Systemic risk in derivative markets: a graph-theory analysis

D. Lautier & F. Raynaud
University Paris-Dauphine
Objectives

• Empirical study on systemic risk in derivative markets
• Integration as a necessary condition for systemic risk to appear
• Previous literature on integration
• Holistic approach, in three dimensions
  - Observation time
  - Spatial integration
  - Maturity
• Influence of physical as well as derivative markets
Selected markets

- Choice directed by:
  - Concerns about speculation in commodities
    Energy products
  - Development of bio fuels
    Agricultural products
  - Portfolio management / Commodities as a new class of assets
    Financial instruments
  - Markets with the highest transaction volumes

- 14 markets ( > 750 000 daily futures prices (settlement))
- 1998 - 2011
Methodology

- Huge volume of data
- 3 dimensional analysis: complex evolving system
- Use of methods originated from statistical physics
- Graph-theory
- Full connected graph: all possible connections between N nodes (time series of price returns) with \((N(N-1)/2)\) links
- Filtered graph: Minimum Spanning Trees (MST)
1. Minimum Spanning Trees (MST)
2. Topology of the MST
   2.1 Maturity dimension
   2.2 Spatial dimension
   2.3 Three-D analysis
   2.4 Allometric coefficients
3. Dynamical studies
   3.1 Correlation coefficients
   3.2 Node’s strength
   3.3 Normalized tree’s length
   3.4 Survival ratios
   3.5 Pruning the trees
1. Minimum spanning trees

- Synchronous correlation coefficients $\rho$ of prices returns $r$:

$$
    r_i = \frac{\ln F_i(t) - \ln F_i(t - \Delta t)}{\Delta t}
$$

$$
    \rho_{ij}(t) = \frac{\langle ri \cdot r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\langle r_i^2 \rangle - \langle r_i \rangle^2} \sqrt{\langle r_j^2 \rangle - \langle r_j \rangle^2}}
$$

- With: $F(t)$, futures prices at $t$
- Correlation matrix $C$, $(N \times N)$, symmetric
From correlation to distances

- Non linear transformation
- Distances $d$ between two nodes defined as follows:
  \[ d_{ij} = \sqrt{2(1 - \rho_{ij})} \]

- Distance matrix $D$, ($NxN$)
- Full connected graph
  - represents all the possible connections between $N$ nodes (vertices)
  - can be weighted (by the distances)
Minimum spanning tree

• All the nodes of the graph are spanned
• No loops
• Result: links of the MST are a subset of the initial graph
• The information space is reduced from \((N(N-1)/2)\) to \((N-1)\)
• In this study: shortest path linking all nodes
  Easiest path for the transmission of prices shocks
2. Topology of the MST

2.1 Maturity dimension
2.2. Spatial dimension
2.3. Three-D analysis
2.4. Allometric coefficients
2.1. The maturity dimension

- General comments (heating oil)
- Evolution of integration through time:
  - The case of the eurodollar
  - The case of the American gaz market
Maturity dimension

Heating oil – Month 1 to 18

Samuelson effect
Evolution of the integration through time
Eurodollar, 1998-2001
Evolution of the integration through time, Eurodollar, 2001-2004
Evolution of the integration through time
Eurodollar, 2004-2009
Evolution of the integration through time, US natural gas, 2001-2004
2.2. Spatial dimension

Agriculture
- Wheat
- Corn
- Soybean

Energy
- Crude, US
- Crude, UK
- Heating oil, US
- Nat. Gas, US
- Nat. Gas, UK
- Gas oil, UK

Finance
- Exchange rates
- Interest rates

Other:
- S&P 500
- Soy oil
- Gas oil, UK

Commodities:
- Wheat
- Corn
- Soybean
- Oil
- S&P 500
- Crude, US
- Crude, UK
- Heating oil, US
- Nat. Gas, US
- Nat. Gas, UK
- Gas oil, UK
2.4 Allometric coefficients

- Quantifying the degree of linearity in the tree
- Maturity, Spatial, 3-D

- The root is the node with the highest connectivity
- Starting from this root, two coefficients $A_i$ and $B_i$ are assigned to each node $i$:

\[
A_i = \sum_j A_j + 1 \quad \text{and} \quad B_i = \sum_j B_j + A_i
\]

\[
B \sim A^\eta
\]

Where $\eta$ is the allometric exponent

$\eta$ stands between 1+ (star-like) and 2- (chain-like)
<table>
<thead>
<tr>
<th>Name</th>
<th>Static coefficient</th>
<th>Dynamical coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>IED</td>
<td>1.927 ± 0.056</td>
<td>1.913 ± 0.011</td>
</tr>
<tr>
<td>LNG</td>
<td>1.874 ± 0.002</td>
<td>1.886 ± 0.059</td>
</tr>
<tr>
<td>LLE</td>
<td>1.88 ± 0.003</td>
<td>1.943 ± 0.02</td>
</tr>
<tr>
<td>NNG</td>
<td>1.75 ± 0.037</td>
<td>1.774 ± 0.018</td>
</tr>
<tr>
<td>LLC</td>
<td>1.889 ± 0.003</td>
<td>1.904 ± 0.095</td>
</tr>
<tr>
<td>NCL</td>
<td>1.994 ± 0.045</td>
<td>1.906 ± 0.013</td>
</tr>
<tr>
<td>NGC</td>
<td>1.732 ± 0.092</td>
<td>1.908 ± 0.013</td>
</tr>
<tr>
<td>CBO</td>
<td>1.889 ± 0.003</td>
<td>1.886 ± 0.032</td>
</tr>
<tr>
<td>CS</td>
<td>1.848 ± 0.095</td>
<td>1.822 ± 0.095</td>
</tr>
<tr>
<td>CW</td>
<td>1.864 ± 0.13</td>
<td>1.761 ± 0.125</td>
</tr>
<tr>
<td>CC</td>
<td>1.88 ± 0.003</td>
<td>1.834 ± 0.024</td>
</tr>
<tr>
<td>Spatial</td>
<td>1.493 ± 0.056</td>
<td>1.621 ± 0.024</td>
</tr>
<tr>
<td>3-D</td>
<td>1.757 ± 0.023</td>
<td>1.85 ± 0.009</td>
</tr>
</tbody>
</table>
3. Dynamical studies

3.1. Correlation coefficients
3.2. Node’s strength
3.3. Normalized tree’s length
3.4. Survival ratios
3.5. Pruning the trees

Rolling time windows : 480 trading days
3.1. Mean correlations and their variances (3-D)
3.2. Node’s strength

- Full connected graph
- The node’s strength $S_i$ indicates the closeness of one node $i$ to all others:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}$$
3.3. Normalized tree’s length

- Sum of the lengths of the links belonging to the MST:

\[ L(t) = \frac{1}{N-1} \sum_{(i,j) \in MST} d_{ij} \]

- The more the length shortens, the more integrated the system is.
3.4. Survival ratios

- Robustness of the topology over time
- The survival ratio $S_R$ refers to the fractions of edges in the MST, that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} |E(t) \bigcap E(t-1)|$$

$E(t)$ : set of edges at date $t$
Survival ratios in the spatial dimension
Length of the trees and survival ratios in the maturity dimension
3.5. Pruning the trees

• Analysis of inter-market and inter-sectors reorganizations
• Consider only the links between markets, whatever the maturity is considered
• Survival ratios on the basis of market links, in the MST
  - Robustness of the topology over time
  - The survival ratio $S_R$ refers to the fraction of links that survives between two consecutive trading days:

\[
S_R(t) = \frac{1}{N-1} \left| E(t) \cap E(t-1) \right|
\]

$E(t)$ : set of market links at date $t$
Pruned trees, length and survival ratios
Length of the re-configurations
Pathological configuration: an example
Links responsible for reorganizations

(a)

(b)

(c)
Main results & conclusions

1. Topology
   - Chain-like trees in the maturity dimension
   - Star-like trees in the spatial and 3-D dimensions

2. Emerging taxonomy
   - Trees organized around the three sectors of activity
   - Center of the graph: two crude oils

3. Integration
   - Increases in all dimensions (spatial, maturity, 3D)
   - Progresses at the heart of the system

Text available on ssrn:

21/07/2011 Université d’été - Strasbourg