weight given to the absurd ages.

II.1.2.1. Presentation of graphics

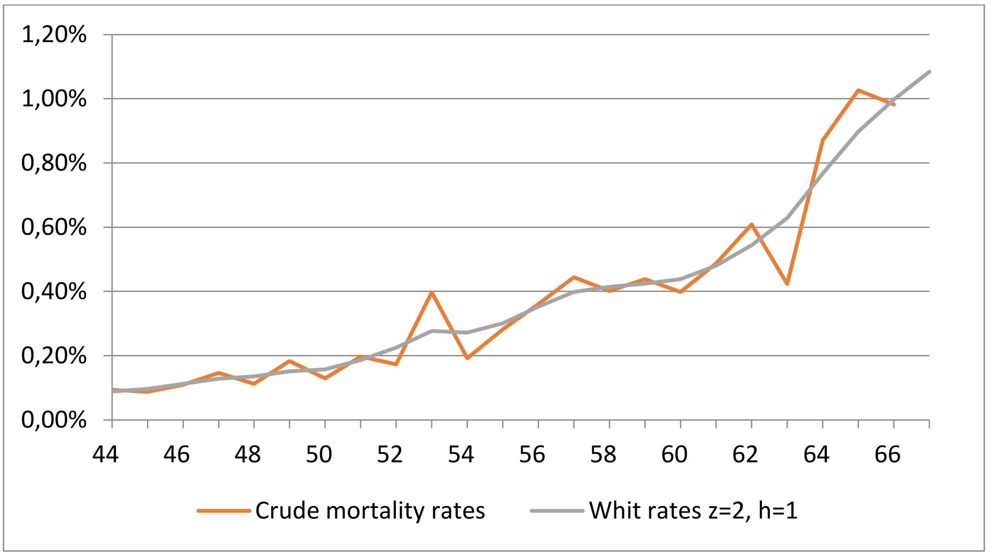
We chose Kaplan-Meier estimators for the application of Whittaker-Henderson graduation method because the results were slightly better than with Hoem estimators.
A lot of additional simulations were realized but we present here the more interesting ones with the best results.

Graphic 1 - ($\forall x, w_x = 1$) / $z = 2$ $h = 1$



Graphic 2 - ($\forall x, w_x = 1$) / $z = 2$ $h = 100$

Graphic 3 - (weighted w_x) / $z = 2\ \ h = 1$



Graphic 4 - (weighted w_x) / $z = 2\ \ h = 100$

Graphic 5 - (weighted w_x) / $z = 3$ $h = 100$

Graphic 6 - (weighted w_x) / $z = 3$ $h = 1000$

II.1.2.2. Choice of the best curves

In order to keep only the best graduated data for the following step of the smoothing process, we applied the chi-squared test to confirm there is no significant difference between the crude mortality rates and the graduated mortality rates. We note:

$$\chi_0^2 = \sum_{x=44}^{67} \frac{(\hat{q}_x - q_x^*)^2}{q_x^* / E_x}$$

$\hat{q}_x$ being the crude mortality rates, q_x^* the smoothed rates and E_x the exposure at age x.

This variable follows a chi-squared law with 23 degrees of freedom (24 ages – 1=23). We will accept the smoothing if $\chi_0{}^2 < \chi^2$ where χ^2 is given by a table. The different values for $\chi_0{}^2$ calculated are:

	Critical threshold	**35,17**	
	Graphics	**Chi-2 Value**	**Validated?**
Unweighted data	**Graphic 1**	17,24	OK
	Graphic 2	26,92	OK
Weighted data	**Graphic 3**	16,91	OK
	Graphic 4	27,24	OK
	Graphic 5	24,39	OK
	Graphic 6	26,30	OK

The 6 smoothing are validated. The best curves in terms of fit are the ones presented in graphics 1 and 3 ($z = 2\ h = 1$). Nevertheless, we decided to keep the curve presented in **graphic 5** to perform the comparison with other methods (Makeham and Brass) because:

- the curves in graphics 1 and 3 are not enough smoothed and still present some irregularities that are not satisfactory and
- the curve of the graphic 5 has obtained the third best result after the curves of graphics 1 and 3.

The rates presented in graphics 1 and 3 will be used to continue and apply Makeham method.

II.2. *MAKEHAM GRADUATION METHOD*

II.2.1. Reminder of Makeham graduation method

On several populations, it was observed that a person's probability of dying increases at a constant exponential rate as age increases. Thus, we can say that the law describes exponential aging. Gompertz (1825) proposed a parametric model to represent this trend:

$$\mu_x = BC^x \text{ with } B > 0,\ C > 1$$

Makeham (1960) generalized the Gompertz model by assuming:

$$\mu_x = A + BC^x \text{ with } A > 0, B > 0, C > 1$$

Where the term A represents age-independent mortality, e.g. because of accidents.

Therefore:

$$p_x = exp\left(-\int_x^{x+t} \mu_s \mathrm{d}_s\right)$$

$$= exp\left(-\int_x^{x+t} (A + BC^s)\mathrm{d}_s\right)$$

$$= exp\left(-\left[As+\frac{B}{\ln C}exp(s\ln C)\right]_x^{x+t}\right)$$

$$= exp\left(-\left(A(x+t-x)+\frac{B}{\ln C}(C^{x+t}-C^x)\right)\right)$$

$$= exp\left(-\left(At+\frac{B}{\ln C}C^x(C^t-1)\right)\right)$$

In order to apply Makeham model, we will:

- check that Makeham model is applicable to the set of data and
- then apply Makeham model by using maximum likelihood method

II.2.2. Application to credit life portfolio with BDK

II.2.2.1. Conditions of application of Makeham model

In a first step, we will represent the curve $\ln|q_{x+1}-q_x|$ from the ages 44 yo to 67 yo where q_x is the mortality rate calculated with both methods (Hoem and Kaplan-Meier).

$$\ln(p_x) = -\left(A+\frac{B}{\ln C}C^x(C-1)\right)$$

With Taylor series of degree 1, we know that:

$\ln(p_x) = \ln(1-q_x) \approx -q_x$ for q_x closed to 0

Therefore:

$q_{x+1}-q_x = \ln(p_x)-\ln(p_{x+1}) = C^x(C-1)^2\left(\frac{B}{\ln C}\right)$ and:

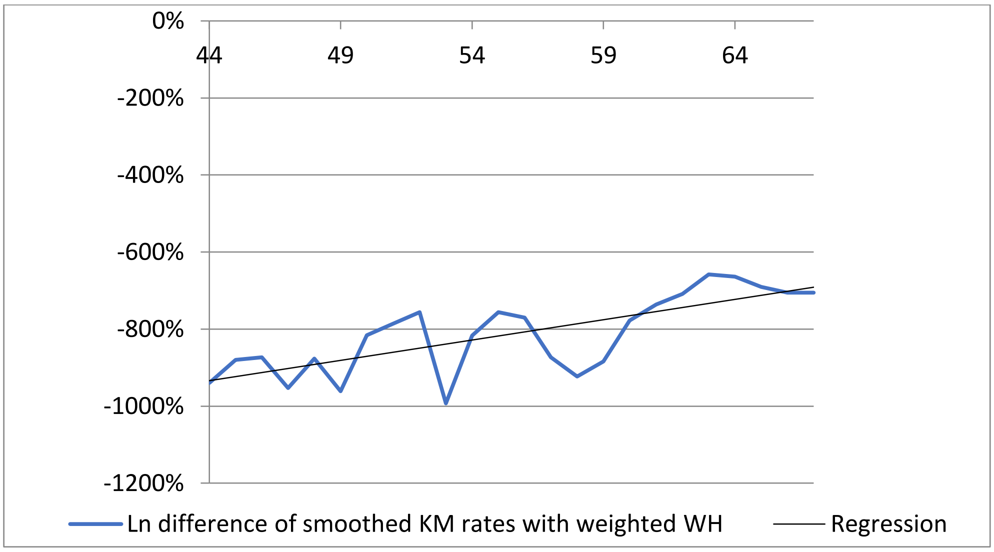
$$\ln|q_{x+1}-q_x| = x\ln C + \ln\left((C-1)^2\left(\frac{B}{\ln C}\right)\right)$$

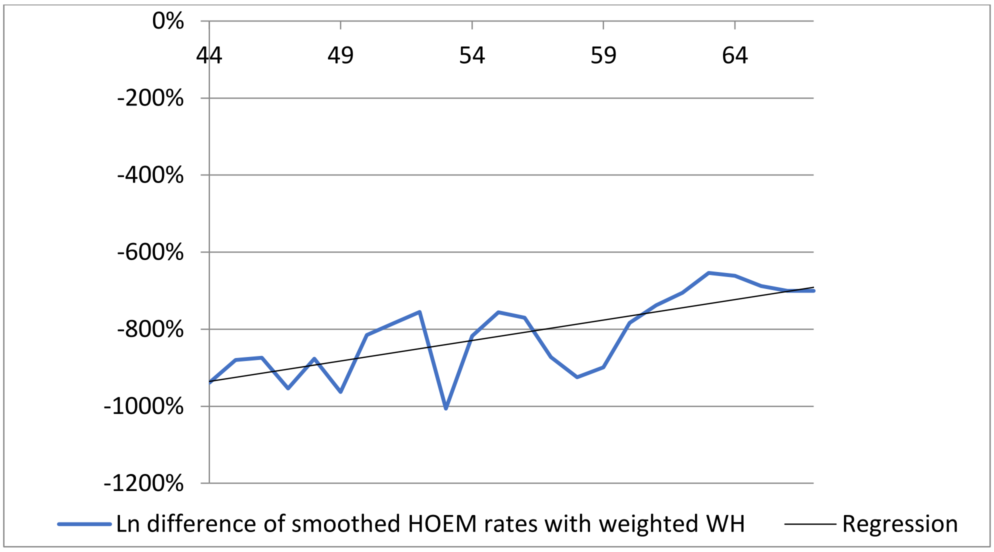
We can conclude with this equation that the values $(x, \ln|\mathrm{q}_{x+1}-\mathrm{q}_x|)$ are on the same straight line with a slope equals to $\ln C$.

In order to select the better set of data for Makeham model, we did several linear regressions on the following data:

- Crude mortality rates calculated with Kaplan-Meier method
- Crude mortality rates calculated with Hoem method
- Mortality rates coming from graphic 1 (application of unweighted Whittaker-Henderson on Kaplan-Meier and Hoem estimators)
- Mortality rates coming from graphic 3 (application of weighted Whittaker-Henderson on Kaplan-Meier and Hoem estimators)

We have observed that the better set of data for the application of Makeham model are the mortality rates coming from the graphic 3 because the function $\ln|q_{x+1}-q_x|$ present a more important linear trend with a better balance (R^2 close to 60%).





The level of R^2 is a little weak but we will nevertheless continue the process and apply Makeham model to check if this method is meaningful for BDK portfolio.

To assess the parameters of the model, we apply the maximum likelihood method.

II.2.2.2. Application of Makeham model

II.2.2.2.1. Estimation of parameters of Makeham model

Let's suppose that we have a set of observations of mortality annual rates between the ages x_0 and x_M. We note: N_x the number of observed people at the beginning of the period and D_x the number of deaths between x and $x+1$, $\forall x \quad x_0 \leq x \leq x_M$

The general formula of the maximum likelihood method is written by:

$$L = \prod_{x=x_0}^{x_M} (q_x^{D_x}) \cdot (p_x)^{(N_x - D_x)}$$

If $\ln(p_x) = -\alpha - b \cdot e^{\gamma x}$

With $b = B \cdot \frac{(c-1)}{(\ln(c))}$ and $\gamma = \ln(c)$

We have the following function:

$$\Phi = \ln(L) = \sum_{x=x_0}^{x_M} (D_x \cdot \ln(q_x) + (N_x - D_x) \cdot \ln(p_x))$$

The best parameters are the ones that maximize the likelihood function Φ.

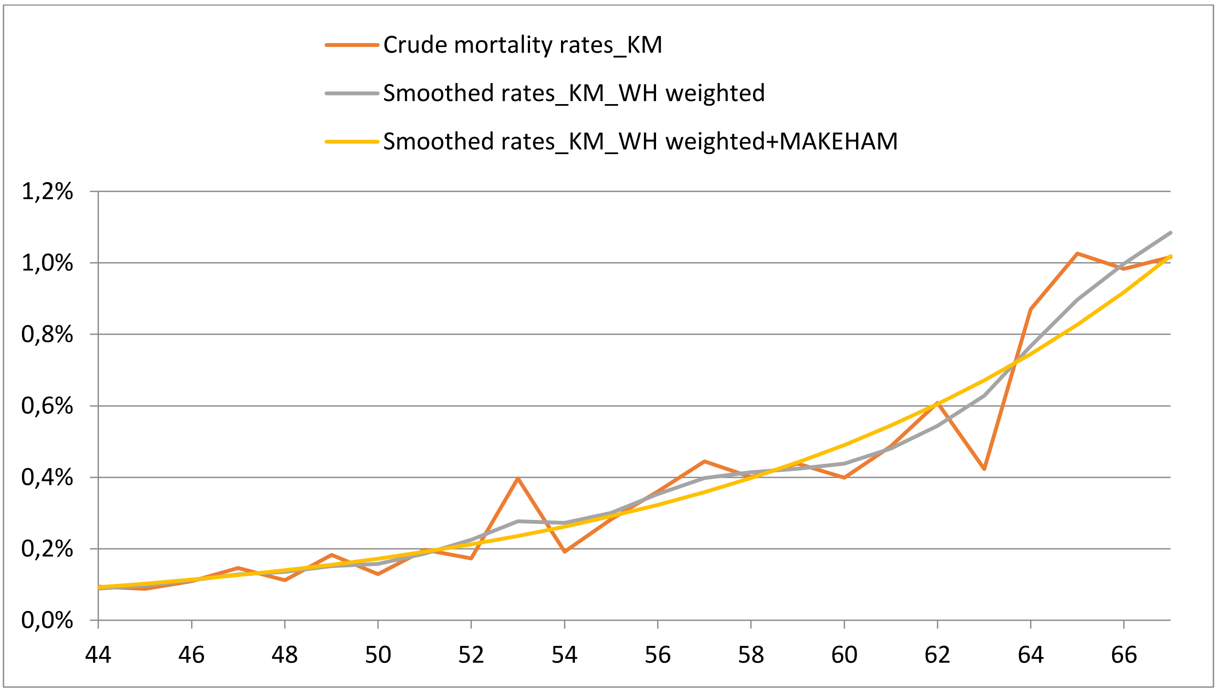
Hence, we must solve the system of equations described below:

$$\begin{cases} d = \dfrac{\partial \Phi}{\partial \alpha} = -\sum N_x + \sum \dfrac{D_x}{1-p_x} & = 0 \\ e = \dfrac{\partial \Phi}{\partial b} = -\sum e^{\gamma x} \cdot N_x + \sum \dfrac{e^{\gamma x}}{1-p_x} \cdot D_x & = 0 \\ f = \dfrac{\partial \Phi}{\partial \gamma} = -b \cdot \sum x \cdot e^{\gamma x} N_x + b \cdot \sum \dfrac{e^{\gamma x}}{1-p_x} \cdot D_x & = 0 \end{cases}$$

There exist several methods to solve this system of equation. The latter will not be detailed in this research method but the curious reader can read the work of KING and HARDY or TAYLOR.

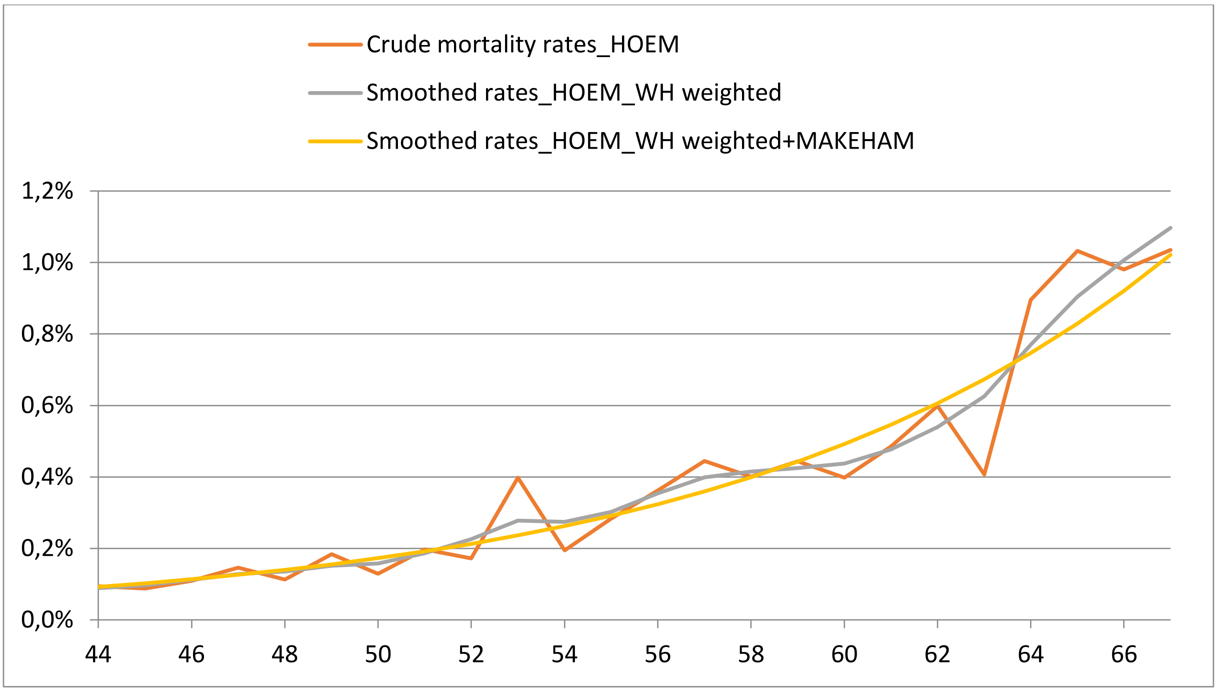
II.2.2.2.2. Presentation of graphics

Graphic 7 – Comparison before and after application of weighted Whittaker-Henderson and after Makeham / Kaplan-Meier rates

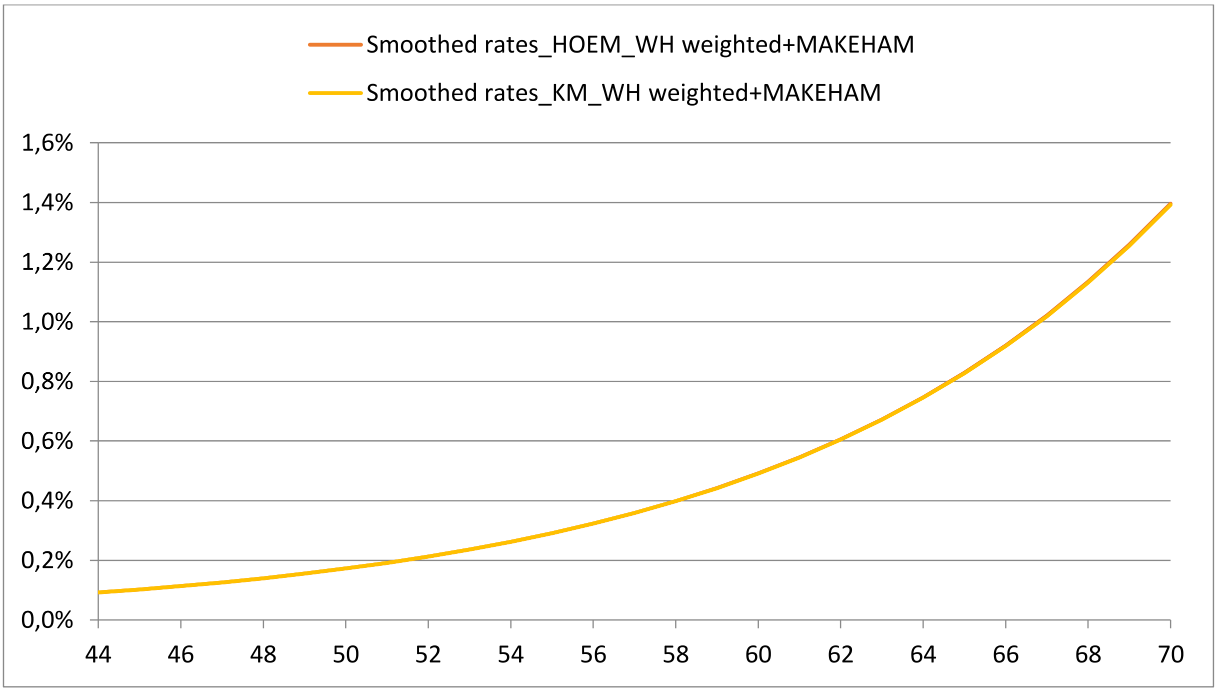


We observe that the curves are closed and meet in several points. Makeham model smoothes the rates provided by WH model.

Graphic 8 – Comparison before and after application of weighted Whittaker-Henderson and after Makeham / Hoem rates



Graphic 9 – Comparison of Makeham smoothing on Kaplan-Meier and Hoem rates



It is difficult to distinguish each curve because Makeham model produce almost the same results for Kaplan-Meier and Hoem estimators: it is logical as the original sample of data is the same for both estimations.

II.2.2.2.3. Choice of the best curve

In order to keep the best graduated data for the comparison with other methods, we applied the chi-squared test as we did with Whittaker-Henderson method.

This variable follows a chi-squared law with 20 degrees of freedom (24 ages –3- 1=20). The different values for ${\chi_0}^2$ calculated are:

	Critical threshold	31,41	
	Graphics	**Chi-2 Value**	**Validated?**
Weighted data	**Graphic 7**	3,40	OK
	Graphic 8	3,72	OK

Both smoothing are validated but we will keep the curve coming from **the graphic 7** (Kaplan-Meier estimators) for the comparison with other methods (Whittaker-Henderson and Brass).

II.3. *BRASS GRADUATION METHOD*

II.3.1. Reminder of Brass graduation method

Brass model (1971) is a relational model: this system provides a greater degree of flexibility than the empirical models. It rests on the assumption that two distinct age-patterns of mortality can be related to each other by a linear transformation of the logit of their respective survivorship probabilities. Hence, the hypothesis in this system is to admit a strong linear relationship between the logit of the observed population and the logit of the reference population.

The big advantage of Brass model is to improve the knowledge of the observed population by using an external mortality table of a population with similar characteristics. It is particularly useful when we do not have many observations and we lack of experience.

For reminder, the definition of the logit is:

$$\text{logit}(x) = \ln\left(\frac{x}{1-x}\right)$$

As said above, we make the hypothesis that it exists a linear relationship between the logit of the observed population and the logit of the reference table:

$$\text{logit}(q_x) = a \cdot \text{logit}\left(q_x^{ref}\right) + b$$

II.3.2. Application to credit life portfolio with BDK

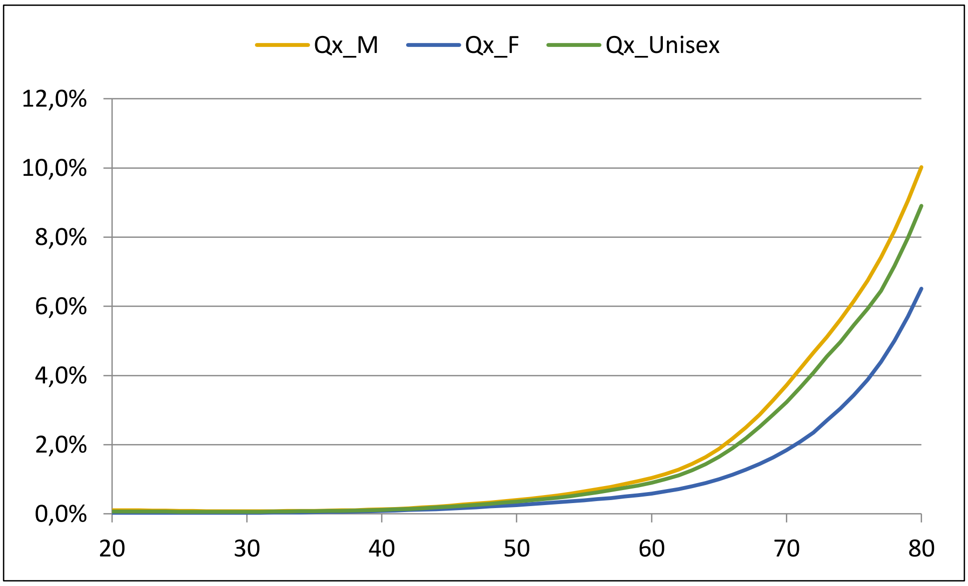
II.3.2.1. Choice of the reference table

For the application of Brass model, we chose the reference tables DAV 2008 T for men and DAV 2008 T for women because they are the most recent tables (2008) describing the risk of death published by the German Actuarial Association. Moreover, the latter are also used by the German actuary in SGI subsidiaries for the modeling of pricing and reserves so it is reasonable to take them as reference.

To build the unisex reference table, we applied the following formula:

$$tab_x^{ref} = DAV - 2008M_x \times \frac{Men_x}{(Men + Women)_x} + DAV - 2008F_x \times \left(1 - \frac{Men_x}{(Men + Women)_x}\right)$$

Comparison of reference tables



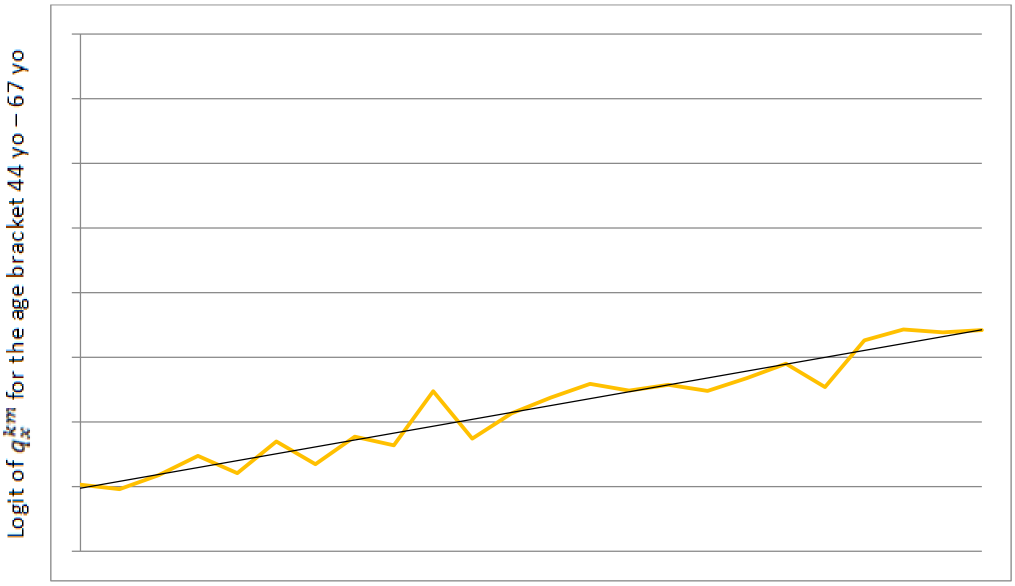
As the observation done about the crude mortality rates, the curve describing the men population is very close to the unisex curve as it is linked to the profile of BDK portfolio (69% of men for the age selection 44 yo – 67 yo / 66% of men for the entire portfolio).

II.3.2.2. Conditions of application of Brass

As said previously, it exists a linear relationship between the logit of the observed population and the logit of the reference table, so we can do a linear regression from the function “logit” of the crude mortality rates to the “logit” of the mortality rates of the reference table to calculate the coefficients a and b of the following equation:

$$\text{logit}(q_x) = a \cdot \text{logit}(q_x^{ref}) + b$$

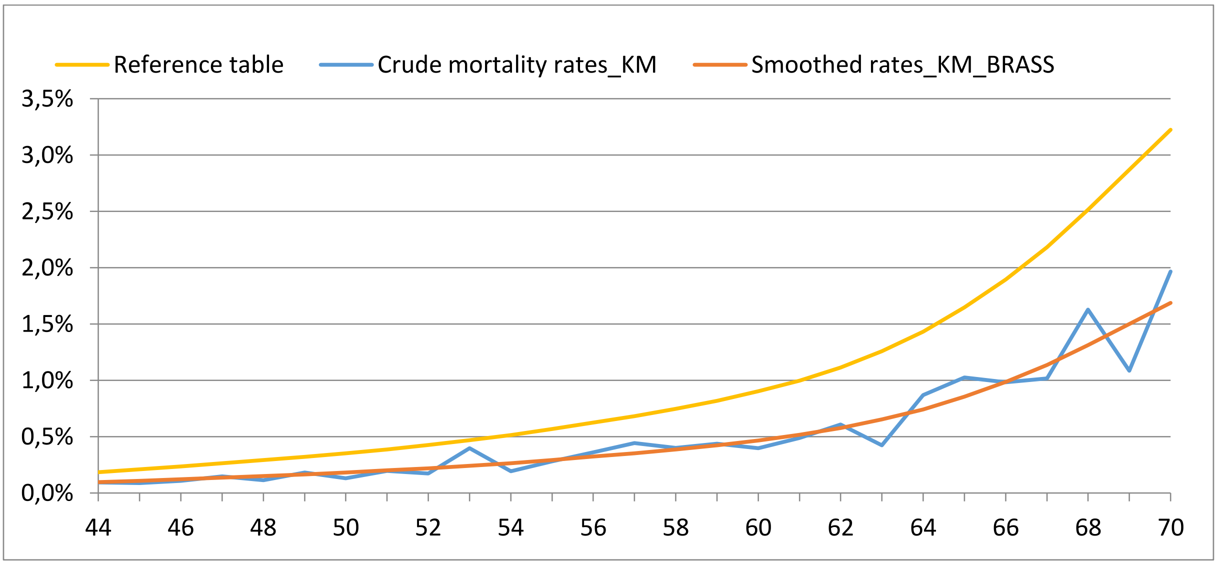
As previously, we did the study on the age selection [44 yo– 67 yo].



Logit of q_x^{ref} for the age bracket 44 yo – 67 yoR^2 coefficient is around 93% so it confirms that there is a strong linear relationship between the logit of both samples of data (crude mortality rates and rates of the reference table).

II.3.2.3. Presentation of graphics

Graphic 10 – Comparison of mortality rates of the reference table and after Brass smoothing / Kaplan-Meier rates



We observe on this graphic that the gap between the reference table and the rates calculated after Brass smoothing is increasing with the age.

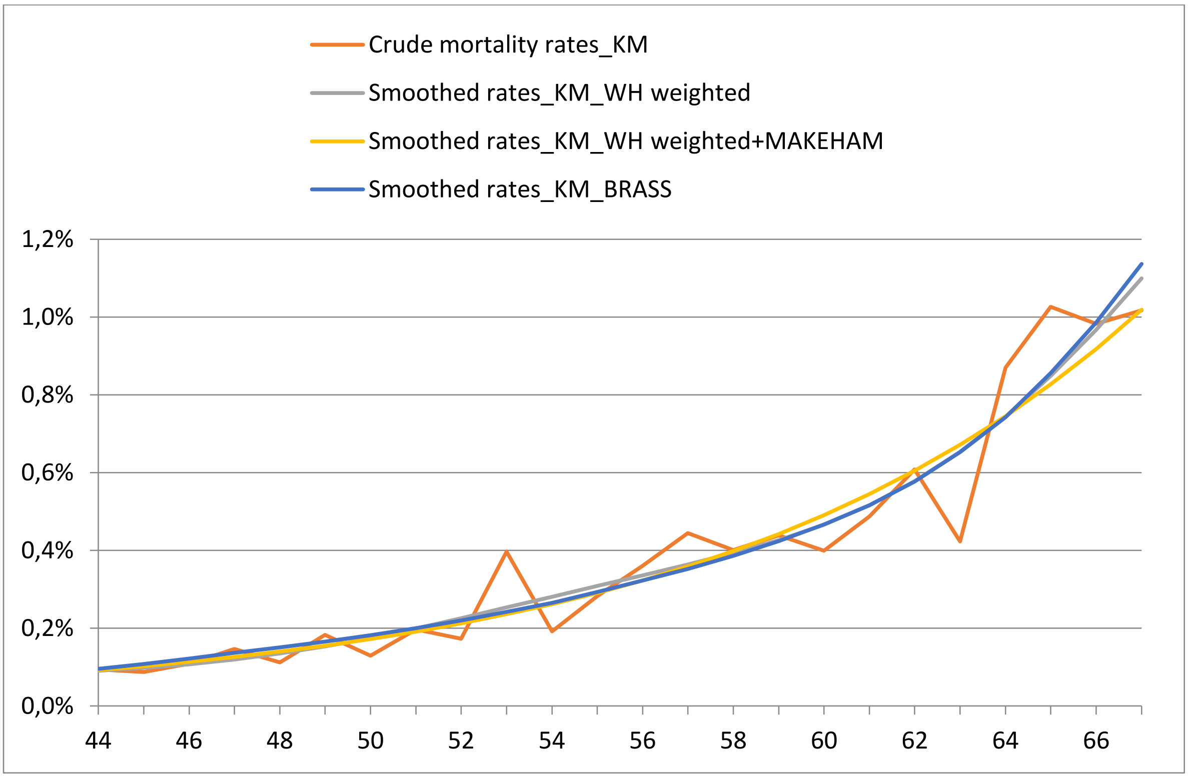
II.4. *COMPARISON OF THE DIFFERENT SMOOTHING*

To summarize, three different methods of smoothing were applied on the credit life portfolio of BDK (Whittaker-Henderson, Makeham and Brass) for the age bracket [44 yo – 67 yo]. It is now time to validate each of them and select the best method for this portfolio.

To do so, we are going to compare each curve and perform different tests for each smoothing.

II.4.1. Comparison of the different curves

Graphic 11 – Comparison of the quality of the different smoothing



We observe that the smoothing are very closed the ones from each other. Nevertheless, we can see on the graphic that the curves move away from the age of 65 years old. We can anticipate a relatively high volatility for the aged people (until the maturity age: 75 years old).

II.4.2. Validation of the different smoothing

II.4.2.1. Fit to the crude mortality rates

This paragraph presents some methods that enable to estimate the fit between the estimated and the observed mortality. That will give us some elements how the different methods are transposing the observed mortality of the portfolio.

II.4.2.1.1. Comparison between the observed deaths / expected deaths:

This table presents the gap between the expected number of deaths with each method (Whittaker-Henderson, Makeham and Brass) and the real number of deaths.

Age brackets	Observed deaths	Whittaker-Henderson		Makeham		Brass	
		Estimated death	Ratio observed deaths / estimated	Estimated death	Ratio observed deaths / estimated	Estimated death	Ratio observed deaths / estimated
Total	**553,00**	**551,76**	**100,22%**	**548,37**	**100,84%**	**554,83**	**99,67%**

All methods are correct and transpose efficiently the observed mortality. We can nevertheless notice that **Brass and Whittaker-Henderson** methods are a little better. Whittaker-Henderson underestimates a little bit the mortality (-1,2 deaths) and Brass overestimates a bit little the mortality (+1,8 deaths). Makeham is less efficient because the mortality with this method is underestimated by 4,6 deaths.

II.4.2.1.2. Chi-squared test

It is also important to apply the chi-squared test to check if there is a significant discrepancy between the observed mortality and the expected mortality. As defined previously, we have applied the following formula:

$$\chi_0^2 = \sum_{x=44}^{67} \frac{(D_x - q_x^* * E_x)^2}{q_x^* * E_x}$$

D_x being the real number of deaths, q_x^* the smoothed rates and E_x the exposure at age x.

This table presents the results of the chi-squared test with each method:

	Chi-squared test	Critical threshold
Whittaker-Henderson	25,24	35,17
Makeham	28,29	31,41
Brass	27,26	32,67

All smoothing are correct and below the critical threshold.

Whittaker-Henderson is the most performing method with this test whereas Makeham is once again the less efficient one.

Nevertheless, it is noticeable that the good results of WH are linked to the peak of mortality observed at 53 years old that is mainly due to a high mortality rate in 2011 and 2012. Indeed, the Whittaker-Henderson curve is following the peak at this age whereas Makeham and Brass model are smoothing more significantly the mortality rate at this age.

If we correct the mortality rates of the years 2011 and 2012 with the average mortality rate observed on the 5 years of observation (0,398% instead of 0,748% for 2011 and 0,455% for 2012), the results of the test would be:

	Chi-squared test	Critical threshold
Whittaker-Henderson	23,07	35,17
Makeham	25,66	31,41
Brass	24,79	32,67

With the correction of the mortality rate at 53 years old, the gap between Whittaker-Henderson method and the other ones is reduced.

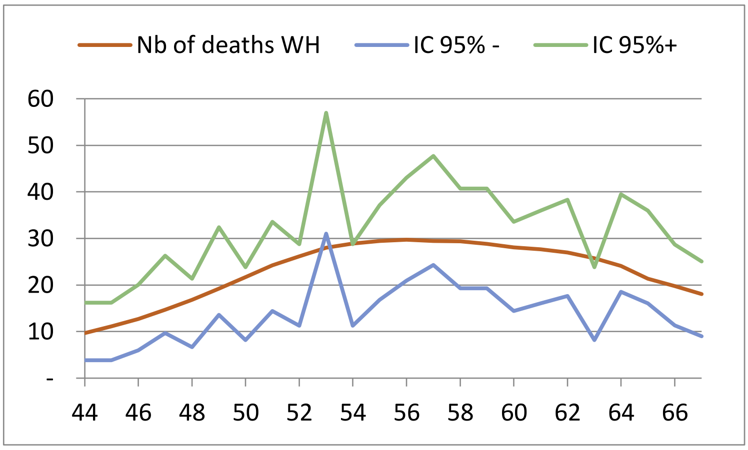
II.4.2.1.3. Confidence intervals

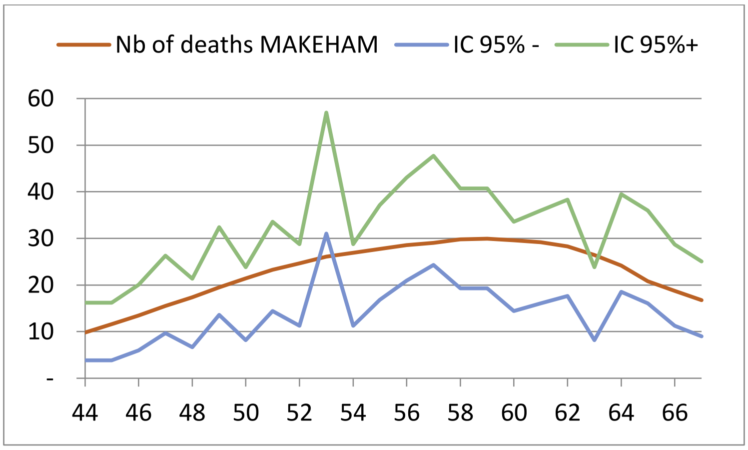
The last check of fit with real mortality is to calculate the number of ages that are out of the confidence intervals.

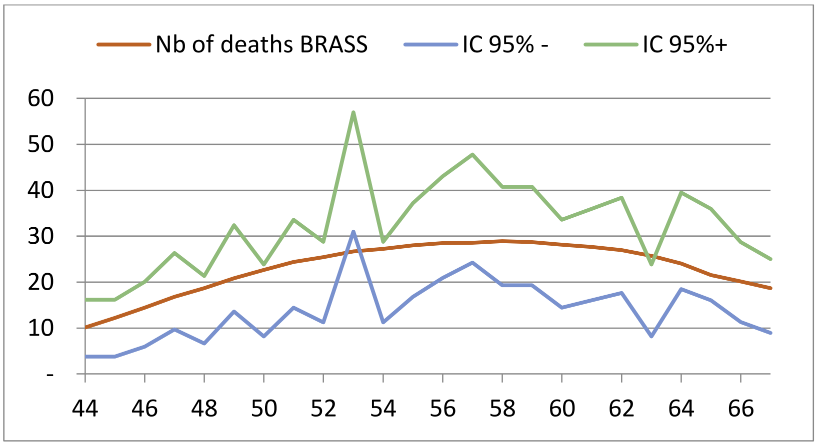
	Nb of ages out of the confidence interval
Whittaker-Henderson	3
Makeham	2
Brass	2

For all smoothing, it is noticeable that only a little number of ages (between 2 and 3 about 24) is out of the confidence intervals: it confirms the strength of the different smoothing. The ages that are out of the confidence intervals are 53 yo and 63 yo for all methods. There is an additional age for Whittaker-Henderson (54 yo).

On the contrary to the other tests, Whittaker-Henderson method is this time the less performing smoothing with 3 ages out of the confidence intervals compared to 2 ages for Brass and Makeham models.







II.4.2.2. Quality of smoothness

This paragraph presents one criteria to estimate the quality of the regularity of the different methods of smoothing.

The smoothed rates are considered as regular when $S = \sum_{x=x\ \mathrm{inf}}^{x\ \mathrm{sup}-z} (\Delta^z q_x)^2$ is close to 0, q_x representing the smoothed mortality rates.

For $z = 1$, the criteria is: $\sum_{x=44}^{x=66} (q_{x+1} - q_x)^2$

This table presents the results of this test:

	Smoothness criteria $z = 1$
Whittaker-Henderson	0,000007
Makeham	0,000005
Brass	0,000008

For all methods of smoothing, the result is close to 0 that confirms the quality of the three methods used (Whittaker-Henderson, Makeham and Brass). Nevertheless, on the contrary to the tests done to challenge the fit, it is Makeham method that has here the best results.

II.4.2.3. Selection of method of smoothing

After the analysis of the different tests done on the fit and on the smoothness, we are now confident that the different methods of smoothing are satisfactory. The results of the tests are different so we cannot easily say that one method is better than the other ones: for the tests on the fit, it was Whittaker-Henderson and Brass methods that had the best results whereas it is Makeham method that is the more efficient for the test on the smoothness.

For pragmatic reasons, we decided to select Brass method of smoothing because:

- it is the only method that does not underestimate the mortality (indeed the estimated number of deaths for Whittaker-Henderson and Makeham is inferior to the observed mortality),
- for the projections of the age brackets [18 yo – 43 yo] and [68 yo – 75 yo] where we lack data and experience, Brass model presents strong advantages because it helps improving

the knowledge of the observed population by using an external mortality table of a population with similar characteristics,

- the conditions of application of Makeham model (already borderline) would not be met if we had extended the age bracket to other ages.

III. EXTRAPOLATION OF MORTALITY RATES FOR YOUNG AND OLD AGES

III.1. *CONSTRUCTION OF THE UNISEX EXPERIENCE TABLE WITH BRASS METHOD*

As said previously, we decided to extend the mortality curve to other age brackets [18 yo – 43 yo] and [68 yo – 75 yo] with Brass model. As we defined the coefficients a and b of the following equation for the age selection [44 yo – 67 yo]:

$$\text{logit}(q_x) = a \cdot \text{logit}(q_x^{ref}) + b$$

We just need to recalculate the new q_x for the other age brackets.

$$\text{logit}(x) = \ln\left(\frac{x}{1-x}\right)$$

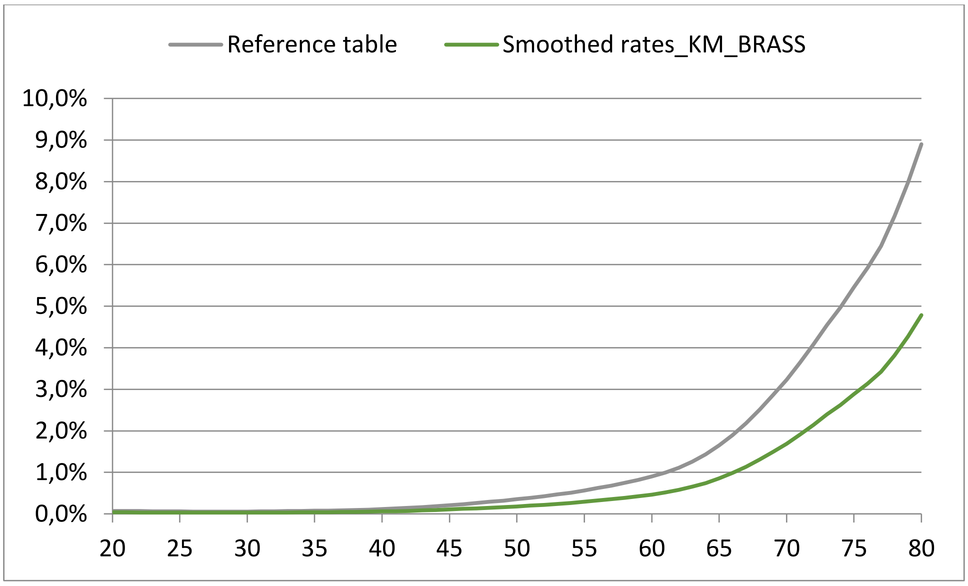
$$\text{logit}(q_x) = a \cdot \text{logit}(q_x^{ref}) + b$$

$$\Leftrightarrow \frac{q_x}{1-q_x} = exp(b) \cdot \left(\frac{q_x^{ref}}{1-q_x^{ref}}\right)^a$$

$$\Leftrightarrow q_x \cdot \left(1 + exp(b) \cdot \left(\frac{q_x^{ref}}{1-q_x^{ref}}\right)^a\right) = exp(b) \cdot \left(\frac{q_x^{ref}}{1-q_x^{ref}}\right)^a$$

$$\Leftrightarrow q_x = \frac{exp(b) \cdot \left(\frac{q_x^{ref}}{1-q_x^{ref}}\right)^a}{1 + exp(b) \cdot \left(\frac{q_x^{ref}}{1-q_x^{ref}}\right)^a}$$

Graphic 12 – Presentation of the final mortality curve



We observe on this graphic that the curves are moving further away from 45 yo.

In this table, we compare the expected number of deaths with Brass model with the real number of deaths on the age bracket [18 yo – 75 yo].

Age brackets	Observed deaths	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	66	78,35	-12,35	84,23%
[44 - 67]	553	554,83	-1,83	99,67%
[68 - 75]	130	159,50	-29,50	81,50%
Total	**749**	**792,69**	**-43,69**	**94,49%**

We observe a gap of 12 claims for the age selection [18 yo – 43 yo] and 30 claims for the age selection [68 yo – 75 yo].

III.2. *APPLICATION OF REBATES ON THE EXPERIENCE TABLE BUILT WITH BRASS METHOD*

In order to see if we have some room of maneuver and if we can reduce the gap for these age selection, we made a comparison for each observation year (from 2012 to 2015).

Age bracket – [18 yo – 43 yo]

Observation year	Observed deaths	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
2012	13	9,98	3,02	130,31%
2013	13	15,89	-2,89	81,83%
2014	18	21,46	-3,46	83,87%
2015	20	27,14	-7,14	73,70%
Total	**66**	**78,35**	**-12,35**	**84,23%**

From 2013 to 2015, the observed mortality is lower than the estimated one with Brass model. As this rule is not general (i.e. we observe the opposite trend for 2012), we decided to keep the estimated mortality rates with Brass model without applying any rebate rate.

Age bracket – [68 yo – 75 yo]

Observation year	Observed deaths	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
2012	13	14,76	-1,76	88,07%
2013	28	30,07	-2,07	93,13%
2014	33	46,00	-13,00	71,75%
2015	55	64,21	-9,21	85,65%
Total	**130**	**159,50**	**-29,50**	**81,50%**

For each observation year, the observed mortality is inferior to the estimated one with the experience table. Therefore, we decided to apply a rebate rate of **7%** (lower rebate rate observed in 2013) on the rates estimated with Brass method. For the intermediary age (68 yo) between the age bracket [44 yo – 67 yo] and [68 yo – 75 yo], we considered that the mortality rate at 68 yo was equal to the average of the mortality rates at the ages 66, 67, 69 and 70.

The new comparison table is the following:

		Brass		
Age brackets	**Observed deaths**	**Estimated death**	**Gap nb of death**	**Ratio observed deaths / estimated**
[18 - 43]	66	78,35	-12,35	84,23%
[44 - 67]	553	554,83	-1,83	99,67%
[68 - 75]	130	149,07	-19,07	87,21%
Total	**749**	**782,25**	**-33,25**	**95,75%**

The mortality remain slightly overestimated **(4,2%)** with the model compared to the observed mortality but thanks to the application of a rebate rate of 7% on the age bracket [68 yo – 75 yo], the difference between the observed deaths and the estimated ones with our model has decreased from 44 deaths to 33 deaths. As we must take into account the fact that the operational risk (processing of big database) exists when such study is done, this experience table can be considered as relatively close *to a best estimate of mortality*.

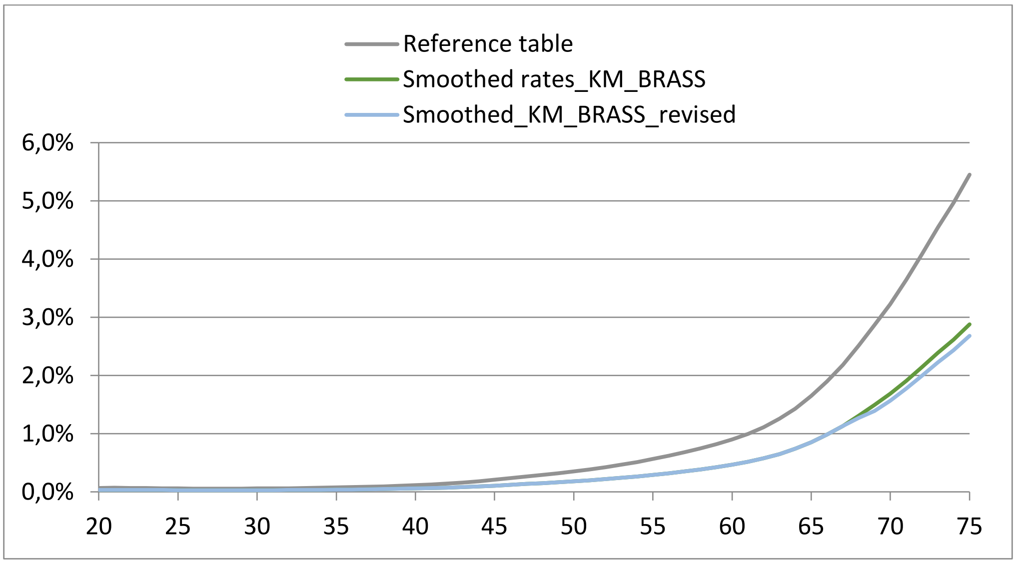
To have an idea of what would be the estimated mortality for one year of observation, we present here the results for 2015:

		Brass		
Age brackets	**Observed deaths**	**Estimated death**	**Gap nb of death**	**Ratio observed deaths / estimated**
[18 - 43]	20	27,14	-7,14	73,70%
[44 - 67]	219	217,82	1,18	100,54%
[68 - 75]	55	60,04	-5,04	91,60%
Total	**294**	**305,00**	**-11,00**	**96,39%**

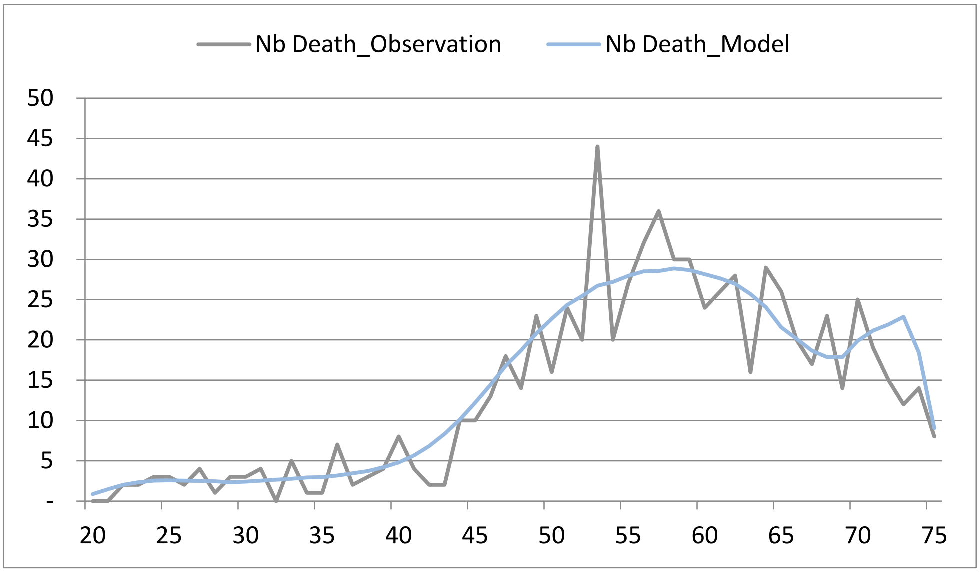
The gap of claims is equal to +11 claims (3,6%) that is satisfactory.

We will now present the *best estimate* mortality curve.

Graphic 12 – Presentation of the unisex *best estimate* mortality curve



Graphic 13 – Comparison of the number of deaths observed and the estimated number of deaths with the experience table



III.3. *Construction of the experience tables by gender with Brass method*

Now that the best method is chosen for the unisex mortality curve, we are going to apply the same method to build the experience tables by gender.

As the women population is not enough important to apply the same process, the table of mortality of women will be deducted from the unisex and the men mortality tables.

$$q_x^{women} = \frac{(q_x^{unisex} - \frac{Men_x}{(Men+Women)_x} * q_x^{men})}{1 - \frac{Men_x}{(Men + Women)_x}}$$

III.3.1. Experience table of the men population

For the men population, we apply exactly the same process. In this table, we compare the expected number of deaths of the men population with the real number of deaths of the men population for the age bracket [18 yo – 75 yo].

		Brass		
Age brackets	**Observed death**	**Estimated death**	**Gap nb of death**	**Ratio observed deaths / estimated**
[18 - 43]	47	63,40	-16,40	74,13%
[44 - 67]	455	456,16	-1,16	99,74%
[68 - 75]	115	140,57	-25,57	81,81%
Total	**617**	**660,13**	**-43,13**	**93,47%**

The gap represents +43 deaths and is almost equal to the gap observed for the entire population (44 deaths for the men and women included). So it means here that the estimated number of deaths for the women population is close to the real mortality.

If we look into details, we see that, once again, the issue concerns the young and the old ages, the estimation of the mortality of the heart of the population being close to the real mortality (a gap of one death is observed for this age bracket). As per the unisex mortality, we have detailed the mortality of the men population by observation year and by age bracket [18 yo – 43 yo] and [68 yo – 75 yo].

Age bracket – [18 yo – 43 yo]

Observation year	Observed death	Brass Estimated death	Brass Gap nb of death	Brass Ratio observed deaths / estimated
2012	8	8,39	-0,39	95,36%
2013	7	13,11	-6,11	53,38%
2014	14	17,26	-3,26	81,12%
2015	17	21,34	-4,34	79,64%
Total	**47**	**63,40**	**-16,40**	**74,13%**

The estimated mortality is superior to the mortality observed (+26%) but we observe a high volatility that depends on the observation year. Therefore, we decided to apply a rebate rate of **5%** that corresponds to the lower rebate rate observed for the observation year 2012. For the intermediary age (43 yo) between the age bracket [18 yo – 43 yo] and [44 yo – 67 yo], we considered that the mortality rate at 43 yo was equal to the average of the mortality rates at the ages 41, 42, 44 and 45.

Age bracket – [68 yo – 75 yo]

Observation year	Observed death	Brass Estimated death	Brass Gap nb of death	Brass Ratio observed deaths / estimated
2012	12	13,07	-1,07	91,83%
2013	23	26,50	-3,50	86,80%
2014	30	40,53	-10,53	74,02%
2015	49	56,64	-7,64	86,52%
Total	**115**	**140,57**	**-25,57**	**81,81%**

As for the unisex mortality, the expected mortality is superior to the mortality observed. We decided to apply a rebate rate of **8%** (lower rebate rate observed in 2012)**.** For the intermediary age (68 yo) between the age bracket [44 yo – 67 yo] and [68 yo – 75 yo], we considered that the mortality rate at 68 yo was equal to the average of the mortality rates at the ages 66, 67, 69 and 70.

The new comparison table is the following:

Age brackets	Observed death	Brass Estimated death	Brass Gap nb of death	Brass Ratio observed deaths / estimated
[18 - 43]	47	60,58	-13,58	77,58%
[44 - 67]	455	456,16	-1,16	99,74%
[68 - 75]	115	130,11	-15,11	88,39%
Total	**617**	**646,85**	**-29,85**	**95,38%**

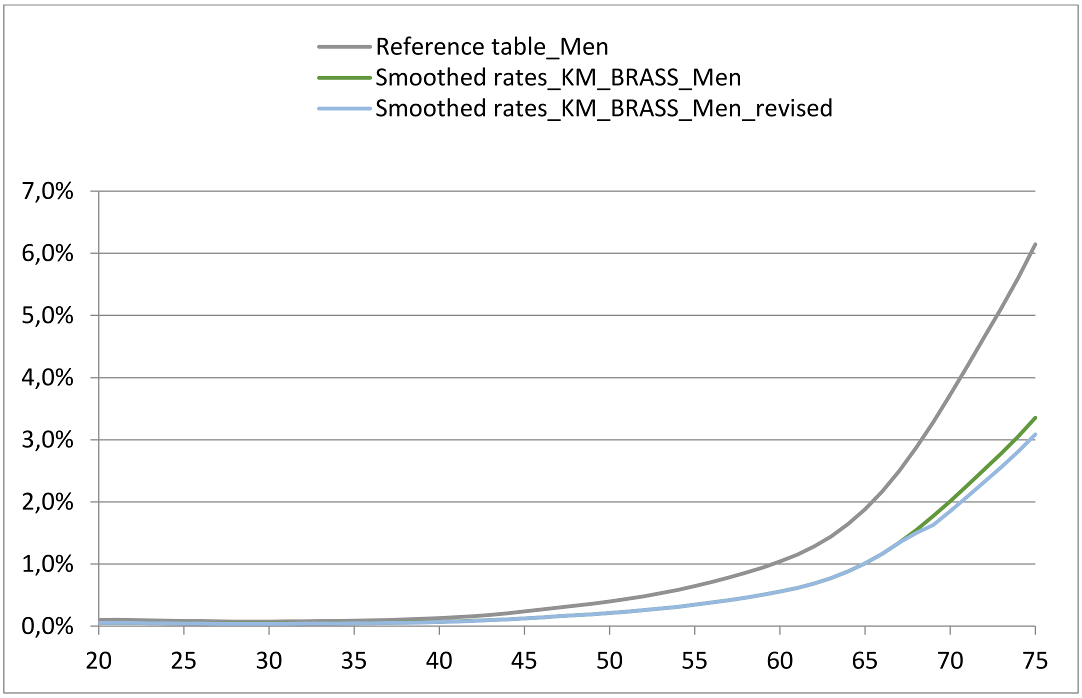
The mortality remain slightly overestimated **(+4,6%)** with the model compared to the observed mortality but thanks to the application of the rebate rates, the difference between the observed deaths and the estimated ones with our model has decreased from 43 deaths to 30 deaths. This experience table can be considered as relatively close *to a best estimate of mortality*.

To challenge this experience table, let's look at the generation 2015:

		Brass		
Age brackets	**Observed death**	**Estimated death**	**Gap nb of death**	**Ratio observed deaths / estimated**
[18 - 43]	17	20,40	-3,40	83,35%
[44 - 67]	182	176,71	5,29	102,99%
[68 - 75]	49	52,45	-3,45	93,42%
Total	**248**	**249,56**	**-1,56**	**99,38%**

The gap between the expected mortality and the observed one is less than 2 deaths (+0,6%) that is very satisfactory.

Graphic 14 – Presentation of the men *best estimate* mortality curve



III.3.2. EXPERIENCE TABLE OF THE WOMEN POPULATION

In this table, we compare the number of deaths expected with the number of deaths observed of the women population before the application of the rebate rates on the men population (respectively 5% and 8% for the age brackets [18 yo – 43 yo] and [68 yo – 75 yo]):

Age brackets	Observed deaths	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	19	14,96	4,04	127,04%
[44 - 67]	98	98,67	-0,67	99,32%
[68 - 75]	15	18,93	-3,93	79,23%
Total	**132**	**132,56**	**-0,56**	**99,58%**

As we said previously, the expected mortality is in line with the mortality observed (+0,6 deaths). Nevertheless, if the mortality is well assessed for the heart of the population (+0,7 deaths), we observe that the mortality is overestimated (+21%) for the age bracket [68 yo – 75 yo] and underestimated (-27%) for the age bracket [18 yo – 43 yo]. There is compensation between the underestimation of the young segment and the overestimation of the old segment.

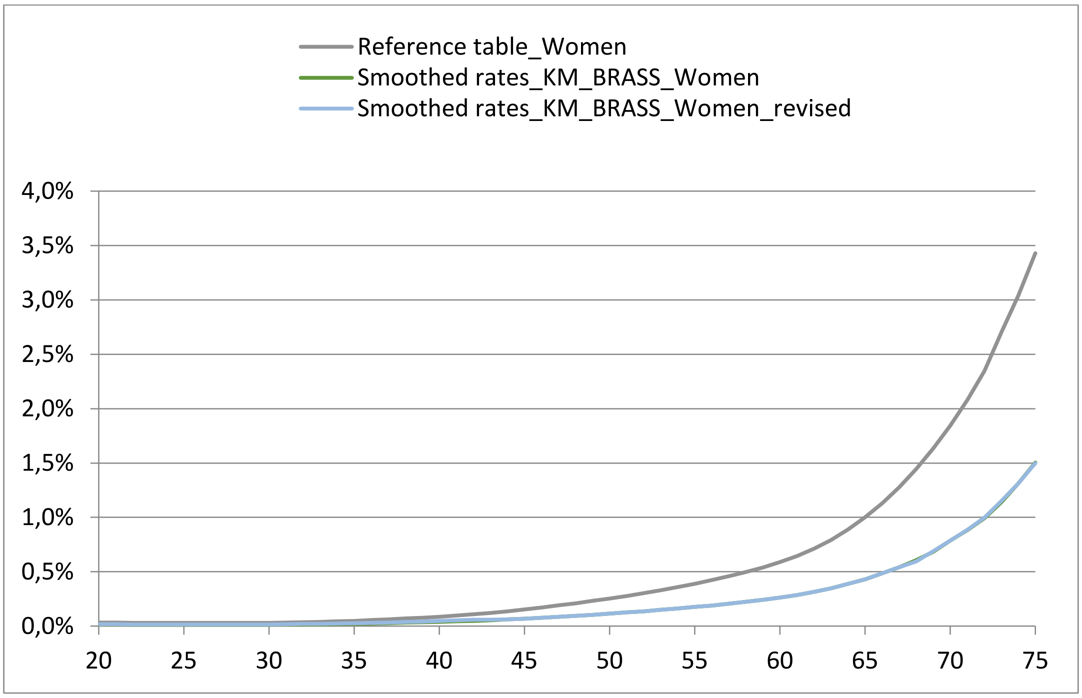
With the application of the rebate rates on the men population (building of the men *best estimate* mortality table), the women *best estimate* experience table gives the following results:

Age brackets	Observed deaths	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	19	17,77	1,23	106,92%
[44 - 67]	98	98,67	-0,67	99,32%
[68 - 75]	15	18,96	-3,96	79,12%
Total	132	135,40	-3,40	97,49%

With the adjustment realized on the mortality of the men population modeled, the estimated mortality of the women population for the young population is much closer to the reality (gap of **6,9%** instead of **27%** / one death instead of 4 deaths). For the old population, no change is observable because the entire adjustment done on the unisex population (-10 deaths expected) is absorbed by the adjustment we did on the men population (-10 deaths expected). For the intermediary age (43 yo) between the age bracket [18 yo – 43 yo] and [44 yo – 67 yo], we considered that the mortality rate at 43 yo was equal to the average of the mortality rates at the ages 41, 42, 44 and 45.

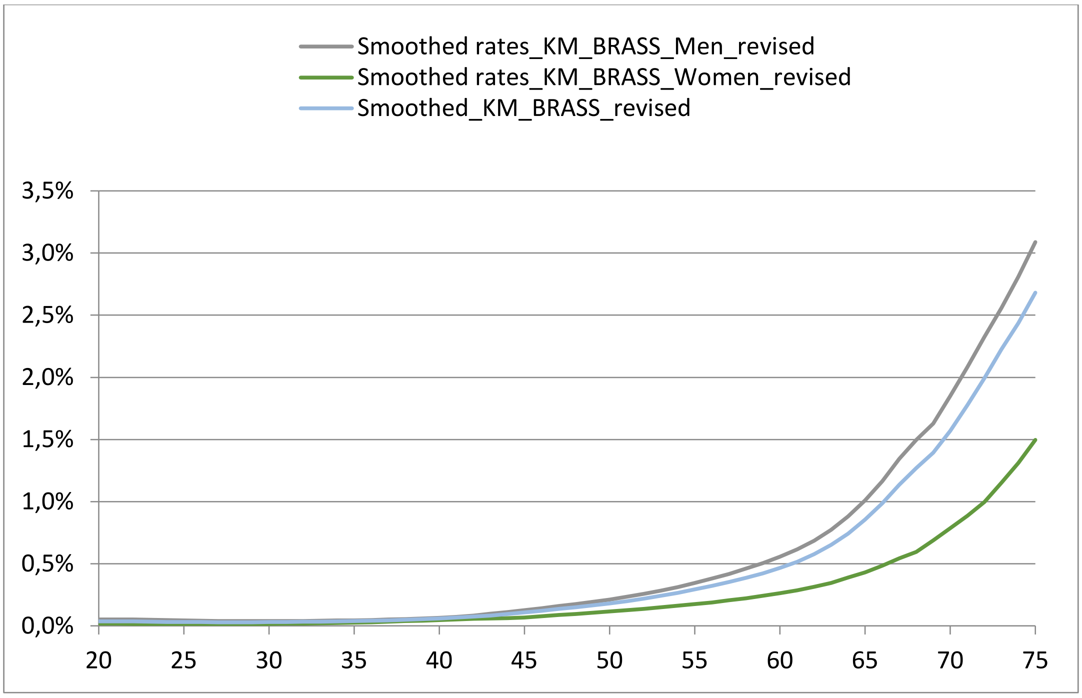
In total, the estimated mortality is a little bit overestimated (+3 deaths) compared to the estimated mortality (+0,5 deaths) without application of the rebates rates on the men population. However, we see that the estimation for the young people is more reliable once a rebate rate is applied on the men population.

Graphic 15 – Presentation of the women *best estimate* mortality curve



The difference between the original and the revised curves is not readable on the graphic because the difference between both curves is only present for the young ages where the q_x are small.

Graphic 16 – Comparison of *best estimate* mortality curve



Chapter 3 – Impacts on the pricing and reserving when we use the experience tables instead of the regulatory tables

The objective of this research paper was to build *best estimate* experience tables as well as experience tables to be used for the pricing. Now that we built the *best estimate* experience tables, it is important to define the safety margin we want to apply for the building of the experience tables used for the pricing activities. Then, we need to evaluate the impacts on the pricing and the reserving after using the experience tables.

I. PRICING ACTIVITIES

In this section, we are going to build a unisex experience table for the pricing starting from the *best estimate* unisex experience table and we will then compare the tariff coming out from the experience table with the current tariff.

I.1. *EXPERIENCE TABLE – TARIFF PURPOSE*

On March, 1st 2011, the European Court of Justice decided to forbid the discrimination of tariff upon the gender. This decision had to be applied from December, 21st 2012. As a consequence, it is important to build a unisex experience table for the pricing activities.

To assess the margin we must add to the *best estimate* experience table, we will compare the number of deaths observed with the number of death estimated by the *best estimate* experience table for each observation year.

Observation year: 2012

		Brass		
Age brackets	**Observed death**	**Estimated death**	**Gap nb of death**	**Ratio observed deaths / estimated**
[18 - 43]	13	9,98	3,02	130,31%
[44 - 67]	61	55,44	5,56	110,03%
[68 - 75]	13	13,80	-0,80	94,17%
Total	87	79,22	7,78	**109,82%**

Observation year: 2013

Age brackets	Observed death	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	13	15,89	-2,89	81,83%
[44 - 67]	104	103,37	0,63	100,61%
[68 - 75]	28	28,09	-0,09	99,67%
Total	145	147,35	-2,35	**98,41%**

Observation year: 2014

Age brackets	Observed death	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	18	21,46	-3,46	83,87%
[44 - 67]	142	158,93	-16,93	89,35%
[68 - 75]	33	42,95	-9,95	76,83%
Total	193	223,34	-30,34	**86,42%**

Observation year: 2015

Age brackets	Observed death	Brass		
		Estimated death	Gap nb of death	Ratio observed deaths / estimated
[18 - 43]	20	27,14	-7,14	73,70%
[44 - 67]	219	217,82	1,18	100,54%
[68 - 75]	55	60,04	-5,04	91,60%
Total	294	305,00	-11,00	**96,39%**

The number of death observed divided by the number of death estimated by the model is around 1. However, it depends on the observation year. That is why it is important to define a safety margin. To fine-tune the latter, we considered the biggest gap between the estimated and the observed mortality (it corresponds to the observation year 2014). So the margin of uncertainty is equal to **14%**.

The experience table used for pricing will integrate a margin of **14%** on the *best estimate* rates to be systematically prudent.

I.2. CALCULATION OF TARIFF IN CASE OF DEATH

We applied the classical actuarial model for the pricing of the risk of death.

The single risk premium corresponds to the commitment of the insurer, i.e. the outstanding amount of loan at the date of death.

$$SPR_{DC} = \sum_{i=0}^{n} C_i \times {}_{\frac{i}{12}}p_x \times q_{x+\frac{i}{12}} \times \frac{1}{(1+j)^{\frac{i}{12}}}$$

Notation	Definition
q_x	Probability of dying between age x and $x+1$
p_x	Probability for aged x person to be still alive at age $x+1$: $p_x = 1 - q_x$
${}_{\frac{k}{12}}p_x$	Probability for aged x person to be still alive at age $x + \frac{k}{12}$
$q_{x+\frac{k}{12}}$	Probability of dying between age $x + \frac{k}{12}$ and $x + \frac{k+1}{12}$
C_0	Initial loan amount
C_i	Outstanding amount after i credit instalments: $C_i = C_0 \times \dfrac{1 - \frac{1}{(1+r)^{\frac{n-k}{12}}}}{1 - \frac{1}{(1+r)^{\frac{n}{12}}}}$
j	Technical interest rate (2%)
r	Interest rate of the loan
n	Duration of the loan in month
x	Age at inception
SPR_{DC}	Single risk premium for the death guarantee

The commercial premium is the premium paid by the insured:

$$CPT = \frac{RP \times (1+tm)}{(1-c-f)} \times (1+t)$$

Notation	Definition
CPT	Commercial premium including taxes = premium paid by the insured
RP	Risk premium
tm	Technical margin rate
f	Management fees rate
c	Commission rate paid to the bank
t	Taxes rate

To test the experience table, we launched the pricing model with the experience table instead of the regulatory table without changing any other parameter.

As example, for loan duration of 60 months, we could decrease the commercial tariff by 25%.

%KI	60 months	
Current tariff - Death	2,437%	
Tariff revised with experience table	1,815%	-25,51%

It is therefore interesting for SGI Germany to use the new experience table for pricing purpose:

- The experience table do not overestimate or underestimate the risk in case of death. In consequence, the pricing is very close to the real risk.
- The tariff calculated with the experience table is lower than the one calculated with the regulatory table so SGI Germany has room for manoeuvre to be more aggressive and competitive.

II. RESERVING ACTIVITIES

In this section, we will evaluate the impacts of calculating the mathematical reserves with the *best estimate* experience tables. The calculated amount does not integrate any risk margin. As of today, SGI Germany is not applying the standard formula for calculating the mathematical reserves (MR):

$$MR_t = \sum_{i=t}^{n} C_i \times {}_{\frac{i}{12}}p_x \times q_{x+\frac{i}{12}} \times \frac{1}{(1+j)^{\frac{i}{12}}}$$

But a proxy that we call unearned premium reserve – rule 78:

$$UPR_{Death} = Premium \times \frac{Nb\_days\_left \times (Nb\_days\_left + 1)}{Nb\_days \times (Nb\_days + 1)}$$

Notation	Definition
$Premium$	Premium amount
Nb_days_left	Number of days left to cover by the premium at the calculation date (accounting cutoff date)
Nb_days	Number of days covered by the insurer

To compare the difference between the current calculation (UPR-78), the MR calculation with the regulatory table (DAV 2008 – T) and without application of any rebate rate, the MR calculation with the regulatory table (DAV 2008 – T) and with the application of a average rebate rate of 40% (rebate rate defined by the German actuary) and the MR calculation with the *best estimate* table, we have performed the calculation at 31/12/2016.

The calculation was limited to the loans without balloon because the amortization plan of these loans is not known:

Loans without balloon

Methods of reserving	31/12/2016
UPR - 78	**3 298 399,00**
MR	
DAV 2008 -T	8 163 495,01
DAV 2008 - T & rebate rate of 40%	4 931 896,85
Best estimate table	**4 216 787,14**

To meet Solvency 2 norms, for the loans without balloon, the amount that should be booked end of 2016 with the *best estimate* table is **4 216 787,14 €.** The amount missing in the reserves is equal to **918 k€.** Nevertheless, to have a complete overview of the amount that should be booked in the premium reserve, we should do the same comparison for the loans with balloon (loans where the amortization is not linear). As we did not receive the amortization plan of these loans for our study, we did not integrate them.

So, with this calculation, we see that the building of the *best estimate* experience table may have a significant impact on the levels of reserves of SGI Germany. With the new standards defined by Solvency 2, it is therefore important to create such tables for the calculation of reserves.

Conclusion

The objective of this study was to build experience tables for a credit life portfolio insured by SGI Germany. The objective was double because we had to build two experience tables: one for the calculation of the *best estimate* and the other one for pricing purpose. An important workload was realized ahead to clean-up the data and to strengthen the quality of data. Indeed, it is crucial to be comfortable with the raw data used otherwise all the models developed later on are worthless.

- We first studied BDK portfolio, cleaned-up the input files (exposure and claims files) and constituted the database. We selected the relevant observation period (observation years between 2011 and 2015) and the appropriate segmentation (age selection between 44 yo and 67 yo) to have consistent figures for the calculation of the crude mortality rates.
- Then, for the calculation of the gross mortality rates, we applied two methods (Hoem and Kaplan-Meier): the results being very close to each other, we decided to keep the Kaplan-Meier estimators for the smoothing because the latter was meeting better conditions for the application of the graduation methods.
- We applied three graduation methods on Kaplan-Meier estimators (Whittaker-Henderson, Makeham and Brass) to smooth the gross mortality rates where some irregularities were present. For a double check, we also applied these methods on the Hoem estimators to be sure that the results coming out from one or the other method were close to each other. Different tests on the levels of fit and smoothing were realized and it showed that all the smoothing methods gave good results on the age bracket [44 yo – 67 yo]. Nevertheless, we decided to keep the rates smoothed by Brass method because of the big advantage of this model, i.e. improving the knowledge of the observed population by using an external mortality table of a population with similar characteristics. It was particularly useful for the extrapolation of the young ages [18 yo – 43 yo] and the old ages [+68 yo] where we lack data and experience.
- To finalize the *best estimate* table, we compared the real and the expected mortality on the young and the old segments for each observation year to check if some rebates could be applied on the rates calculated by the Brass model.
- Once the *best estimate* table was finalized, we defined the safety margin (+14%) to add to the *best estimate* table to build a prudential table that might be used for future pricing.
- Finally, the experience tables were tested for the pricing and the reserving (best estimate calculation without any risk margin), and interesting conclusions were raised:
  - ✓ The current offer of BDK could be decreased by 25% if the experience tables were applied instead of the regulatory table (DAV 2008 – T).
  - ✓ The building of the *best estimate* experience table may have an impact on the level of reserves of SGI Germany. Indeed, when we do the calculation for the loans without balloon (loans where the amortization is linear), we have observed that the premium reserving is underestimated by 918 k€ when we use the mathematical reserves formula

with the experience table instead of the current proxy method – UPR 78. To know whether the total amount booked in the premiums reserves end of 2016 is correct, we should do the same comparison for the loans with balloon (loans where the amortization is not linear). This calculation is not presented in this document because the amortization plans of the loans without balloon were not available.

In the future, it is important to make a yearly risk monitoring to check that the potential modifications on the product does not affect the mortality, there is no deviation of the mortality, the profile of the insured population is stable, the mix between men and women is stable for the building of the unisex table, etc.

It will also be interesting to get additional data with the new observation years, in particular for the tail distribution (young and old ages) because the volatility between the observation years on these segments is currently high and additional data would enable to strengthen the projection for these ages.

Then, these experience tables must be tested on other credit life portfolio insured by SGI Germany where we have fewer contracts to see whether these tables could be considered as a standard on all credit life portfolio distributed by financing institutions in Germany.

Finally the programs and the methodology developed for BDK product will be used for other countries such as Italy, Poland, Romania, Serbia, Morocco, etc.

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  - ✓ http://donnees.banquemondiale.org/pays/allemagne

Appendix

APPENDIX 1 – REGULATORY TABLES DAV 2008 T

Age	Men		Women	
	Lx	Qx	Lx	Qx
0	100 000	0,00611	100 000	0,00509
1	99 389	0,00042	99 491	0,00038
2	99 347	0,00034	99 453	0,00032
3	99 313	0,00028	99 421	0,00025
4	99 285	0,00022	99 396	0,00020
5	99 263	0,00018	99 376	0,00017
6	99 245	0,00015	99 359	0,00013
7	99 230	0,00014	99 346	0,00011
8	99 216	0,00013	99 335	0,00011
9	99 203	0,00012	99 324	0,00010
10	99 191	0,00013	99 314	0,00010
11	99 178	0,00014	99 304	0,00011
12	99 164	0,00017	99 293	0,00012
13	99 147	0,00022	99 281	0,00016
14	99 125	0,00030	99 265	0,00018
15	99 095	0,00042	99 247	0,00023
16	99 053	0,00056	99 224	0,00027
17	98 998	0,00071	99 197	0,00030
18	98 928	0,00085	99 167	0,00033
19	98 844	0,00095	99 134	0,00032
20	98 750	0,00101	99 102	0,00033
21	98 650	0,00102	99 069	0,00032
22	98 549	0,00100	99 037	0,00031
23	98 450	0,00096	99 006	0,00030
24	98 355	0,00090	98 976	0,00029
25	98 266	0,00085	98 947	0,00029
26	98 182	0,00081	98 918	0,00029
27	98 102	0,00076	98 889	0,00029
28	98 027	0,00075	98 860	0,00029
29	97 953	0,00075	98 831	0,00030
30	97 880	0,00076	98 801	0,00031
31	97 806	0,00077	98 770	0,00032
32	97 731	0,00079	98 738	0,00035
33	97 654	0,00082	98 703	0,00038
34	97 574	0,00086	98 665	0,00044

Age	Men		Women	
	Lx	Qx	Lx	Qx
35	97 490	0,00089	98 622	0,00049
36	97 403	0,00094	98 574	0,00056
37	97 311	0,00101	98 519	0,00062
38	97 213	0,00108	98 458	0,00070
39	97 108	0,00118	98 389	0,00078
40	96 993	0,00130	98 312	0,00087
41	96 867	0,00145	98 226	0,00097
42	96 727	0,00162	98 131	0,00109
43	96 570	0,00183	98 024	0,00121
44	96 393	0,00209	97 905	0,00136
45	96 192	0,00236	97 772	0,00152
46	95 965	0,00267	97 623	0,00170
47	95 709	0,00299	97 457	0,00191
48	95 423	0,00330	97 271	0,00211
49	95 108	0,00363	97 066	0,00232
50	94 763	0,00398	96 841	0,00255
51	94 386	0,00438	96 594	0,00278
52	93 973	0,00481	96 325	0,00303
53	93 521	0,00531	96 033	0,00331
54	93 024	0,00585	95 715	0,00358
55	92 480	0,00647	95 372	0,00390
56	91 882	0,00712	95 000	0,00423
57	91 228	0,00783	94 598	0,00459
58	90 514	0,00861	94 164	0,00497
59	89 735	0,00945	93 696	0,00540
60	88 887	0,01041	93 190	0,00588
61	87 962	0,01150	92 642	0,00645
62	86 950	0,01281	92 044	0,00713
63	85 836	0,01443	91 388	0,00793
64	84 597	0,01642	90 663	0,00890
65	83 208	0,01883	89 856	0,01003
66	81 641	0,02170	88 955	0,01132
67	79 869	0,02502	87 948	0,01279
68	77 871	0,02873	86 823	0,01447
69	75 634	0,03283	85 567	0,01633
70	73 151	0,03722	84 170	0,01844
71	70 428	0,04187	82 618	0,02082
72	67 479	0,04659	80 898	0,02347
73	64 335	0,05119	78 999	0,02704
74	61 042	0,05611	76 863	0,03040
75	57 617	0,06147	74 526	0,03430
76	54 075	0,06744	71 970	0,03874
77	50 428	0,07415	69 182	0,04394
78	46 689	0,08182	66 142	0,05000

Age	Men		Women	
	Lx	Qx	Lx	Qx
79	42 869	0,09048	62 835	0,05702
80	38 990	0,10026	59 252	0,06511
81	35 081	0,11120	55 394	0,07429
82	31 180	0,12328	51 279	0,08460
83	27 336	0,13649	46 941	0,09608
84	23 605	0,15090	42 431	0,10905
85	20 043	0,16649	37 804	0,12361
86	16 706	0,18335	33 131	0,14002
87	13 643	0,20135	28 492	0,15825
88	10 896	0,22026	23 983	0,17817
89	8 496	0,24011	19 710	0,19970
90	6 456	0,26053	15 774	0,22252
91	4 774	0,28152	12 264	0,24641
92	3 430	0,30321	9 242	0,27115
93	2 390	0,32469	6 736	0,29558
94	1 614	0,34696	4 745	0,31949
95	1 054	0,36907	3 229	0,34345
96	665	0,39098	2 120	0,36792
97	405	0,41481	1 340	0,39254
98	237	0,43882	814	0,41769
99	133	0,45865	474	0,44304
100	72	1,00000	264	1,00000

APPENDIX 2 – FINAL MORTALITY TABLES

Age	Qx WH	Qx Makeham	Qx Brass
44	0,00091	0,00092	0,00095
45	0,00098	0,00102	0,00108
46	0,00107	0,00113	0,00122
47	0,00120	0,00126	0,00137
48	0,00135	0,00140	0,00150
49	0,00153	0,00155	0,00166
50	0,00175	0,00172	0,00182
51	0,00199	0,00191	0,00200
52	0,00226	0,00213	0,00219
53	0,00253	0,00236	0,00242
54	0,00281	0,00262	0,00265
55	0,00309	0,00291	0,00293
56	0,00336	0,00323	0,00322
57	0,00364	0,00359	0,00353
58	0,00393	0,00398	0,00386
59	0,00426	0,00442	0,00424
60	0,00466	0,00491	0,00467
61	0,00516	0,00545	0,00516
62	0,00578	0,00605	0,00577
63	0,00654	0,00671	0,00653
64	0,00744	0,00745	0,00743
65	0,00849	0,00827	0,00856
66	0,00967	0,00918	0,00986
67	0,01099	0,01019	0,01136

APPENDIX 3 – BEST ESTIMATE MORTALITY TABLE

Age	Unisex Lx Best estimate	Unisex qx Best estimate	Men Lx Best estimate	Men qx Best estimate	Women Lx Best estimate	Women qx Best estimate
0	100 000	0,00293	100 000	0,00310	100 000	0,00275
1	99 707	0,00021	99 690	0,00022	99 725	0,00021
2	99 686	0,00017	99 668	0,00017	99 705	0,00017
3	99 669	0,00014	99 651	0,00014	99 687	0,00014
4	99 655	0,00011	99 636	0,00011	99 674	0,00011
5	99 644	0,00009	99 625	0,00009	99 663	0,00009
6	99 635	0,00007	99 616	0,00008	99 654	0,00007
7	99 628	0,00007	99 608	0,00007	99 647	0,00006
8	99 621	0,00006	99 601	0,00007	99 641	0,00006
9	99 615	0,00006	99 594	0,00006	99 635	0,00005
10	99 609	0,00006	99 588	0,00007	99 629	0,00005
11	99 603	0,00007	99 581	0,00007	99 624	0,00006
12	99 596	0,00008	99 574	0,00009	99 618	0,00007
13	99 588	0,00010	99 565	0,00011	99 611	0,00009
14	99 579	0,00013	99 554	0,00015	99 603	0,00010
15	99 566	0,00018	99 539	0,00022	99 593	0,00013
16	99 548	0,00022	99 517	0,00028	99 581	0,00015
17	99 526	0,00028	99 489	0,00036	99 566	0,00016
18	99 498	0,00032	99 453	0,00043	99 550	0,00018
19	99 466	0,00034	99 410	0,00048	99 532	0,00017
20	99 432	0,00037	99 362	0,00052	99 515	0,00018
21	99 396	0,00037	99 311	0,00052	99 497	0,00017
22	99 360	0,00036	99 259	0,00051	99 479	0,00017
23	99 324	0,00035	99 208	0,00049	99 463	0,00017
24	99 290	0,00032	99 159	0,00046	99 446	0,00016
25	99 258	0,00031	99 114	0,00044	99 431	0,00016
26	99 227	0,00030	99 071	0,00042	99 415	0,00015
27	99 197	0,00029	99 029	0,00039	99 400	0,00016
28	99 167	0,00029	98 991	0,00038	99 384	0,00016
29	99 138	0,00029	98 953	0,00038	99 368	0,00016
30	99 109	0,00030	98 915	0,00039	99 352	0,00016
31	99 079	0,00032	98 877	0,00039	99 336	0,00018
32	99 048	0,00033	98 838	0,00040	99 318	0,00020
33	99 015	0,00035	98 799	0,00042	99 299	0,00021
34	98 981	0,00037	98 758	0,00044	99 278	0,00024
35	98 944	0,00039	98 714	0,00045	99 254	0,00026
36	98 905	0,00042	98 669	0,00048	99 229	0,00030
37	98 863	0,00045	98 622	0,00051	99 199	0,00033
38	98 818	0,00050	98 571	0,00055	99 166	0,00038
39	98 769	0,00054	98 517	0,00060	99 128	0,00042

Age	Unisex Lx Best estimate	Unisex qx Best estimate	Men Lx Best estimate	Men qx Best estimate	Women Lx Best estimate	Women qx Best estimate
40	98 716	0,00060	98 458	0,00066	99 086	0,00047
41	98 657	0,00067	98 393	0,00074	99 040	0,00053
42	98 591	0,00075	98 320	0,00083	98 988	0,00058
43	98 517	0,00084	98 239	0,00099	98 930	0,00060
44	98 434	0,00095	98 142	0,00112	98 870	0,00061
45	98 341	0,00108	98 033	0,00126	98 810	0,00069
46	98 235	0,00122	97 909	0,00143	98 742	0,00078
47	98 115	0,00137	97 769	0,00160	98 665	0,00087
48	97 981	0,00150	97 613	0,00177	98 579	0,00095
49	97 834	0,00166	97 440	0,00194	98 485	0,00105
50	97 671	0,00182	97 251	0,00213	98 382	0,00115
51	97 493	0,00200	97 044	0,00234	98 268	0,00126
52	97 298	0,00219	96 817	0,00257	98 144	0,00137
53	97 085	0,00242	96 568	0,00284	98 009	0,00150
54	96 850	0,00265	96 294	0,00313	97 862	0,00162
55	96 593	0,00293	95 993	0,00346	97 704	0,00176
56	96 310	0,00322	95 661	0,00381	97 532	0,00189
57	95 999	0,00353	95 296	0,00419	97 348	0,00206
58	95 661	0,00386	94 897	0,00460	97 148	0,00223
59	95 292	0,00424	94 460	0,00506	96 932	0,00242
60	94 888	0,00467	93 983	0,00557	96 697	0,00262
61	94 445	0,00516	93 459	0,00616	96 443	0,00287
62	93 958	0,00577	92 884	0,00686	96 167	0,00314
63	93 416	0,00653	92 246	0,00773	95 864	0,00347
64	92 806	0,00743	91 533	0,00880	95 532	0,00389
65	92 117	0,00856	90 727	0,01010	95 161	0,00430
66	91 329	0,00986	89 811	0,01166	94 751	0,00485
67	90 428	0,01136	88 764	0,01345	94 292	0,00543
68	89 401	0,01271	87 570	0,01497	93 779	0,00595
69	88 264	0,01394	86 259	0,01629	93 222	0,00689
70	87 034	0,01569	84 854	0,01850	92 579	0,00787
71	85 668	0,01777	83 284	0,02085	91 851	0,00886
72	84 146	0,01994	81 547	0,02325	91 037	0,00998
73	82 468	0,02226	79 651	0,02559	90 128	0,01151
74	80 632	0,02436	77 613	0,02811	89 091	0,01313
75	78 668	0,02681	75 431	0,03087	87 922	0,01498
76	76 559	0,02919	73 102	0,03396	86 605	0,01707
77	74 324	0,03185	70 620	0,03745	85 127	0,01936
78	71 957	0,03551	67 975	0,04147	83 478	0,02203
79	69 401	0,03971	65 157	0,04604	81 639	0,02513
80	66 645	0,04451	62 157	0,05125	79 587	0,02869
81	63 679	0,04996	58 971	0,05713	77 304	0,03274

Age	Unisex Lx Best estimate	Unisex qx Best estimate	Men Lx Best estimate	Men qx Best estimate	Women Lx Best estimate	Women qx Best estimate
82	60 497	0,05608	55 602	0,06371	74 773	0,03728
83	57 105	0,06288	52 060	0,07098	71 985	0,04234
84	53 514	0,07049	48 365	0,07902	68 937	0,04806
85	49 742	0,07892	44 543	0,08786	65 625	0,05447
86	45 816	0,08834	40 629	0,09757	62 050	0,06171
87	41 769	0,09869	36 665	0,10813	58 221	0,06974
88	37 646	0,10991	32 701	0,11943	54 161	0,07852
89	33 509	0,12210	28 795	0,13154	49 908	0,08800
90	29 417	0,13506	25 008	0,14426	45 516	0,09806
91	25 444	0,14879	21 400	0,15763	41 053	0,10859
92	21 658	0,16332	18 027	0,17176	36 595	0,11949
93	18 121	0,17809	14 930	0,18609	32 222	0,13026
94	14 894	0,19358	12 152	0,20133	28 025	0,14080
95	12 011	0,20899	9 705	0,21682	24 079	0,15136
96	9 501	0,22541	7 601	0,23257	20 434	0,16214
97	7 359	0,24337	5 833	0,25017	17 121	0,17299
98	5 568	0,26061	4 374	0,26839	14 159	0,18407
99	4 117	0,27884	3 200	0,28384	11 553	0,19524
100	2 969	1,00000	2 292	1,00000	9 297	1,00000

APPENDIX 4 – MORTALITY TABLE FOR PRICING PURPOSE

Age	Unisex Lx Pricing	Unisex qx Pricing	Men Lx Pricing	Men qx Pricing	Women Lx Pricing	Women qx Pricing
0	100 000	0,00334	100 000	0,00354	100 000	0,00313
1	99 666	0,00024	99 646	0,00025	99 687	0,00024
2	99 642	0,00020	99 622	0,00020	99 663	0,00020
3	99 623	0,00016	99 602	0,00016	99 644	0,00015
4	99 607	0,00012	99 585	0,00013	99 628	0,00012
5	99 595	0,00011	99 573	0,00011	99 616	0,00011
6	99 584	0,00008	99 562	0,00009	99 605	0,00008
7	99 576	0,00008	99 553	0,00008	99 597	0,00007
8	99 568	0,00007	99 545	0,00008	99 590	0,00007
9	99 561	0,00007	99 537	0,00007	99 584	0,00006
10	99 554	0,00007	99 530	0,00008	99 577	0,00006
11	99 547	0,00008	99 523	0,00008	99 571	0,00007
12	99 540	0,00009	99 515	0,00010	99 565	0,00007
13	99 531	0,00011	99 505	0,00013	99 557	0,00010
14	99 520	0,00015	99 492	0,00018	99 547	0,00011
15	99 505	0,00020	99 474	0,00025	99 536	0,00014
16	99 485	0,00026	99 450	0,00032	99 522	0,00017
17	99 459	0,00031	99 418	0,00041	99 505	0,00019
18	99 428	0,00037	99 377	0,00049	99 487	0,00020
19	99 391	0,00039	99 328	0,00055	99 466	0,00019
20	99 353	0,00042	99 273	0,00059	99 447	0,00021
21	99 312	0,00042	99 215	0,00059	99 426	0,00020
22	99 270	0,00041	99 156	0,00058	99 407	0,00019
23	99 230	0,00039	99 098	0,00056	99 388	0,00019
24	99 191	0,00037	99 042	0,00053	99 369	0,00018
25	99 154	0,00036	98 990	0,00050	99 351	0,00018
26	99 119	0,00035	98 941	0,00047	99 333	0,00018
27	99 085	0,00033	98 894	0,00044	99 316	0,00018
28	99 051	0,00033	98 850	0,00044	99 298	0,00018
29	99 018	0,00034	98 807	0,00043	99 280	0,00018
30	98 985	0,00035	98 764	0,00044	99 262	0,00019
31	98 951	0,00036	98 721	0,00045	99 243	0,00020
32	98 915	0,00038	98 677	0,00046	99 223	0,00023
33	98 878	0,00040	98 632	0,00048	99 201	0,00024
34	98 839	0,00043	98 585	0,00050	99 177	0,00028
35	98 796	0,00044	98 535	0,00052	99 150	0,00029
36	98 753	0,00048	98 484	0,00055	99 121	0,00034
37	98 705	0,00052	98 430	0,00058	99 087	0,00038
38	98 654	0,00057	98 373	0,00063	99 050	0,00044
39	98 598	0,00062	98 311	0,00069	99 006	0,00048

Age	Unisex Lx Pricing	Unisex qx Pricing	Men Lx Pricing	Men qx Pricing	Women Lx Pricing	Women qx Pricing
40	98 537	0,00068	98 244	0,00075	98 959	0,00053
41	98 470	0,00076	98 170	0,00084	98 906	0,00060
42	98 395	0,00085	98 087	0,00094	98 847	0,00066
43	98 311	0,00096	97 995	0,00112	98 781	0,00069
44	98 217	0,00109	97 885	0,00127	98 713	0,00070
45	98 110	0,00123	97 760	0,00144	98 644	0,00078
46	97 990	0,00139	97 620	0,00163	98 567	0,00089
47	97 854	0,00156	97 461	0,00182	98 480	0,00099
48	97 701	0,00171	97 283	0,00201	98 382	0,00108
49	97 534	0,00189	97 087	0,00221	98 275	0,00120
50	97 349	0,00208	96 872	0,00243	98 157	0,00132
51	97 147	0,00228	96 637	0,00267	98 028	0,00144
52	96 926	0,00250	96 380	0,00293	97 887	0,00156
53	96 683	0,00276	96 097	0,00324	97 734	0,00171
54	96 417	0,00302	95 785	0,00357	97 567	0,00184
55	96 125	0,00335	95 444	0,00394	97 387	0,00201
56	95 804	0,00368	95 068	0,00434	97 191	0,00215
57	95 452	0,00402	94 655	0,00477	96 982	0,00234
58	95 068	0,00440	94 203	0,00525	96 755	0,00254
59	94 650	0,00483	93 709	0,00576	96 509	0,00276
60	94 193	0,00532	93 169	0,00635	96 243	0,00299
61	93 691	0,00588	92 577	0,00702	95 955	0,00327
62	93 140	0,00658	91 927	0,00782	95 641	0,00358
63	92 528	0,00744	91 208	0,00882	95 299	0,00395
64	91 839	0,00847	90 404	0,01003	94 922	0,00443
65	91 062	0,00976	89 497	0,01152	94 501	0,00490
66	90 174	0,01124	88 466	0,01329	94 038	0,00553
67	89 160	0,01295	87 290	0,01533	93 518	0,00619
68	88 005	0,01449	85 952	0,01707	92 939	0,00678
69	86 730	0,01589	84 485	0,01857	92 309	0,00786
70	85 352	0,01789	82 916	0,02109	91 583	0,00897
71	83 825	0,02025	81 167	0,02377	90 762	0,01010
72	82 127	0,02273	79 238	0,02650	89 845	0,01138
73	80 260	0,02538	77 138	0,02917	88 823	0,01312
74	78 223	0,02777	74 887	0,03205	87 658	0,01496
75	76 051	0,03056	72 487	0,03519	86 346	0,01707
76	73 726	0,03328	69 936	0,03871	84 872	0,01946
77	71 273	0,03631	67 229	0,04269	83 220	0,02208
78	68 685	0,04049	64 359	0,04727	81 383	0,02512
79	65 904	0,04527	61 316	0,05249	79 339	0,02865
80	62 921	0,05075	58 098	0,05842	77 066	0,03271
81	59 728	0,05695	54 704	0,06513	74 545	0,03732
82	56 326	0,06393	51 141	0,07262	71 763	0,04250

Age	Unisex Lx Pricing	Unisex qx Pricing	Men Lx Pricing	Men qx Pricing	Women Lx Pricing	Women qx Pricing
83	52 725	0,07168	47 427	0,08091	68 713	0,04827
84	48 946	0,08036	43 589	0,09009	65 396	0,05478
85	45 012	0,08997	39 663	0,10016	61 814	0,06210
86	40 963	0,10071	35 690	0,11123	57 975	0,07034
87	36 837	0,11251	31 720	0,12326	53 897	0,07951
88	32 693	0,12530	27 810	0,13615	49 612	0,08951
89	28 596	0,13920	24 024	0,14995	45 171	0,10032
90	24 616	0,15397	20 421	0,16446	40 639	0,11179
91	20 826	0,16962	17 063	0,17970	36 096	0,12379
92	17 293	0,18619	13 997	0,19581	31 628	0,13622
93	14 073	0,20302	11 256	0,21215	27 319	0,14849
94	11 216	0,22068	8 868	0,22951	23 263	0,16051
95	8 741	0,23825	6 833	0,24718	19 529	0,17254
96	6 658	0,25697	5 144	0,26513	16 159	0,18484
97	4 947	0,27744	3 780	0,28519	13 172	0,19721
98	3 575	0,29709	2 702	0,30596	10 575	0,20984
99	2 513	0,31788	1 875	0,32357	8 356	0,22258
100	1 714	1,00000	1 268	1,14000	6 496	1,14000