Scenario Testing for Flatrated Fleets during the yearly price adjustment process – a practical example

Michael Klamser,
Allianz Versicherungs AG

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About the speaker

Name
- Michael Klamser (Senior Actuary)

1994: Entering Allianz Insurance Company (Actuarial Department)
1994-2000: Actuarial Department (Motor business – retail and commercial)
2000-today: Commercial Motor Department
Since 1999: Actuary at the German Association of Actuaries (DAV)

Allianz Group (Non-Life) - 2019
- Turnover: 59,2 bln. €
- Operating profit: 5,0 bln €
- Loss ratio: 68,0 % (German fleet market/before run-off: 92,0 %)
- Combined Ratio: 95,5 % (German fleet market/after run-off: 102,0 %)
Disclaimer:

All the figures/KPIs in the following slides which are connected with the Allianz fleet portfolio, do not correspond with the figures in reality.

Still, the deductions done in the presentation respectively during the session are the same as the ones based on the real figures.
Glossary:

TP: (actuarially correct) technical premium

CP: commercial premium (before any adjustments)

AP: actual respectively offered premium

LR: loss-ratio (not lapse-ratio!!)

MRP: Manual Renewal-Probability
Overview

1. The flatrate model (cred.) / Bonus-Malus

2. The MRP- / Lapse-Ratio-Model:
The Build-up of the database

3. Modelling the MRP

4. Prediction of the Loss-Ratio through Multinominal Approach

5. The lapse ratio model: 9-field-analysis / scenario-analysis
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1. The flatrate model (cred.) / Bonus-Malus - 1

**Basics:**

-Introduced in 2013;
-An essential model to increase the profitability of the overall flatrate portfolio;
-As of end of 2018: approx. 1.000 fleets with an AP of 70 Mio.;
-Includes an **optional** premium adjustment-clause
  (➔ to compensate for the loss in GWP due to automatic renewal).
-Enables a new calculation of the fleet if certain criteria are met.
1. The flatrate model (cred.) / Bonus-Malus - 2

**Rules for automatic renewal (dependent of 8 LR-classes):**

- LR<45 %  ➔ -15 % discount,
- LR in (45%,55%) ➔ -10 % discount,
- LR in (85%,95%) ➔ +15 % loading,
- LR > 95 % ➔ new calculation on the basis of credibility.
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Whence comes the need to model the probability for manual renewal?

(1) Direct impact on the top- and bottom-line through...
   - shunning the premium adjustment and/or
   - avoiding the lapse of a customer,
   - Portfolio-cleaning.

(2) To answer the question:
   - What’s the impact on the lapses?

(3) To estimate separately the rate change because of manual renewal.
2. The MRP-/Lapse-Ratio-Model: The Build-up of the database - 2

Variables to be examined conc. significance of the risk variables for the ...

- **MRP-Model**
  - fleets flagged for ptf-cleaning,
  - LR (grouped) as of end of July,
  - individual premium adjustment (dBAK),
  - installment,
  - distribution channel,
  - fleet size....

- **Lapse-Ratio-Model**
  - Customer tenure,
  - number of large claims in the previous years,
  - fleet mix,
  - distribution channel,
  - fleets flagged for ptf-cleaning,
  - fleets flagged for MRP (!)
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3. Modelling the MRP - 1

Selection Procedure (for single and 2x2-effects):

- Model assumptions:
  
  - $\alpha$: maximum significance level (5%),
  
  - $p_i$: lapse ratio for fleet i,
  
- $g(p_i) = \log\left(\frac{p_i}{1-p_i}\right)$: Link-function, $p = \frac{e^\mu}{1+e^\mu}$

- distribution: bin(1,$p_i$).

- Out of the pool of m possibly significant predictors, the most significant factor is selected.

- 2nd step: the 2nd most significant factor is selected (and so forth)…

- Stop criterion: The sum of all single $\alpha$ surpasses the maximum significance level $\alpha$. 
3. Modelling the MRP - 2

Old result (through Cluster Method by Ward):

<table>
<thead>
<tr>
<th>Cluster</th>
<th>loss-ratio (as of 31st of July)</th>
<th>MRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>from 170 %</td>
<td>93,0%</td>
</tr>
<tr>
<td>2</td>
<td>120 % to 170 %</td>
<td>63,0%</td>
</tr>
<tr>
<td>3</td>
<td>60 % to 120 %</td>
<td>45,0%</td>
</tr>
<tr>
<td>4</td>
<td>up to 60 %</td>
<td>25,0%</td>
</tr>
</tbody>
</table>

Shortcomings:

- Dependency of the MRP merely on one predictor.
- Though organic behaviour was achieved, the result is not too helpful (see rules for automatic renewal above).
3. Modelling the MRP - 3

New Approach through GLM:

Selected Variables:

<table>
<thead>
<tr>
<th>predictor</th>
<th>1st degree freedom</th>
<th>F-statistics</th>
<th>alpha*</th>
</tr>
</thead>
<tbody>
<tr>
<td>portfolio cleaning (flagged)</td>
<td>1</td>
<td>19,02</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>LR as of 31/7 (grouped)</td>
<td>4</td>
<td>22,38</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>fleet size</td>
<td>2</td>
<td>3,77</td>
<td>0,0233</td>
</tr>
</tbody>
</table>

The shortcomings of Ward were all taken care of.

Parameter Estimator-Statistic:

<table>
<thead>
<tr>
<th>predictor</th>
<th>level</th>
<th>estimate (lin. pred.)</th>
<th>Standard-error</th>
<th>alpha (Chi-square)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td></td>
<td>3,5133</td>
<td>0,4307</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>ptf cleaning (flagged)</td>
<td>not flagged</td>
<td>-0,9625</td>
<td>0,3231</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td></td>
<td>flagged</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>LR as of 31/7 (grouped)</td>
<td>&lt;45%</td>
<td>-2,2192</td>
<td>0,3177</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td></td>
<td>45-65%</td>
<td>-1,712</td>
<td>0,31</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td></td>
<td>65-95%</td>
<td>-1,3232</td>
<td>0,3253</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td></td>
<td>95-125%</td>
<td>-0,9518</td>
<td>0,3819</td>
<td>0,0007</td>
</tr>
<tr>
<td></td>
<td>above 125%</td>
<td>0</td>
<td>0</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>fleetsize</td>
<td>30-60</td>
<td>-0,2097</td>
<td>0,2136</td>
<td>0,0088</td>
</tr>
<tr>
<td></td>
<td>60-100</td>
<td>-0,1399</td>
<td>0,2104</td>
<td>0,0199</td>
</tr>
<tr>
<td></td>
<td>above 100</td>
<td>0</td>
<td>0</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>
## 3. Modelling the MRP - 4

### Validation (20% of sample)

Flagged for ptf-cleaning:

<table>
<thead>
<tr>
<th>ptf cleaning</th>
<th># fleets (validation sample)</th>
<th>MRP (observed)</th>
<th>MRP (estimated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>not flagged</td>
<td>223</td>
<td>41,1%</td>
<td>41,4%</td>
</tr>
<tr>
<td>flagged</td>
<td>26</td>
<td>79,8%</td>
<td>94,1%</td>
</tr>
</tbody>
</table>

LR as of 31st of July:

<table>
<thead>
<tr>
<th>LR as of 31/7 (grouped)</th>
<th># fleets (validation sample)</th>
<th>MRP (observed)</th>
<th>MRP (estimated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;45%</td>
<td>83</td>
<td>21,3%</td>
<td>26,1%</td>
</tr>
<tr>
<td>45-65%</td>
<td>79</td>
<td>42,0%</td>
<td>38,9%</td>
</tr>
<tr>
<td>65-95%</td>
<td>28</td>
<td>50,6%</td>
<td>57,5%</td>
</tr>
<tr>
<td>95-125%</td>
<td>21</td>
<td>63,0%</td>
<td>68,0%</td>
</tr>
<tr>
<td>above 125%</td>
<td>38</td>
<td>90,4%</td>
<td>90,0%</td>
</tr>
</tbody>
</table>

### Fleetsize:

<table>
<thead>
<tr>
<th>fleet size</th>
<th># fleets (validation sample)</th>
<th>MRP (observed)</th>
<th>MRP (estimated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-60</td>
<td>89</td>
<td>32,1%</td>
<td>39,5%</td>
</tr>
<tr>
<td>60-100</td>
<td>100</td>
<td>47,5%</td>
<td>47,0%</td>
</tr>
<tr>
<td>above 100</td>
<td>60</td>
<td>60,8%</td>
<td>57,9%</td>
</tr>
</tbody>
</table>
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4. Prediction of the Loss-Ratio through Multinomial Approach - 1

Predicament:

An eventual overall premium-adjustment in addition to the automatic renewal has to be decided no later than in August (due to technical restraints).

→ Prediction of the loss-ratio as of 31st of December on the basis of 31st of July is of paramount importance.

Possible solution (see also SAS/STAT – PROC GENMOD, examples):

Application of the Generalized Linear Model with

- the **multinomial distribution** and
- the **cumulative logit function**.

In a nutshell:

(1) Creating an **ordinal-scaled predictor** “LR as of 31st of July” -
grouped into classes “up to 15 %“, „15 to 25 %“,.....till “higher than 195 %“

(Attention: Further grouping should be envisaged in the modelling process!)

(2) Defining the **ordinal scaled response** “LR as of 31st of December“
on the basis of the „rules for automatic renewal“ (see chapter 1 ➔ 7 LR-classes).

(3) For each of the k LR-classes as of 31st of July (k=1 to 7), be \( p_i \) the probability
that the fleet falls into the i-th LR-class as of 31st of December (i=1 to 7).
Then the cumulative logit function for the i-th LR-class is
\[
g(p_1, p_2, ..., p_7) = \log\left(\sum_{j=1}^i p_j / \left(1 - \sum_{j=1}^i p_j\right)\right)
\]

(4) Finally, through a simple recursion all the estimates for the \( p_i \) can be determined –
and this in dependence of the respective linear predictor \( \eta \).
4. Prediction of the Loss-Ratio through Multinomial Approach - 3

Can be any (statistically sensible) grouping

LR-class as of 31st of July

- up to 45 %
- 45 till 55 %
- 85 till 95 %
- from 95 % onward

Should be the same as for automatic renewal

LR-class as of 31st of December

- up to 45 %
- 45 till 55 %
- 85 till 95 %
- from 95 % onward

$p_1^1$, $p_2^1$, $p_6^1$, $p_7^1$, $p_1^2$, $p_2^2$, $p_6^2$, $p_7^2$
4. Prediction of the Loss-Ratio through Multinomial Approach - 4

General result:

Predictor „LR as of 31st of July (grouped)“ highly significant with regard to the Response „LR as of 31st of December (grouped according to rules for automatic renewal)“

<table>
<thead>
<tr>
<th>Difference between observed/estimated prob.</th>
<th>F- and Chi-Square-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std-Deviance</td>
<td>Std. Error of Mean</td>
</tr>
<tr>
<td>0.1392</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

Parameter Estimates Statistic:

The intercepts behave very organic and there is no overlapping of the conf. limits with the former/latter parameter estimate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>unteres Konf.limit</th>
<th>Estimate</th>
<th>oberes Konf.limit</th>
<th>Std.error</th>
<th>ProbChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept1</td>
<td>0,43</td>
<td>0,75</td>
<td>1,07</td>
<td>0,16</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept2</td>
<td>1,21</td>
<td>1,56</td>
<td>1,91</td>
<td>0,18</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept3</td>
<td>1,96</td>
<td>2,35</td>
<td>2,75</td>
<td>0,20</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept4</td>
<td>2,61</td>
<td>3,04</td>
<td>3,47</td>
<td>0,22</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept5</td>
<td>3,15</td>
<td>3,61</td>
<td>4,08</td>
<td>0,24</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept6</td>
<td>3,61</td>
<td>4,10</td>
<td>4,59</td>
<td>0,25</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Intercept7</td>
<td>4,04</td>
<td>4,55</td>
<td>5,06</td>
<td>0,26</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>LR (31/7) – grouped</td>
<td>-5,12</td>
<td>-4,53</td>
<td>-3,94</td>
<td>0,30</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>
4. Prediction of the Loss-Ratio through Multinomial Approach - 6

Validation:
The median of the difference between estimated transfer-prob. (test sample) and the observed one (validation sample) is very close to zero. But the tendency is clearly towards a bigger observed value than estimated ones.

<table>
<thead>
<tr>
<th>max</th>
<th>q99</th>
<th>q95</th>
<th>q90</th>
<th>q75</th>
<th>q50</th>
<th>q25</th>
<th>q10</th>
<th>q5</th>
<th>q1</th>
<th>min</th>
</tr>
</thead>
<tbody>
<tr>
<td>18,4%</td>
<td>18,4%</td>
<td>6,7%</td>
<td>4,7%</td>
<td>2,1%</td>
<td>0,5%</td>
<td>-4,6%</td>
<td>-9,1%</td>
<td>-13,4%</td>
<td>-29,0%</td>
<td>-29,0%</td>
</tr>
</tbody>
</table>

Final transfer probabilities (2 examples):

Confidence limits show the high reliability of the estimators for the transfer probability.

<table>
<thead>
<tr>
<th>LR (as of 31/7) grouped</th>
<th>LR (as of 31/12) grouped</th>
<th>transfer-prob. (single)</th>
<th>lower conf.limit</th>
<th>transfer-prob. cumulative</th>
<th>upper conf.limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-45%</td>
<td>0-45%</td>
<td>67,9%</td>
<td>60,5%</td>
<td>67,9%</td>
<td>74,5%</td>
</tr>
<tr>
<td>0-45%</td>
<td>45-55%</td>
<td>14,7%</td>
<td>77,0%</td>
<td>82,6%</td>
<td>87,1%</td>
</tr>
<tr>
<td>0-45%</td>
<td>55-65%</td>
<td>8,7%</td>
<td>87,7%</td>
<td>91,3%</td>
<td>94,0%</td>
</tr>
<tr>
<td>0-45%</td>
<td>65-75%</td>
<td>4,1%</td>
<td>93,1%</td>
<td>95,4%</td>
<td>97,0%</td>
</tr>
<tr>
<td>0-45%</td>
<td>75-85%</td>
<td>1,9%</td>
<td>95,9%</td>
<td>97,4%</td>
<td>98,3%</td>
</tr>
<tr>
<td>0-45%</td>
<td>85-95%</td>
<td>1,0%</td>
<td>97,4%</td>
<td>98,4%</td>
<td>99,0%</td>
</tr>
<tr>
<td>0-45%</td>
<td>higher than 105%</td>
<td>1,0%</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>95-105%</td>
<td>0-45%</td>
<td>3,5%</td>
<td>2,3%</td>
<td>3,5%</td>
<td>5,1%</td>
</tr>
<tr>
<td>95-105%</td>
<td>45-55%</td>
<td>4,0%</td>
<td>5,4%</td>
<td>7,5%</td>
<td>10,3%</td>
</tr>
<tr>
<td>95-105%</td>
<td>55-65%</td>
<td>7,7%</td>
<td>11,6%</td>
<td>15,1%</td>
<td>19,5%</td>
</tr>
<tr>
<td>95-105%</td>
<td>65-75%</td>
<td>11,0%</td>
<td>21,2%</td>
<td>26,2%</td>
<td>31,8%</td>
</tr>
<tr>
<td>95-105%</td>
<td>75-85%</td>
<td>12,4%</td>
<td>32,6%</td>
<td>38,6%</td>
<td>45,0%</td>
</tr>
<tr>
<td>95-105%</td>
<td>85-95%</td>
<td>11,9%</td>
<td>44,0%</td>
<td>50,5%</td>
<td>57,0%</td>
</tr>
<tr>
<td>95-105%</td>
<td>95-105%</td>
<td>11,1%</td>
<td>55,0%</td>
<td>61,6%</td>
<td>67,8%</td>
</tr>
<tr>
<td>95-105%</td>
<td>higher than 105%</td>
<td>38,4%</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
Though competition advances us forward, only by cooperation can we manage to master the real challenges ahead – „dog eats dog“ is doomed to fail.

“The only thing worse than fighting with allies is fighting without them.”
(by Winston Churchill, in the 1940-ies)
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5. The lapse ratio model: 9-field-analysis / scenario-analysis - 1

9-field-analysis: Categorization of the AP/TP-ratio and the Lapse-Ratio - graph:

- possibly higher discount
- possibly higher loading
- Lapse-ratio vs. AP/TP (flatrate/bonus-malus)
5. The lapse ratio model: 9-field-analysis / scenario-analysis - 2

scenario-analysis: 20 scenarios (prem. adjustment 0 % to 20 %) - table

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>1.157</td>
<td>63.4</td>
<td>11.1%</td>
<td>89.6%</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>1.150</td>
<td>63.1</td>
<td>11.3%</td>
<td>89.9%</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>1.150</td>
<td>62.9</td>
<td>11.4%</td>
<td>90.4%</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>1.153</td>
<td>63.7</td>
<td>11.6%</td>
<td>90.9%</td>
<td></td>
</tr>
<tr>
<td>4%</td>
<td>1.153</td>
<td>64.1</td>
<td>11.7%</td>
<td>91.5%</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.150</td>
<td>64.9</td>
<td>11.8%</td>
<td>92.1%</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>1.142</td>
<td>64.6</td>
<td>12.0%</td>
<td>92.4%</td>
<td></td>
</tr>
<tr>
<td>7%</td>
<td>1.140</td>
<td>64.7</td>
<td>12.2%</td>
<td>93.0%</td>
<td></td>
</tr>
<tr>
<td>8%</td>
<td>1.136</td>
<td>65.2</td>
<td>12.4%</td>
<td>93.6%</td>
<td></td>
</tr>
<tr>
<td>9%</td>
<td>1.135</td>
<td>65.5</td>
<td>12.6%</td>
<td>94.2%</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>1.106</td>
<td>63.9</td>
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</table>

premises:

- x up to 9 % increase in premium adjustment ➔ x % increase in overall lapse-ratio.
- x from 10 % to 20 % increase in premium adjustment ➔ 3 times x % increase in overall lapse-ratio.
5. The lapse ratio model: 9-field-analysis / scenario-analysis - 3

scenario-analysis: 21 scenarios (prem. adjustment 0 % to 20 %) - graph

result: With maximum AP being the requirement by the Board of Management, 9 % would be the optimal premium adjustment factor.
Backup
The lapse-ratio-model: Build-up of database (creation of fleet mix through clustering)

Cluster method by Ward (source: SAS/STAT guide):

The distance between two clusters is defined by

\[ d(x, y) = \frac{1}{2} ||x - y||^2 \]

then the combinatorial formula is

\[ D_{KL} = B_{KL} = \frac{||\bar{x}_K - \bar{x}_L||^2}{\frac{1}{N_K} + \frac{1}{N_L}} \]

\[ D_{JK} = \frac{(N_J + N_K)D_{JK} + (N_J + N_L)D_{KL} - N_JD_{KL}}{N_J + N_K} \]

In Ward’s minimum-variance method, the distance between two clusters is the ANOVA sum of squares between the two clusters added up over all the variables. At each generation, the within-cluster sum of squares is minimized over all partitions obtainable by merging two clusters from the previous generation.

The sums of squares are easier to interpret when they are divided by the total sum of squares to give proportions of variance (squared semipartial correlations).

Ward’s method joins clusters to maximize the likelihood at each level of the hierarchy under the following assumptions:

- multivariate normal mixture,
- equal spherical covariance matrices,
- equal sampling probabilities.

Peculiarities:

- Ward’s method tends to join clusters with a small number of observations;
- It is strongly biased toward producing clusters with roughly the same number of observations;
- It is also very sensitive to outliers.
The calculation of the TP by credibility (here: the risk premium)

\[ z_i^d = \frac{w_i^d}{(w_i^d + k)} \]

\( w_i^d \): e. g. expected number of claims (for the KPI “overall claims frequency”)

\( k = \sigma / \tau \), where \( \sigma \): variability of the fleet over time, \( \tau \): variability between the fleets

\( c_f \): claims frequency, \( ca \): claims average

Thus, for dimension \( d \) and KPI \( i \), we get:

\[ \text{credibility factor for claims-layer } d \text{ and KPI } i \]

\[ \rightarrow \text{risk}_i \text{ premium} = c_f^{\text{cred}} \ast c_a^{\text{cred}} + c_f^{\text{cred} \geq 25k} \ast c_a^{\text{cred} \geq 25k} + \text{loading}^{\text{acc} \geq 50k} \]

\[ \rightarrow \text{net premium} = \text{risk}_i \text{ premium} \quad \text{(incl. cost loadings).} \]
Thank you for your attention

Contact details:

Michael Klamser
Königinstr. 28
80802 Munich
Germany

Michael.Klamser@allianz.de

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