Vision conditionnelle du monde dans les stress tests et révision des hypothèses actuarielles

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Flashback: Examen ERM 2012

SCR cat vie

- Choc: accroissement instantané (non permanent) de 0,15% du taux de mortalité (pendant 12 mois)

- Si décès de l’assuré génère une rente de conjoint survivant ou une rente éducation,

  le choc s’applique-t-il aux rentes (avec du coup un effet atténuation du choc)?

Quelques éléments de réponse

- Choc calibré sur risque pandémie.
- Risque nucléaire mentionné.
- Présence de l’expression « par exemple »

- *The instantaneous increase in mortality rate shall only apply to those policies for which an increase in mortality rates which are used to reflect the mortality experience in the following 12 months leads to an increase in technical provisions.*

  Page 220 of Technical Spec. For Prep. Phase (Part I), EIOPA-14/209, 30 avril 2014

Plus généralement, dans un stress test,

- Comment simuler le « monde conditionnel »?

- Réponse en proba: conditionner sachant que X=x n’est pas pertinent.

- Il est plus pertinent de conditionner par X est pire que x.

- Lien avec allocation de capital économique avec méthode d’Euler:
  - Fait du sens avec TVaR
  - Pas avec VaR

Exemple: Inconvénient de la Co-VaR

- Co-VaR: proposée par Fed il y a quelques années pour mesurer la sensibilité d’une banque au risque systémique
- Idée: sachant que le marché (banques) est dans un état désastreux, que devient la VaR conditionnelle de notre banque?
- Etat désastreux: représenté par le marché est (exactement!) à sa VaR

How to measure sensitivity to systemic risk?

To measure what would happen if the hedge fund market or if the investment banking sector gets into trouble, several approaches may be proposed, including:

- if there are a few big players, remove one big player from the market and see what happens (Lehmann-type scenario): some institutions may have the feeling to be hedged against some risk despite the risk that the counterpart disappears. Example: portfolio of long and short positions into interest rate swaps (with maturities 30 years, 29 years, and so on...). The regulator needs to be able to determine what would happen...
- In addition to the institution's "stand-alone" Value-at-Risk, ask all institutions to report their exposure towards systemic risk. The Federal reserve proposed to measure this by the means of the so-called Co-Value-at-Risk (Co-VaR) or Co-Expected Shortfall (Co-ES).

Co-Value-at-Risk

- For $\alpha \in (0,1)$, define the α-Co-Value-at-Risk of Y given X as

$$CoVaR_\alpha(Y \mid X) = VaR_\alpha(Y \mid X = VaR_\alpha(X)).$$

- Using the copula approach, with $U = F_X(X)$ and $V = F_Y(Y)$, we have

$$CoVaR_\alpha(Y \mid X) = F_Y^{-1}\left(C_{2|1}^{-1}(\alpha \mid \alpha)\right),$$

where C is the copula of X and Y and where

$$C_{2|1}(v \mid u) = \frac{\partial C}{\partial u}(u, v)$$

is the conditional distribution of V given that $U = u$.

- The idea is the following: if Y corresponds to wealth of institution A and if X corresponds to a market index, $CoVaR_\alpha(Y \mid X)$ is the conditional α-VaR of institution A given that the market index is at its α-VaR: if α is small, this corresponds to a conditional VaR in a market distress scenario.

Co-Value-at-Risk

- For $\alpha \in (0,1)$, define the α-Co-Value-at-Risk of Y given X as

$$CoVaR_\alpha\left(Y \mid X\right) = VaR_\alpha\left(Y \mid X = VaR_\alpha(X)\right).$$

- The goal is to take into account systemic risk: if institution A is hedged against some risks using the market, then X and Y should be positively correlated and we expect Co-VaR to take this into account by increasing the risk capital needed to control it (in comparison to the case where X and Y are independent).
- Problem: this is not the case!

Co-Value-at-Risk: Clayton copula

- The Clayton copula with parameter $\theta > 0$ is given by

$$C_\theta(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}}.$$

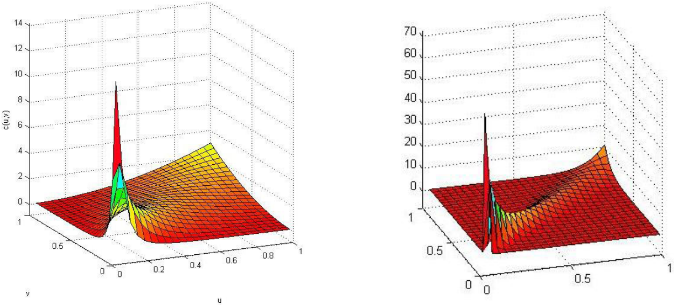
- Keeping in mind that

$$CoVaR_\alpha\left(Y \mid X\right) = F_Y^{-1}\left(C_{2|1}^{-1}(\alpha \mid \alpha)\right),$$

we can compute explicitly $C_{2|1}^{-1}(\alpha \mid \alpha)$ for the Clayton copula:

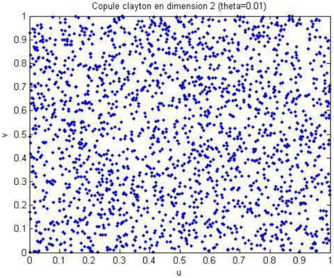
$$C_{2|1}^{-1}(\alpha \mid \alpha) = \left[\alpha^{-\theta}\left(\alpha^{-\frac{\theta}{\theta+1}} - 1\right) + 1\right]^{-\frac{1}{\theta}}.$$

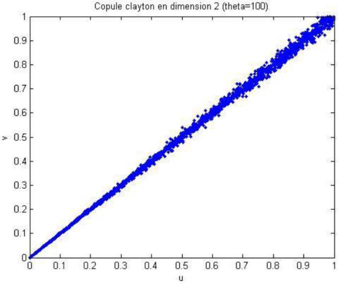
Clayton copula



Density of the Clayton copula, $\theta = 2$ (left) and $\theta = 100$ (right).
Correlation increases with θ.

Clayton copula





Simulations of the Clayton copula, $\theta = 0.01$ (left, close to independence) and $\theta = 100$ (right, close to comonotonicity).

Co-Value-at-Risk: Clayton copula

- $$CoVaR_\alpha\left(Y \mid X\right) = F_Y^{-1}\left(C_{2|1}^{-1}(\alpha \mid \alpha)\right),$$
- We can compute explicitly $C_{2|1}^{-1}(\alpha \mid \alpha)$ for the Clayton copula:

$$C_{2|1}^{-1}(\alpha \mid \alpha) = \left[\alpha^{-\theta}\left(\alpha^{-\frac{\theta}{\theta+1}} - 1\right) + 1\right]^{-\frac{1}{\theta}}.$$

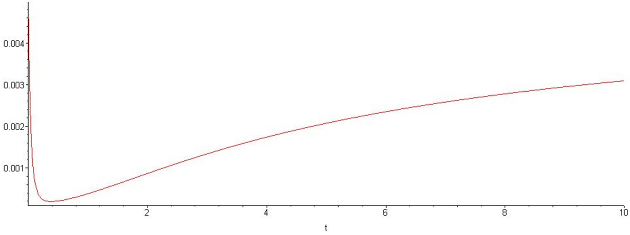
- Problem: as $\theta \to 0$ and as $\theta \to +\infty$, we have

$$C_{2|1}^{-1}(\alpha \mid \alpha) \to \alpha.$$

- This means that if Y is independent from the market index, or if Y is perfectly correlated with the market index X, then

$$CoVaR_\alpha\left(Y \mid X\right) = VaR_\alpha(Y).$$

Co-Value-at-Risk: Clayton copula



β such that $CoVaR_\alpha(Y \mid X) = VaR_\beta(Y)$ as a function of correlation parameter θ for the Clayton copula. The lower β, the higher the required capital.

Conclusion: one must condition w.r.t. $X < VaR_{\alpha'}(X)$ instead of $X = VaR_\alpha(X)$.

Et pour des stress tests:

- Faut-il / Devrait-on / Peut-on réestimer le SCR ?
- Probabilités et périodes de retour réévaluées:
  - 25 déc. 1999: Période de retour d'un événement comme Lothar dans un certain modèle CAT = 120 ans
  - Le lendemain soir: on ne sait plus!
  - Janvier 2000: Période de retour d'un événement comme Lothar = 70 ans
  - A. Mornet, M. Luzi, T. Opitz, S. Loisel, **Construction of an Index that links Wind Speeds and Strong Claim Rate of Insurers after a Storm in France**, *Risk Analysis* (2015).
- Pour d'autres risques: retour à la moyenne -> réduction du risque en univers choqué
- Loss absorbing capacity!!!
- Article en préparation avec F. Borel-Mathurin (ACPR) et J. Segers (Louvain)

Partie 2: Révision des hypothèses actuarielles

- Supposons qu’on observe une déviation de la fréquence de sinistres ou d’autres événements par rapport aux hypothèses actuarielles.
- D’un point de vue statistique, comment réagir le plus vite possible sans faire trop de fausses alarmes?