INSTITUT DES ACTUAIRES

Lyon 1

I.S.F.A.

Mémoire présenté le :

pour l'obtention du Diplôme Universitaire d'actuariat de l'ISFA et l'admission à l'Institut des Actuaires

Par : Burhan Butun

Titre Property Excess of Loss Reinsurance Pricing in the Lloyd's market

Confidentialité : ☐ NON ☒ OUI (Durée : ☐ 1 an ☒ 2 ans)

Les signataires s'engagent à respecter la confidentialité indiquée ci-dessus

Membre présents du jury de l'Institut des Actuaires signature

Entreprise :ANV Syndicates

Nom :

Signature :

Directeur de mémoire en entreprise :

Nom : Alois Rouffiac

Signature :

Invité :

Nom :

Signature :

***Autorisation de publication et de mise en ligne sur un site de diffusion de documents actuariels** (après expiration de l'éventuel délai de confidentialité)*

Signature du responsable entreprise

Signature du candidat

Membres présents du jury de l'ISFA

Secrétariat

Bibliothèque :

Institut de Science Financière et d'assurances

Property Excess of Loss Reinsurance Pricing in the Lloyd's market

Burhan Butun

Actuarial Dissertation

Tutors: Stephane Loisel (ISFA) & Alois Rouffiac (ANV)

Acknowledgement

I would like to thank ANV Syndicates for allowing me to undertake actuarial studies in France and the support provided to me during my studies at the Institute of Financial and Insurance Sciences over the last 2 years.

I would like to thank Alois Rouffiac, Head of Pricing, for his availability and support.

I would like to thank Stephane Loisel and the teachers of ISFA for the quality of their teachings and feedbacks.

Abstract

The managing agent ANV operates in the Lloyd's market and writes excess of loss reinsurance (XoL RI) contracts through its syndicate 1861. The Lloyd's market operates on a subscription basis. Syndicates participates in a share of the risk. ANV syndicate 1861 therefore takes a share of the XoL RI contracts placed in the Lloyds market with other syndicates. In most of the cases ANV syndicate 1861 is a follower and on a few occasions a lead. XoL RI has the particularity of presenting a few losses and can also be very volatile. As a consequence it is hard for underwriters to make a decision on whether to write them or not. The aim of this work is to build a pricing tool to assist the underwriters in their decision making.

I. Introduction to the Lloyd’s market

A. Lloyd’s Brief History

In the 17th century, London's importance as a trade centre led to an increasing demand for ship and cargo insurance. Edward Lloyd's coffee house became recognised as the place for obtaining marine insurance and this is where the Lloyd’s that we know today began.

From those beginnings in a coffee house in 1688, Lloyd’s has been a pioneer in insurance and has grown over 325 years to become the world’s leading market for specialist insurance.

<u>1871 First Lloyd’s Act</u>

The first Lloyd's Act was passed in Parliament incorporating the Society of Lloyd’s as a statutory corporation for the first time and, making it illegal for anyone not a recognised Lloyd’s underwriting member to sign his name to a Lloyd’s policy.

<u>1877 The talented visionary</u>

Non-marine policies were introduced to Lloyd’s by Cuthbert Heath, one Lloyd’s most famous and illustrious members. Throughout this decade and for years to come, Heath would forge a brand new, highly adventurous path for Lloyd’s, and establish an astonishing presence for the Society in America and on a global scale.

<u>1905 Risk based pricing</u>

By 1905, non-marine insurance had become embedded in Lloyd‘s. But this period was also noteworthy as the birth of risk based pricing.

Underwriters Cuthbert Heath and Christopher Head began to collect a wide range of data on Gulf of Mexico hurricanes, study it in detail, and identify the exact level of risk based on what they’d found. They did the same for earthquakes – Heath’s Earthquake Book can still be found in Lloyd‘s treasures. Heath was scrupulous, continuing to bring together a huge wealth of information on any risk he considered underwriting.

<u>1903 An even more international Lloyd’s</u>

Amidst all this, Heath found time to give delegated authority to Alfred Schroder in Amsterdam to write insurance on his behalf – the first time such a thing had been done. By the 1930s, Heath had an international contracts department, delegating to agents in India, New Zealand, Belgium, Denmark and Norway. In 1930, Heath sent John Cope to Shanghai, Calcutta, Alexandria and Athens; in some cases, the first agent from London to visit these places. Eventually, Lloyd’s would be licensed to accept business from more than 200 countries and territories worldwide.

1923 Harrison's Folly – and the creation of the Central Fund

The Lloyd's of today is financially sound – but in addition to this, policyholders have the added security of knowing that, should an underwriter or syndicate fail and be unable to pay its claims, there is a central fund to fall back on.

The Central Fund is a fund of last resort. It comes into play if a member has insufficient assets to cover a claim on an individual basis; in this case, and at the discretion of the Council of Lloyd's, assets in the Central Fund can be allocated to cover that member's liabilities.

The idea for the Central Fund came about after underwriter Stanley Harrison had pursued a complex motor/credit insurance line and run up debts of over £360,000. He confessed to the Chairman, Raymond Sturge, who called a meeting, telling underwriters that if Harrison's debts weren't paid in full, the name of Lloyd's 'will never recover in our lifetime'. For the first time, the principle of mutuality appeared, the combined members agreeing unanimously to pay a share of the debts proportionate to their premium income. Shares ranged from £10,000 to eight pence. Sturge described it as an 'heroic conclusion'. Harrison's Folly had laid the foundations for what came to be known as the Central Fund, which was officially created four years later.

1965 Further international expansion

Lloyd's first admitted foreign members, although the far-sighted Cuthbert Heath had proposed American and French underwriters in the pre-war era.

Around now, the jet age began to change the way business was conducted, and the market rapidly became more international. Just one example: cover for the Delaware River Port Authority was brokered in the 1960s at Washington's Dulles airport – the London broker arriving and leaving on the same Concorde plane.

There were eye-opening South American risks, too – such as the giant hydroelectric Salto Grande Dam on the Argentine-Uruguayan border, and the Itaipu dam on the border of Brazil and Paraguay.

1985 An eye to the East

Peter Miller became the first Lloyd's chairman to visit China. Brokers accompanied him; early reinsurance included nuclear power stations.

1980s and 1990s Scandal and fraud

At this time, the Society entered the most turbulent and traumatic time in its history, facing up to a series of problems which would, in time, lead to the birth of the modern, robust and secure Lloyd's we know today. Lloyd's had long known that some types of cover if offered were very risky. Equally, Names knew that being an underwriter at Lloyd's involved unlimited liability for the risks they underwrote - and that meant putting at risk everything they had. But suddenly, unexpectedly large legal awards made in US courts on asbestos, pollution and health hazard claims, some dating back 40 years or more, resulted in huge losses to Names. Many Names suffered severe financial loss and unfortunately, some faced bankruptcy.

The financial challenges faced by the market were compounded when, between 1987 and 1989, a series of gigantic oil, wind and fire claims, including the loss of the North Sea oil rig Piper Alpha, came into Lloyd's. Costs were in their billions.

<u>1993 The game changer – and saviour</u>

David Rowland was appointed as first full-time remunerated Chairman of Lloyd's, initiating sweeping reforms to save Lloyd's after the catastrophes that had threatened its collapse. He spearheaded the establishment in 1996 of Equitas, a special vehicle into which all pre-1993 business would be transferred by reinsurance-to-close – at a cost of over $21bn. There were losses for many Names, but Lloyd's had survived.

<u>1994 Corporate members introduced</u>

The first corporate members began underwriting – with capacity of £1,595 million.

<u>1998 Financial Services authority introduced</u>

On 21 January, the government announced that Lloyd's would no longer be self-regulating but would be subject to the oversight of the new Financial Services Authority, effective from midnight on 30 November 2001.

<u>2003 Creation of Realistic Disaster Scenarios</u>

The establishment of the Franchise Board and the appointment of Rolf Tolle as Lloyd's first Franchise Performance Director. Tolle set about developing a range of information tools to gauge Lloyd's performance and identify trends across the wider insurance market. Minimum underwriting standards were laid down and a number of new risk management procedures introduced.

One of these innovations was the creation of Realistic Disaster Scenarios (RDS), in which syndicates would be required to model their expected losses in the event of a range of major disasters, such as a Japanese earthquake, a US hurricane or an act of terrorism, to ensure they hadn't taken on too much exposure to a single event.

<u>2012 Vision 2025 launched</u>

In May 2012, Prime Minister David Cameron visited Lloyd's to help launch Vision 2025 – a brand new strategy for its further development, positioning Lloyd's to take advantage of opportunities presented by the world's developing economies.

Central to Vision 2025 is the need for Lloyd's to be larger than today, so that it can target profitable growth from both developing and developed economies. The aim is to ensure that Lloyd's remains the global centre for specialist insurance and reinsurance.

B. The Lloyd's market today

Nowadays Lloyd's accepts business from over 200 countries and territories worldwide, supported by a network of local offices and cover holders across the world.

The Lloyd's market is one of the top non-life reinsurer in the world, ranked 3rd based on 2012 gross reinsurance premiums written[1].

Top 10 Global Non-Life Reinsurance Groups

Ranked by gross non-life premium written in 2012 (USD Millions)

		Reinsurance Premiums Written Non-Life Only						Non-Life Combined Ratios		
2013 Ranking	Company	Gross 2012	Net 2012	Gross 2011	Net 2011	Gross % Change	Net % Change	2012	2011	Change
1	Munich Reinsurance Co.	$22,539	$22,038	$21,441	$20,539	5.1%	7.3%	91.2	114.2	(23.0)
2	Swiss Reinsurance Co. Ltd.	19,468	15,117	17,181	13,571	13.3%	11.4%	83.1	101.6	(18.5)
3	Lloyd's	15,770	11,358	13,621	10,015	15.8%	13.4%	91.0	130.6	(39.6)
4	Hannover Rueckversicherung AG	10,201	9,060	8,840	7,719	15.4%	17.4%	96.0	104.5	(8.5)
5	Berkshire Hathaway Inc.	9,668	9,668	9,867	9,867	-2.0%	-2.0%	99.9	99.9	0.0
6	SCOR S.E.	6,146	5,558	5,157	4,650	19.2%	19.5%	94.3	104.5	(10.2)
7	Korean Reinsurance Co.	5,113	3,390	4,551	3,043	12.3%	11.4%	97.9	102.8	(4.9)
8	Everest Re Group Ltd.	4,311	4,081	4,286	4,109	0.6%	-0.7%	93.8	118.5	(24.7)
9	China Reinsurance (Group) Corp.	4,184	4,090	3,652	3,526	14.6%	16.0%	100.4	100.1	0.3
10	PartnerRe Ltd.	3,910	3,768	3,831	3,688	2.1%	2.2%	87.8	125.4	(37.6)

Sources: AM Best data & research, Aug. 26, 2013 Special Report; 2011 figures from A.M. Sept. 3, 2012 report

Notes: NEP used for Hannover; Korean Re information for fiscal year ended Mar. 31, 2013 and 2012

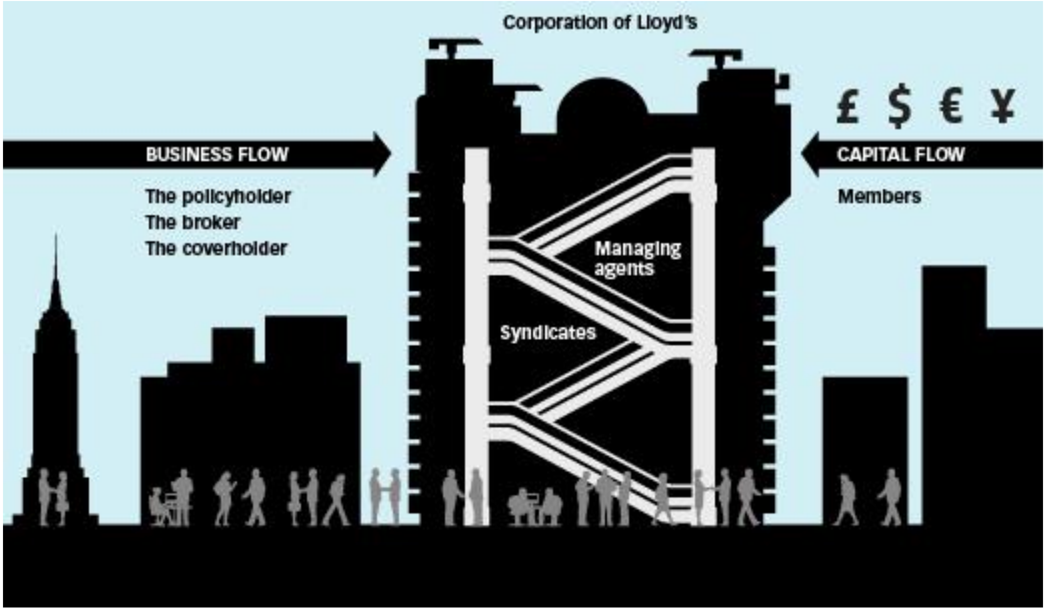
1. How the Lloyd's market works

The Lloyd's market is home to 56 managing agents and 91 syndicates, which offer an unrivalled concentration of specialist underwriting expertise and talent. Business at Lloyd's is still conducted face-to-face. The majority of business written at Lloyd's is placed through brokers who facilitate the risk-transfer process between clients (policyholders) and underwriters. Clients can discuss their risk needs with a broker, a cover holder or a service company.

The diagram below illustrates how the Lloyd's market works[2]:

[1] AM Best data & research, Aug.26,2013 Special report

[2] Definitions of each component of the market can be found in the appendix in the section "How the Loyd's market works"

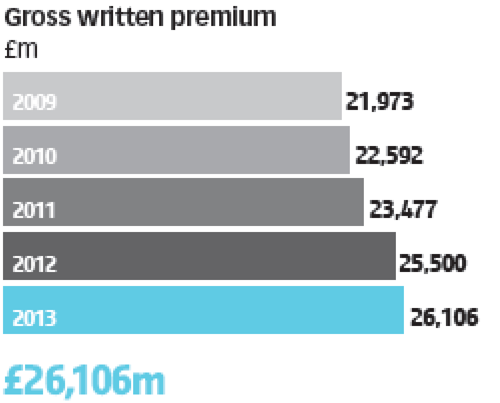


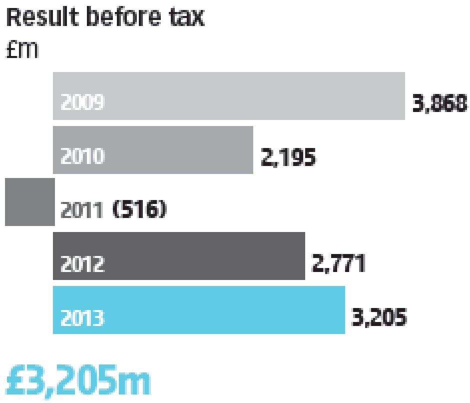
The diagram below illustrates the 56 managing agents that take part in the Lloyd's market.

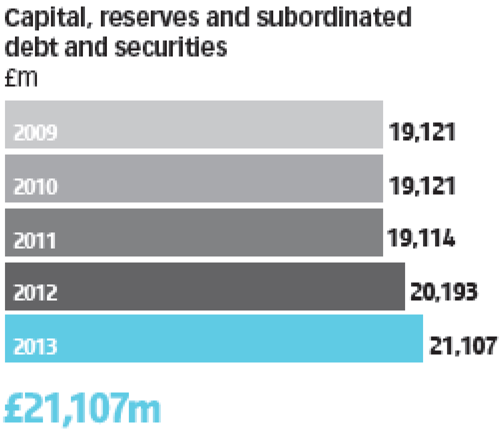
At 31 December 2013, the Lloyd's market consisted of 91 active syndicates (plus five RITC syndicates) managed by the 56 managing agents shown here. The sheer scale of the market offers huge diversity of products; the scope of specialist broking and underwriting expertise found together under the Lloyd's umbrella makes it unique.

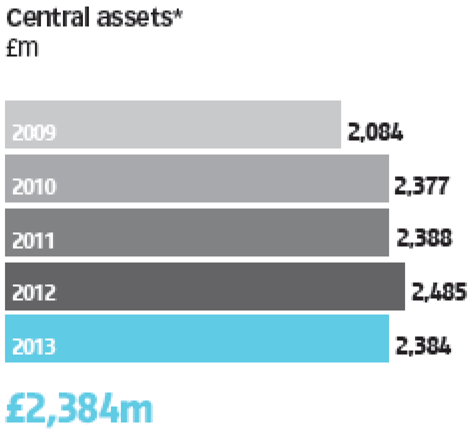
2. Financial highlights

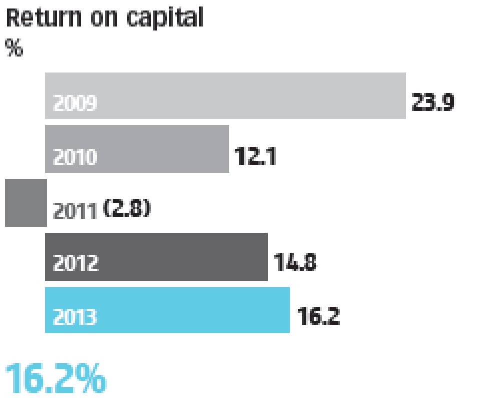
The graphs below show Lloyd's financials for over the last 5 years. Lloyd's made a profit of £3,205m in 2013 (2012: £2,771m) despite a reduction in investment return. In 2013 the combined ratio was of 86.8% (2012: 91.1%). Gross written premium rose by 2.4% in 2013 to £26,106m (2012:£25,500m). The return on capital can be noted as exceptional in comparison to other types of investments.

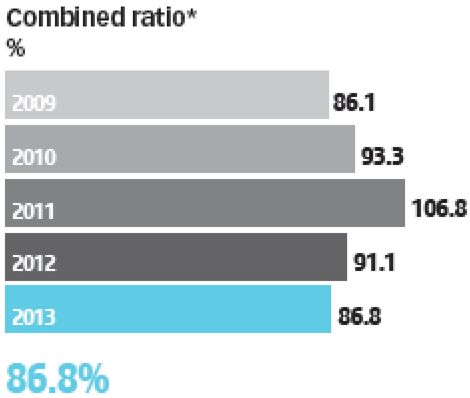












3. Lloyd's financial strength

Lloyd's financial strength derives from its unique capital structure, often referred to as the 'Chain of Security'. It provides excellent financial security to policyholders and capital efficiency for members.

Financial rating

Lloyd's strength and robust capitalisation is reflected in its ratings. Three of the world's leading insurance rating agencies recognise Lloyd's strengths and the financial strength of the market.

All Lloyd's syndicates benefit from Lloyd's central resources, including the Lloyd's brand, its network of global licences and the Central Fund. As all Lloyd's policies are ultimately backed by this common security, a single market rating can be applied. The Lloyd's financial strength ratings apply to every policy issued by every syndicate at Lloyd's since 1993.

The Chain of Security

Lloyd's unique capital structure, often referred to as the Chain of Security, provides excellent financial security to policyholders and capital efficiency to members.

The Corporation is responsible for setting both member and central capital levels to achieve a level of capitalisation that is robust yet also allows members the potential to earn superior returns.

There are three links in the Chain of Security:

- Syndicate level assets
- Members' funds at Lloyd's
- Central assets

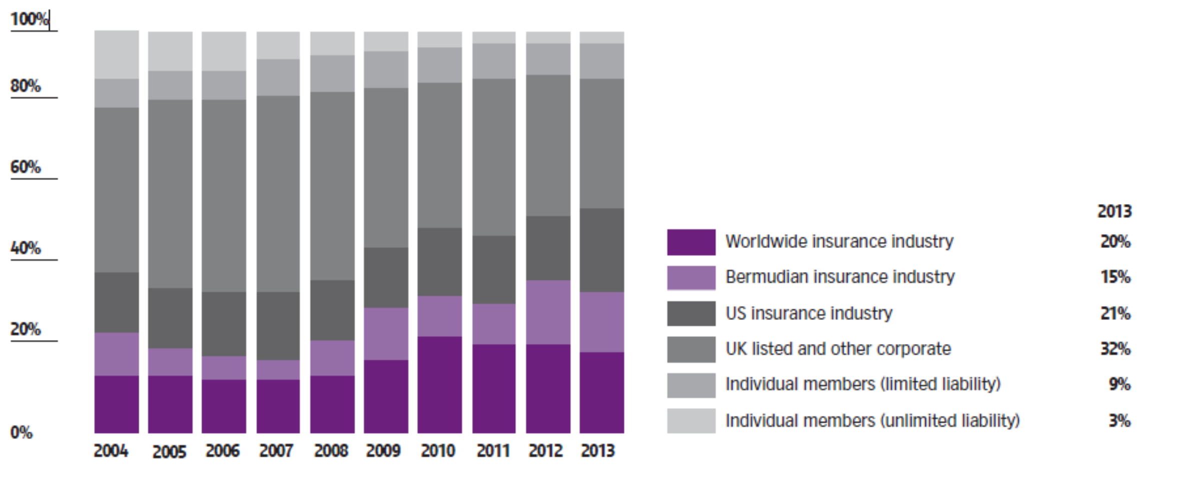
The funds in the first and second links are held in trust, primarily for the benefit of policyholders whose contracts are underwritten by the relevant member. Members underwrite for their own account and are not liable for other members' losses. The third link contains mutual assets held by the Corporation which are available, subject to Council approval, to meet any member's insurance liabilities. The diagram below illustrates the "chain of security" at Lloyds as at 2013:

4. Capital providers

Historically Lloyd's capital providers were individuals, called "names", who were exposed to unlimited liabilities. Due to the scandals of the 80s and 90s where many names went bankrupt, Lloyd's struggled to find individuals who would put their capital at risk; hence corporate members were introduced in 1994.

Nowadays most of the capital providers are corporates. Individuals only represent 12% of the Lloyd's capacity as at 2013 as shown on the graph below.

Lloyd's capacity by source and location (Figure 2)



5. The risks covered in the Lloyd's market

The risks Lloyd's covers can be grouped into 7 main categories: casualty, property, marine, energy, motor, aviation and reinsurance.

<u>Casualty</u>

Casualty risks are particularly specialist and complex and the US accounts for a large proportion of this business. This market includes professional indemnity, medical malpractice, accident and health, directors' & officers' liability and general and employers' liability.

<u>Property</u>

The property sector is hugely varied, encompassing everything from supporting the building of the new World Trade Center to protecting holiday resorts against storm damage. It's also highly competitive.

<u>Marine</u>

Maritime risk is where the Lloyd's story began, over 300 years ago. Today, it's a smaller but still significant part of our business. Most of the cover in this area is for hull, cargo, marine, liability and specie (the insurance of highly valued items, such as fine art, while in transit).

Energy

The energy market is steadily evolving, from onshore and offshore property, oil rigs and refineries to emerging renewable energy ventures.

Motor

In the competitive motor sector, Lloyd’s is focused on company fleet business and non-standard risks. This includes high value, vintage and collectors’ vehicles, high risk drivers and affinity groups. Lloyd’s has insured numerous land speed record attempts and Sir Malcolm Campbell, the first man to break 300mph on land, was a Lloyd’s broker.

Aviation

Lloyd’s is an industry leader within the global aviation market. This includes: airline, general aviation, products, war and terrorist coverage, airports and satellite business. Since 9/11, this sector’s loss experience has been well below the industry’s long-term average. This is thanks to new safety systems, increased security and improved regulation.

Reinsurance

Reinsurance tends to fall into four categories: to protect an insurer against very large claims; to reduce exposure to peaks and troughs; to obtain an international spread of risk; and to increase the capacity of the direct insurer.

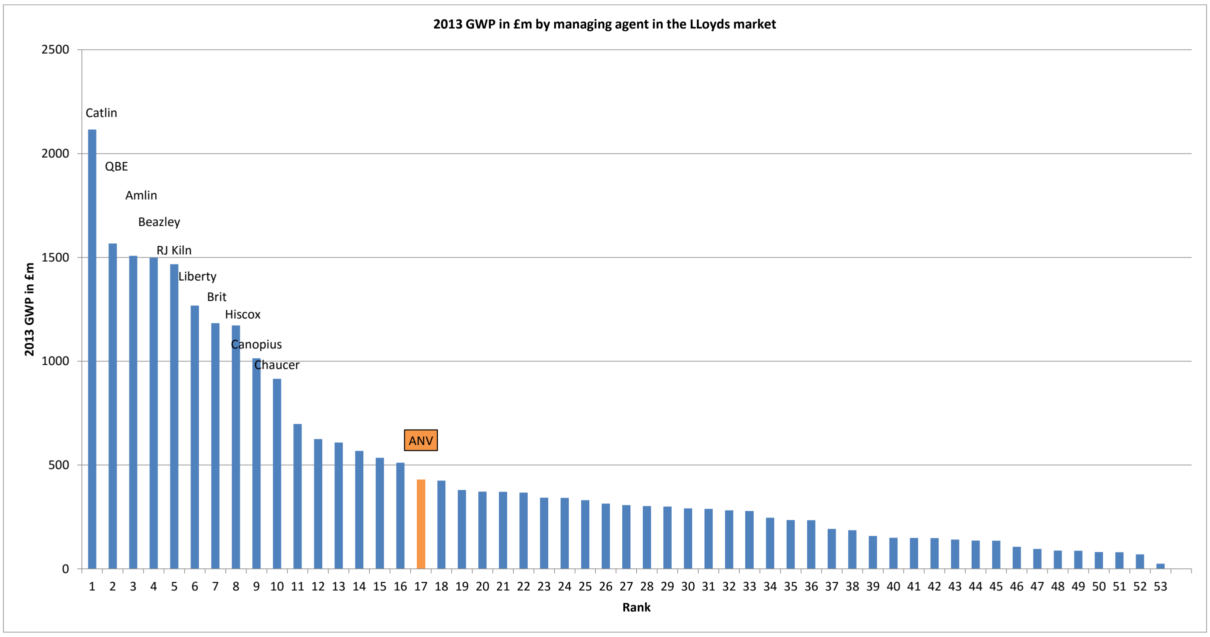
II. About ANV

ANV is structured to operate as an insurer, integrated Lloyd's vehicle, and managing general underwriter (MGU). This three-pillar strategy is designed to build, support, and leverage high quality underwriting through a team of specialists who understand the unique risks of unique markets. Working as a unified and worldwide organisation, ANV is focused on managing business risk in all its forms.

ANV Holdings BV, the parent company of the ANV Group, is a privately held and Dutch registered holding company and its lead investor is the Canadian pension fund Ontario Teachers' Pension Plan.

ANV's Lloyd's managing agency, ASL, manages Syndicates 1861, 5820, and 779 (collectively branded as ANV Syndicates), as well as providing incubator services to Apollo Syndicate 1969, which is expected to move to its own Managing Agency by 2016. ANV Group considers its Lloyd's operation as a key component of its vision and strategy to build a globally integrated specialty insurance and reinsurance company. Its Lloyd's platform allows ANV the opportunity to continue to grow its business profitably and complements its other specialty operations in offering a broad range of products and underwriting expertise.

As of 2013, ANV is the 17th biggest managing agent operating within the Lloyd's market as shown on the chart below.



To build its Lloyd's business, ANV acquired Flagstone Syndicate Management Limited (Subsequently renamed ANV Syndicate Management Limited – "ASML") in 2012, and in December 2013 completed the acquisition of Jubilee Managing Agency Limited (subsequently renamed ANV Syndicates Limited – "ASL"). Following the novation of Syndicates 1861 and 1969 in April 2014, all four Syndicates are now managed by ASL. With managed capacity in 2014 of over £400 million (including Apollo), ASL benefits from greater scale, as well as diversification through a broader range of product lines, all of which improve its value proposition to brokers, clients, and capital providers. ANV Group will continue to strengthen and build its Lloyd's franchise significantly over the years to come.

As noted above, ASL's strategy is to concentrate on up to twelve core lines of business in which it has the profile and capability to lead business. The table below shows the distribution of ASL's core lines (and their respective sub-classes) across the three ANV Syndicates.

Core Line of Business	Sub-Classes Syndicate 1861	Syndicate 5820	Syndicate 779
Aviation	• General Aviation • Apollo Airline & Aerospace Consortium • Satellites		
Energy	• Energy Onshore • Energy Offshore (ex GoM wind) • Energy (GoM wind)		
Property	• Property D&F • Property Binders	• Property D&F • Property Binders	
Political Risks	• Political Risks & Violence	• Political Risk & Credit	
Professional Lines	• Directors & Officers Liability • Professional Indemnity • Bankers Blanket Bond • Commercial Crime • Employment Practices Liability		
Cyber	• Physical damage & business interruption • Liability • Crime		
Marine	• Cargo & Specie • Marine & Energy Liability • Aquaculture • Miscellaneous Marine		
Life, Accident & Health	• Accident & Health • Kidnap & Ransom • Contingency	• Accident & Health • Kidnap & Ransom	• Life
Non-Marine Liability	• Excess Casualty • AEGIS consortium	• Nutraceutical • Commercial General Liability Binders	
Reinsurance	• Marine • Aviation • A&H • Political Risks		
Consumer Products		• Warranty • Creditor • Mortgage Indemnity	

III. Definition of Reinsurance

A. Definition

A brief definition of reinsurance is given by the Lloyd's glossary as follow:

A contract under which a reinsurer agrees to pay specified types and amounts of underwriting loss incurred by an insurer or another reinsurer in return for a premium. Reinsurance serves to 'lay-off' risk. Reinsurance may be proportional or non-proportional and may take the form of a cover in respect of an individual risk exposure (see facultative risk) or cover in respect of multiple risk exposures (see treaty). Reinsurance accounts for more than half of Lloyd's total business.

Another interesting definition is given by David R. Clark in his paper "Basics of Reinsurance Pricing":

We can define reinsurance as a mechanism for spreading risk. A reinsurer takes some portion of the risk assumed by the primary insurer (or other reinsurer) for premium charged. A major difference between reinsurance and primary insurance is that reinsurance program is generally tailored more closely to the buyer; there is no such thing as the "average" reinsured or the "average" reinsurance price. Each contract must be individually priced to meet the particular needs and risk level of the reinsured. This leads to what might be called the pricing paradox:

If you can precisely price a given contract, the ceding company will not want to buy it.

It means that if the historical experience is stable enough to provide data to make a precise expected loss estimate, then the reinsured would be willing to retain that risk. As such any pricing tools are usually only a starting point in determining an adequate premium. The actuary and the underwriter have to understand when the assumptions in the ratemaking process are not met and know how to supplement the results with additional adjustments and judgement.

B. Functions of Reinsurance

The most common reasons for purchasing reinsurance include:

- **Capacity Relief** - It allows the reinsured to write larger amounts of insurance.
- **Catastrophe Protection** - It protects the reinsured against a large single, catastrophic loss or multiple large losses.
- **Stabilization** - It helps smooth the reinsured's overall operating results from year to year.
- **Surplus Relief** - It eases the strain on the reinsured's surplus during rapid premium growth.
- **Market Withdrawal** - It provides a means for the reinsured to withdraw from a line of business or geographic area or production source.
- **Market Entrance** - Helps the reinsured spread the risk on new lines of business until premium volume reaches a certain point of maturity; can add confidence when in unfamiliar coverage areas.
- **Expertise/Experience** - It provides the reinsured with a source of underwriting information when developing a new product and/or entering a new line of insurance or a new market.

C. Forms of Reinsurance

Different forms and types of reinsurance contracts exist. Each form of reinsurance contract defines the rules around how the risk can be ceded to the reinsurer. There are 3 forms of reinsurance which are Treaty, Facultative and Facultative Obligatory Treaty. A definition of each form is given in the sections below.

- **Treaty Reinsurance** - A reinsurance contract under which the reinsured agrees to offer and the reinsurer agrees to accept all risks of certain size within a defined class.

- **Facultative Reinsurance** - A reinsurance risk that is placed by means of separately negotiated contract as opposed to one that is ceded under a reinsurance treaty.

- **Facultative Obligatory Treaty Reinsurance** - A reinsurance contract which allows the reassured to select which risks of a given type are to be ceded to the reinsurer. The reinsurer is obliged to accept all the cessions made by the reassured provided they fall within the scope of the treaty.

D. Types of Reinsurance

Each form of reinsurance contract can exist under 4 different types which are Quota Share, Surplus, Excess of Loss and Stop Loss. Each type of reinsurance contract has different rules and mechanism in places to calculate the amount of premium the reinsured has to pay to the reinsurer and the amount of recoveries the reinsurer has to pay to the reinsured. A definition of each type is given in the section below. These 4 types can be summarised as shown in the diagram below:

<table>
<tr><td colspan="4">Reinsurance Types</td></tr>
<tr><td colspan="2">Non proportional</td><td colspan="2">Proportional</td></tr>
<tr><td>Excess of Loss</td><td>Stop Loss</td><td>Quota Share</td><td>Surplus</td></tr>
</table>

- **Non-Proportional Reinsurance** - Non-Proportional Reinsurance is a type of reinsurance in which the reinsurer does not share similar proportions of the premiums earned and the claims incurred by the reassured plus certain associated expenses. Excess of loss reinsurance is an example of non-proportional reinsurance.
    - ***Excess of Loss*** - A type of reinsurance that covers specified losses incurred by the reassured in excess of a stated amount (the excess) up to a higher amount, for example £5 million excess of £1 million. An excess of loss reinsurance is a form of non-proportional reinsurance.
    - ***Stop Loss*** - Stop Loss is also known as excess of loss ratio reinsurance. This is a form of excess of loss reinsurance which provides that the reinsurer will pay some or all of the reassured’s losses in excess of a stated percentage of the reassured’s premium income in respect of its whole account or a specified account, subject (usually) to an overall limit of liability which may be expressed as a percentage of the relevant premium income or an amount.

- **Proportional Reinsurance** - Proportional Reinsurance is a type of reinsurance in which the reinsurer shares similar proportions of the premiums earned and the claims incurred by the reassured plus certain associated expenses. Quota share treaties and surplus line treaties are examples of proportional reinsurance.
    - ***Quota Share*** - A reinsurance treaty which provides that the reassured shall cede to the reinsurer a specified percentage of all the premiums that it receives in respect of a given section or all of its underwriting account for a given period in return for which the reinsurer is obliged to pay the same percentage of any claims and specified expenses arising on the reinsured account.
    - ***Surplus*** - A type of reinsurance under which bands of cover known as lines are granted above a given retention which is referred to as the cedant's line. Each line is of equivalent size and the capacity of the treaty is expressed as a multiple of the cedant's line so that with a retention of £2 million, a three line treaty would provide reinsurance cover of £6 million (£2 million X 3) excess of £2 million. The reinsurer receives an equivalent proportion of the full risk premium. A surplus treaty is a form of proportional reinsurance.

E. Basis of Reinsurance

The basis of a reinsurance contract refers to the period the contract is applicable to. There are 2 common ways of defining which losses fall within the contract period.

- **Loss Occurring During (LOD)** - LOD basis, does exactly what it says and states that the contract will respond to any losses that occur within the contract period. In property insurance it is quite straightforward, because the losses generally start at a precise time. LOD in Liability contracts can be a little trickier, because some losses cannot be attributed to a sudden event. A good example is seen in Employers' Liability, where an employee could suffer a work-related illness as a result of long-term exposure to hazardous substances or working practices.

- **Risk Attaching During (RAD)** - RAD contracts will cover all policies that incept during the contract period, irrespective of when the losses occur. Depending on how the original policies are worded, the losses could emerge several years after the policy itself has expired.

F. Features of excess of loss

Excess and Limit

An excess of loss reinsurance contract is primarily defined by its attachment point or excess point and its limit. For example a \$5m xs \$5m layer means that the contract will protect the reinsured for each and every loss that is in excess of \$5m and less than \$10m. The recoveries are calculated according to the following formula:

$$\boxed{Recoveries = Min(Max(X - E, 0), L)}$$

Where E: Excess point; L: Limit; X: Loss Amount of a single claim.

ROL and Return Period

An XoL layer is often priced using the terminology ROL (Rate On Line) or return period. For example a 10% ROL for a \$5m xs \$5m layer means that the premium to pay for the reinsured to the reinsurer is \$500,000. The return period is defined as: $\boxed{Return\ Period = \frac{1}{ROL}}$

It basically means that for a 10% ROL, once in 10 years the layer will be totally burnt.

In the Lloyd's market and in general the highest a ROL can be is 45% and in practice a 35% ROL would be the maximum a reinsured would be ready to pay.

Adjustment rate and M&D

A reinsurance contract covers protects against losses arising from an underlying book that the reinsured is writing. As the reinsurance contract covers the prospective year to come and is bought before the reinsured starts writing any business, its price is purely based on an Estimated Premium Income (EPI) provided by the reinsured. The reinsured may write more than what he said he would write. It means that the amount of exposure the reinsured is exposed to is greater hence it is riskier for the reinsurer. Therefore when the contract incept, the reinsured would pay an M&D (Minimum and Deposit) to the reinsurer that is not refundable and is based on the EPI provided by the reinsured. When the contract expires if the reinsured has written more business then additional premium will be paid to the reinsurer. The additional premium is calculated as follow:

$$\boxed{Additional\ Premium = Adjustment\ rate * (Written\ Premium\ Income - EPI)}$$

$$\boxed{where\ Adjustment\ rate = \frac{M\&D}{EPI}}$$

If the reinsured wrote less business than expected then there is no refund of any M&D premium.

AAD (Annual Aggregated Deductible) and AAL (Annual Aggregated Limit)

The amount of recoveries that can be collected by the reinsured are limited in most contract. Most contracts have an AAL (Annual Aggregated Limit). For example a $5m xs $5m contract may have a $15m AAL. It means that if the reinsured has 3 losses and each of them is $10m, he can then recover $15m. However if the reinsured has a fourth losses that is $6m no recoveries will be possible under the contract terms as the AAL would have been exhausted.

The reinsurer may also protect himself and reduce the cost of reinsurance for the reinsured by adding an AAD (Annual Aggregated Deductible). For example a $5m AAD means that the reinsurer will start paying claims only if the recoverable amount is greater than $5m. For example a $5m xs $5m contract may have a $5m AAD and a $15m AAL. If the reinsured has a first loss at $10m no recoveries will be made and the AAD will be exhausted. If a second loss at $10m is reported then $5m recoveries will be made. AAD is often added when the reinsured loss history is poor. It is often added on working layers or bottom layers i.e. layers with a low excess point that attract many losses. AAD helps on these layers to have a reasonable price for the reinsured.

In mathematical words the recoveries can be calculated as follow:

$$Recoveries = min\left[max\left[\left(\sum_{i=1}^{i=n} min(max(X_i - E, 0), L)\right) - AAD, 0\right], AAL\right]$$

Where X is a vector of n losses. E is the excess point, L the Limit.

Reinstatements

If a layer has an AAL that is strictly greater than its limit it means that the reinsured can reinstate the cover. In most cases reinstating the cover has a price. In general reinstatements are achieved at 100% of the initial ROL. For example if a $5m xs $5m layer is priced at 10% ROL and has 1 reinstatement at 100%. If the reinsured has a first loss at $10m then it will entirely consume the layer. In order to reinstate the cover the reinsured will have to pay 10% of $5m i.e. $500,000. If the reinsured has a second loss at $10m then a $5m recovery will be collected and the cover will then be totally exhausted as there is only 1 reinstatement. The number of reinstatement implicitly defines the AAL.

In mathematical words we have:

$$RIP = min(Recoveries, (AAD - L)) * RIPperc * ROL)$$

Where RIP is Reinstatement Premium; RIPperc is the percentage at which the cover is reinstated.

If there is more than 1 reinstatement then it is possible that the reinstatement percentage is different at each reinstatement. We have assumed in the formula above that the reinstatement percentage is the same at each reinstatement which is the case in general.

Other features

Other features such as franchise and drop down exist. These are not in the scope of our project. However, one or two contracts in ANV present these characteristics and methods have been tailored to take these features into account.

If a franchise is in place then a minimum amount of loss must be incurred before reinsurance coverage applies. A franchise deductible differs from an ordinary deductible in that; once it is met the entire amount of the loss is paid, subject to the policy limit.

In mathematical words it means that:

$$\boxed{If\ X > F\ then\ Recoveries = X\ else\ Recoveries = 0}$$

Where X is the loss amount and F is the franchise.

IV. Reinsurance Pricing Techniques

A. Scope

In this paper we will focus on pricing techniques for **Excess of Loss type** reinsurance in the **form of Treaty** as these are the contracts ANV mostly writes.

There are two different perspectives when pricing a XoL reinsurance contract. In the Lloyd's market almost every risk is shared across several syndicates. In this case there will always be a lead syndicate and followers. The lead in general (but not systematically) takes the biggest share of the risk. The lead is also responsible for editing all the policy documentation and dealing with the management of claims and all related administration works related to the policy. The lead will often receive a fee for that service. The lead will also negotiate directly with the broker the terms and price of the reinsurance contract.

Therefore if a **syndicate is a lead** it has to provide a quote to the broker. If a **syndicate is a follower** the broker will offer the terms and price agreed with the lead. Hence a following syndicate would assess if the reinsurance contract is profitable or not. In most cases ANV is a follower and on a few occasions a lead.

We will also focus on pricing **Property business**. Pricing techniques for Casualty are similar to Property ones although they differ in some aspects and further considerations have to be made given the long tailed nature of this kind of business compared to property.

In most cases ANV writes **LOD policies**. However, if **RAD policies** are written then the inputs to the rating model have to be on a different basis. The table below summarises the inputs to the rating model and their nature according to the basis of the contract.

Inputs	RAD	LOD
Historical Losses	Use the year the policy has incepted	Use the year the loss occurred
Historical Premium	Use gross net written premium income (GNWPI) for each YOA (Year of Account)	Use gross net earned premium income (GNEPI) for each calendar year
LDF (Loss Development Factors)	Derived from an underwriting year based triangle The IBNR (Incurred But Not Reported) loss count is likely to be higher than on an LOD basis	Derived from an accident year based triangle

In general there are 2 elements of coverage to consider in a XoL Treaty reinsurance contract. On one hand there is **Per Risk Coverage** and on other hand **CAT Coverage**. The wording of the reinsurance contract can exclude one of these coverage element or include both. A third type of cover exists, called clash cover, although it is less frequent.

Each cover is priced differently, that is the reason why it is important to define them. In this paper we will focus on per risk and CAT cover pricing techniques.

- **Per Risk Coverage** - A Property Per Risk excess treaty provides a limit of coverage in excess of the ceding company's retention. The layer applies on a "per risk" basis, which typically refers to a single property location. This is more narrow than "per occurrence" property excess treaty which applies to multiple risks to provide catastrophe protection.

- **CAT Coverage** - A Property Catastrophe excess treaty provides a limit of coverage in excess of the ceding company's retention for a catastrophic event, such as a hurricane or earthquake. The occurrence may often affect multiple risks and multiple policies. For example a hurricane that goes through Florida and damages most of the properties in that state would trigger recoveries on the XoL treaty layer as the losses are aggregated. Typically, the catastrophe cover applies to the ceding company's retained exposure net of surplus share, per risk excess treaties and facultative certificates. That is, other reinsurance inures to the benefit of the catastrophe cover.

- **Clash covers** - Typically a loss on a single policy will not penetrate the treaty layer. A clash cover will be penetrated due to multiple policies involved in a single occurrence. For example the collision of 2 marine hull insured by the same insurer would trigger a combined loss that can penetrate a XoL treaty layer while a loss on a single policy would not.

B. Overview of pricing techniques

As discussed above reinsurance contract can have 2 types of cover: per risk and catastrophe. It is therefore crucial to price these 2 elements separately.

1. Per risk coverage

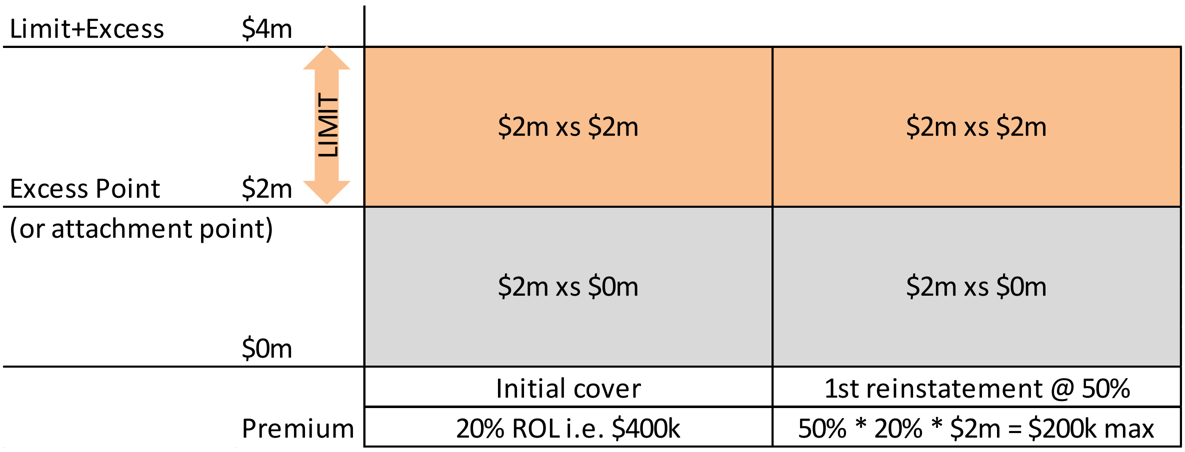
There are mainly 3 methods to price per risk reinsurance contracts.

a) Experience rating

The first one is called "Experience rating". This method is purely based on historical loss information. The basic idea of experience rating is that the historical experience, adjusted properly, is the best predictor of future expectations.

<u>Coverage and available information</u>

Let's assume that a $2m xs $2m has been bought by the reinsured. It is a per risk cover on an LOD basis for a property book. There is a $1m AAD and there is 1 reinstatement @50%. Hence the AAL is $4m. The proposed rate on line by the broker is 20%. The brokerage fee is 10%.



The reinsured provides to the reinsurer the following information:

- Historical gross net earned premium income
- Historical losses in excess of a threshold. The threshold has to be below the attachment point of the layer i.e. $2m in this case.

Premium on levelling

In order to perform the experience rating, the historical premiums and losses have to be on levelled to today's terms. The premium has to be adjusted for two reasons:

- **Rate change** – it reflects how the average base rates and average adjustment factors changed year on year. Hence historical premiums have to be adjusted to reflect today's base rates and adjustment factors of the underlying risk insured. The rate change for the renewing year is not known at the time of pricing however some assumptions have to be made. This is not trivial and relies on expert judgement.
- **Inflation** - In the Lloyds market, in general property premium are derived from a base rate applied to the sum insured. In general if the premium base is insured value or some other inflation-sensitive base, then an exposure inflation factor should also be included in the adjustment of historical premium. Therefore if the historical sum insured is available the historical premium has to be inflated in order to reflect higher sums insured in the underlying book.

Let's assume the reinsured writes a cargo book and it has earned the following premium over the years[3]:

Calendar Year	GNEPI ($USD)	Rate Change	Inflation	Cum. RC. Factor	Cum. Inf. Factor	Adjusted GNEPI ($USD)
Y1	42,000,000	4%	3%	0.95	1.15	45,758,070
Y2	50,000,000	-2%	0%	0.96	1.15	55,359,647
Y3	41,000,000	-3%	3%	0.99	1.11	45,435,803
Y4	35,000,000	1%	2%	0.99	1.09	37,761,806
Y5	29,000,000	0%	5%	0.99	1.04	29,768,663
Y6	34,000,000	-4%	3%	1.03	1.01	35,370,200
Renewal	41,000,000	3%	1%			41,000,000

In mathematical words it means:

$$P_j^a = P_j \left(\prod_j^n (1 + I_j)\right)\left(\prod_j^n (1 + RC_j)\right)$$

Where 1≤j≤n, n is the total number of years, I_j is the inflation rate for year j, RC_j is the rate change for the year j, P_j is the premium income for year j and P_j^a is the on levelled premium income for year j.

[3] The data provided does not reflect any real book and has been purely made up in order to illustrate the point

Claims on levelling

If catastrophe claims exist in the dataset then they have to be excluded as the aim here is to price a per risk cover. Then the claims have to be adjusted for 3 elements

- **IBNER –** Depending on the information provided by the reinsured if a claim is closed then no IBNER would be added. If not, a Loss Development Factor (LDF) will be applied to the incurred loss in order to take it to its ultimate level. The question of deriving an LDF pattern is not trivial to answer. If the reinsured can provide a triangle with sufficient history it can be used to derive an LDF pattern. If not Lloyd's provide market triangles dating back from 1993. However these triangles are for ground up losses. Triangles of claims in excess of certain amounts do not exist. Therefore any LDF pattern derived from these market triangles would not be a true reflection of large claims development. The development of individual claims to ultimate is not easy and often relies on knowledge and understanding of the underlying claims. The reinsured may provide details about each claim. In addition to this if the reinsured is part of the Lloyds market; most of the large losses are shared across a panel of syndicates. Therefore it is possible to refer to our own claims department in order to get a fair estimate of the ultimate outcome for market losses.
- **IBNR-** At the time the reinsurance contract is priced the available information often dates back to a few months earlier. For example for a contract staring on the 1st of January, the data available will date back to November in a best case scenario. Therefore it is possible that some losses have not been reported. This is very important if a frequency-severity model is used as it has a direct impact on the frequency. Due to the low number of reported claims in excess of a certain threshold, an additional claim can easily increase the average claim count assumed per year by 100% in the worst cases. It can also potentially shift the severity distribution fitted to historical losses. If claims that are known to the market have been reported between the cut-off date and the time the contract is priced then it has to be added. Otherwise it is possible to load the average claim count by a percentage in order to allow for IBNR claims. The severity distribution is assumed to remain the same. Finally if the contract is on an RAD basis the likelihood of having IBNR claims is higher than on an LOD basis. This is purely due to the fact that an RAD cover is longer tailed and is linked to the tail of the underlying business whereas an LOD cover would expire after 12 months no matter how long tailed the underlying business is.
- **Inflation –** Claims that occurred 5 years ago won't have the same cost as claims that occurred today. Therefore claims have to be adjusted for inflation. However, inflation is not easy to determine as for some class of business it is almost impossible to know what the real inflation is. In general an inflation rate varying from 1% to 6% is assumed on average across the years.

Let's assume the same cargo book has suffered the following losses over the years. All these losses are in excess of $1.5m which is below the attachment point of $2m.

Accident Year	Incurred Loss ($USD)	LDF	Inflation	Inf. Factor	Ultimate Adj. Loss ($USD)
Y1	5,700,000	1.00	3%	1.19	6,806,098
Y2	3,652,000	1.00	3%	1.16	4,233,669
Y3	4,543,000	1.01	3%	1.13	5,189,107
Y3	2,594,000	1.01	3%	1.13	2,962,920
Y3	3,304,000	1.01	3%	1.13	3,773,896
Y3	3,366,000	1.01	3%	1.13	3,844,714
Y5	25,000,000*	1.06	3%	1.06	28,113,850
Y5	2,901,000	1.06	3%	1.06	3,249,139
Y6	1,694,000	1.26	3%	1.03	2,198,938
Y6	1,526,000	1.26	3%	1.03	1,980,861

*The $25m incurred loss in Y5 is a CAT loss generated by a hurricane. This loss will have to be excluded from the experience rating as the per risk element is priced here.

In mathematical terms this can be written as:

$$C_j^a = C_j \times LDF_j \times \left(\prod_{j}^{n} (1 + I_j) \right)$$

Where 1≤j≤n, n is the total number of years, I_j is the inflation rate for year j, LDF_j is the Loss Development Factor for the year j, C_j is the claim cost for year j and C_j^a is the on levelled claim cost for year j.

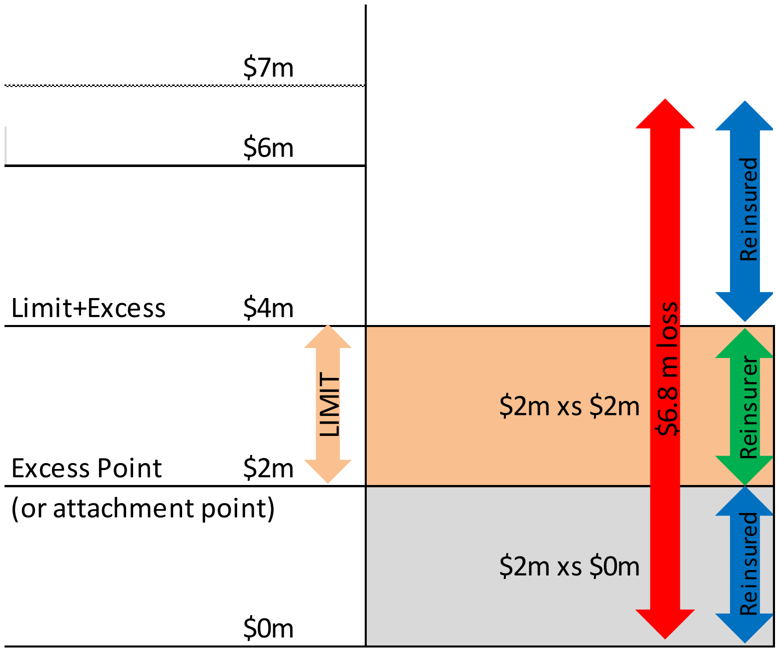
Application of the treaty terms: Excess and Limit

Each on levelled loss is taken through the treaty layer in order to calculate the loss to layer for the reinsurer (or the recoveries for the reinsured). In the table below an on levelled loss of \$6.8m occurred in Y1. The reinsurer will pay \$2m to the reinsured for that loss. The reinsured will pay the first \$2m and the remaining cost from \$4m to \$6.8m as shown on the diagram below.

Treaty Terms	
Excess	2,000,000
Limit	2,000,000
AAD	1,000,000
AAL	4,000,000
1st Reinstatement	50%
ROL	20%

Application of Excess and Limit

Accident Year	**Ultimate Adj. Loss (\$USD)**	**Loss to Layer**
Y1	6,806,098	2,000,000
Y2	4,233,669	2,000,000
Y3	5,189,107	2,000,000
Y3	2,962,920	962,920
Y3	3,773,896	1,773,896
Y3	3,844,714	1,844,714
Y5	3,249,139	1,249,139
Y5	2,198,938	198,938
Y6	1,980,861	-



In mathematical words it means:

$$R_j^i = \min\left(max\left(\left(C_j^i - E\right), 0\right), L\right)$$

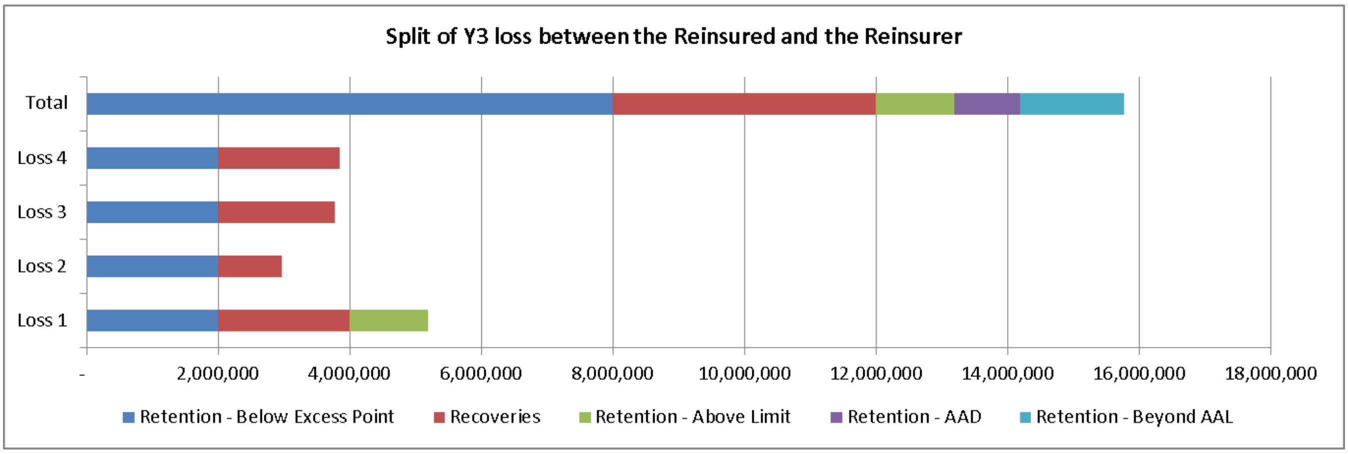
Where R_j^i is the amount of recoveries for the loss i in year j, E is the excess point, L is the limit, C_j^i is the on levelled loss amount in year j for loss i.

Application of the treaty terms: AAD and AAL

On levelled losses and losses to the layer are then summarised by year (Accident year in this case as it is an LOD contract) and additional contract terms are applied i.e. AAD and AAL. In the example above, it can be noticed that Y3 is a very active year. It had 4 losses totalling to an on levelled value of $15.8m. The loss to the layer is of $6.6m. However, the contract terms mention that there is a $1m AAD. Hence the cost to the layer for Y3 is in fact $5.6m. Then as the reinsured can only benefit from one reinstatement, the maximum recoveries that can be collected in a year is capped at $4m. Therefore the loss to the reinsurer in Y3 is not $5.6m but capped at $4m. As for the reinsured the total retained loss for Y3 is $11.8m. The histogram below summarizes this situation.

Summary by year and application of AAD,AAL

Accident Year	Ultimate Adj. Loss ($USD)	Loss to Layer	Application of AAD	Application of AAL
Y1	6,806,098	2,000,000	**1,000,000**	1,000,000
Y2	4,233,669	2,000,000	**1,000,000**	1,000,000
Y3	15,770,637	6,581,530	**5,581,530**	**4,000,000**
Y4	-	-	-	-
Y5	5,448,077	1,448,077	**448,077**	448,077
Y6	1,980,861	-	-	-



This can be written as:

$$R_j = min\left[max\left(\left(\sum_{i=1}^{n_j} R_j^i - AAD\right), 0\right), AAL\right]$$

Where R_j is the total amount of recoveries for losses that occurred in year j, n_j is the number of losses that occurred in year j, R_j^i is the amount of recovery for the on levelled loss I that occurred in year j.

Loss cost selection

Once the final ultimate loss to layer is calculated for the reinsurer for each accident year, a burn rate can be worked out for each year using the reinsured GNEPI. The average burn rate across the 6 years of historical information is taken as a **best estimate** of future expectations regarding the performance of that layer. In this example Y4 and Y6 are clean years. Y3 is the worst year with a burn rate of 8.8%. Overall the burn rate is 2.58%. This means that for every dollar of business written and earned by the reinsured the reinsurer would charge $0.0258 in order to break even.

Summary by accident year and loss cost

Accident Year	Adjusted GNEPI ($USD)	Final Loss to Layer ($USD)	Burning Rate
Y1	45,758,070	1,000,000	2.19%
Y2	55,359,647	1,000,000	1.81%
Y3	45,435,803	4,000,000	8.80%
Y4	37,761,806	-	0.00%
Y5	29,768,663	448,077	1.51%
Y6	35,370,200	-	0.00%
Total	249,454,189	6,448,077	2.58%

As the estimated premium income for Y7 i.e. the renewing year is $41m, the burn rate enables the reinsurer to give an exposure adjusted price to the reinsured. In this case $1,059,798 will be payable to the reinsurer by the reinsured. In general most of the reinsurance contracts start on the first of January. The reinsurance premium is paid in 4 instalments at the beginning of each quarter.

	GNEP ($USD)	Estimated Loss to Layer ($USD)	Burn Rate
Renewal	41,000,000	1,059,798	2.58%

The final burn rate can be written as:

$$\boxed{Burn\ Rate = \frac{\sum_{j=1}^{n} R_j}{\sum_{j=1}^{n} P_j^a}}$$

Where R_j is the total amount of recoveries that can be collected in year j after application of contract terms to each on levelled loss i.e. application of excess, limit, AAD and AAL. N is the total number of years of experience. P_j^a is the on levelled premium income for year j for the reinsured. Burn rate is the price to charge the reinsured for the considered reinsurance layer for every $USD of premium written.

Limitations

There are 2 main limitations of this method:

- The rate change and inflation adjustments made to the premium and loss information are not trivial. They do not take into account the change in risk profile of the reinsured over the years as the risk profile of the renewing book might be totally different from the one of the previous years.
- Other issues can arise with this method if the reinsured has no loss history or is a start-up as the suggested price would be nil while it is obviously not the case. Even if the reinsured has many years of losses issues can arise on top layers where historical losses may not penetrate. In these instances the price is obviously not nil.

Hence alternative methods like the exposure rating have to be used.

b) Exposure rating

Introduction

The second pricing method is called "Exposure rating". It is a pricing technique that only exists for reinsurance contracts. Nothing similar to this approach can be found on primary insurance pricing. The approach was first developed by Ruth Salzmann in 1963 for homeowners business and eventually adapted for commercial property as well. The basic idea of the method is to use a curve called "exposure curve" to allocate the ground up risk premium to different layers. The advantages of this approach over experience rating are:

- The current risk profile of the reinsured is modelled, not what was written years earlier.
- Every layer can be priced and the issue of "free cover"[4] is solved.

Mathematical definition

If the loss distribution of each risk in the reinsured portfolio was known then it would be easy to determine the amount of risk premium that the reinsured would retain below an excess point. Let's assume each risk is distributed according to a probability density function (pdf) noted f(x) and that each risk has an insured value noted IV. Let's assume the reinsured retains a percentage p of the insured value (in this case $p = \frac{Excess\ point}{IV}$) and that anything beyond that percentage is taken by the reinsurer.

The reinsured is then exposed to the following percentage noted G (p) of the mean expected loss cost noted E[x].

$$G(p) = \frac{\int_0^{pIV} xf(x)dx + \int_{pIV}^{\infty} pIVf(x)dx}{\int_0^{\infty} xf(x)dx} = \frac{\int_0^{pIV} [1 - F(x)]dx}{E[x]}$$

[4] See section IV.C.1

The numerator can also be noted L (Excess, x) = E[min(Excess,x)] which is the limited expected value of the loss distribution. The denominator is the expected loss cost. The function G(p) is called exposure curve.

Because [1-F(x)] >0 and F'(x)=f(x)>0, G(p) is an increasing and concave function on the interval [0,1]. By definition of G(p) we have G(0)=0 and G(1)=1.

It is then possible to estimate the portion of the risk premium that the reinsurer will take for a simple XoL contract with an excess point and a limit using the following formula:

$$\boxed{RI\ Risk\ Premium = G\left[\frac{(Excess + Limit)}{IV}\right] - G\left[\frac{Excess}{IV}\right]}$$

Where: $(Excess + Limit) \leq IV$

If the excess point is greater than the insured value then the reinsurer is not exposed to that risk and the risk is entirely retained by the reinsured.

If the excess point is less than the insured value but the excess point plus the limit is less than the insured value then the reinsurer is partially exposed to that risk.

However in practise 2 issues arises:

- It is impossible to know what the probability density function of each risk in a portfolio is.
- A portfolio is also made of various risks that are different from each other's and have different insured values.

Risk profiles

To address the second point defined above, the reinsured summarises its data in bands of insured value. It is assumed that all the risks within each band are homogenous. The reinsured would also provide the number of risks that falls within each band as well as the total premium collected.

Let's take the example of a cargo book. The reinsured summarized the risks into bands of sum insured as shown in the table below which represents the re-insured's risk profile.

For example the reinsured has 763 risks that have an insured value between $0 and $2,000,000.

It is possible that the reinsured also provides the average sum insured in each band. For example in the first band the average insured value of the 763 risks is $426,391. If the average insured value is not provided then the mid-point between the lower and the upper insured value is taken. In this case for the first band it would have been $1m.

The reinsured collected a total premium of $3,350,000 from the risks in the first band.

In order to estimate the expected loss cost or in other words the risk premium of each risk, a loss ratio is assumed or provided by the reinsurer as an a priori expectation. A different loss ratio per band could be assumed. In this case and in general, given the lack of information, the same loss ratio is assumed across the bands.

SI LB	SI UB	Average SI	Premiums	Risk Count	Assumed Loss Ratio	Risk Premium
-	2,000,000	426,391	3,350,000	763	75%	2,512,500
2,000,000	4,000,000	2,950,100	5,370,000	414	75%	4,027,500
4,000,000	6,000,000	4,014,202	8,640,000	357	75%	6,480,000
6,000,000	8,000,000	7,440,570	4,490,000	207	75%	3,367,500
8,000,000	10,000,000	8,904,012	4,020,000	182	75%	3,015,000
10,000,000	15,000,000	13,285,700	7,590,000	284	75%	5,692,500
15,000,000	20,000,000	17,398,169	4,710,000	158	75%	3,532,500
20,000,000	25,000,000	21,694,317	1,990,000	70	75%	1,492,500
25,000,000	30,000,000	25,564,646	1,780,000	50	75%	1,335,000
30,000,000	35,000,000	33,744,388	930,000	19	75%	697,500
35,000,000	40,000,000	36,335,908	450,000	8	75%	337,500
40,000,000	65,000,000	51,079,233	540,000	9	75%	405,000

Therefore if the loss distribution was known, then the exposure curve would have been applied to each band, assuming that each risk within each band has an insured value that is equal to the average insured value.

However the loss distribution is not known. Therefore the question is to find a way of having an exposure curve without knowing the underlying distribution of the losses.

Empirical exposure curves

For large reinsurers with large amount of data it is possible to use historical information in order to build an empirical exposure curve. This is what the reinsurer Swiss Re, the world's second largest reinsurer, has done by establishing the famous Swiss Re exposure curves. Large amount of historical losses are classified by category of risks and of insured value. Then empirical limited expected values are calculated at different thresholds in order to build the exposure curve as shown in the first section earlier.

MBBEFD class of distributions and exposure curves

- Introduction

In his paper "the swiss re exposure curves and the MBBEFD distribution class" Stefan Bernegger managed to find a class of distribution used in physics called MBBEFD for Maxwell Boltzman Bose Einstein Fermi Dirac that can exactly match the Swiss Re curves and more over produce an infinite number of exposure curves that depend on a single parameter.

In his paper, Stefan Bernegger first establishes a few useful equations that link the distribution of the underlying losses to the exposure curve function.

In the following formulas, we define as X the random variable for the underlying losses. M is the insured value or maximum possible loss. x is the normalised random variable i.e. x=X/M.

The derivative of the exposure curve as defined in the section above is:

$$G'(p) = \frac{[1 - F(p)]}{E[x]} \text{ with } F(0) = 0 \text{ and } G'(0) = \frac{1}{E[x]} \text{ so } F(x) = 1 - \frac{G'(x)}{G'(0)} \text{ for } 0 \leq x \leq 1$$

In other word the derivative of the exposure curve at any point p can be seen as the probability of having a loss greater than p% of the insured value. As the exposure curve is concave the probability of having a loss greater than p% of the insured value decrease with p.

- Total loss probability and expected value

The total loss probability is equal to $d = 1 - F(1^-) = \frac{G\prime(1)}{G\prime(0)}$

The expected value is equal to $\mu = E[x] = \frac{1}{G\prime(0)}$

As G(x) is concave and increasing on the interval [0,1] and G(0)=0 and G(1)=1 we have

$$G'(0) > 1 > G'(1) > 0 \text{ which can be written as } 1 > \mu > d > 0$$

- The MBBEFD class of distribution

The MBBEFD curve used in physics is defined as

$$G(x) = \frac{\ln(a + b^x) - \ln(a + 1)}{\ln(a + b) - \ln(a + 1)}$$

The parameters a and b are defined such that the function G is increasing and concave on the interval [0,1]. A new parameter $g = \frac{1}{d}$ defined as the inverse of the total loss probability is introduced. The parameter a is replaced by the parameter g using the relation $a = \frac{(g-1)b}{1-gb}$

The condition 0<d<1 is fulfilled for g>1 and the function G(x) is a real only if b is greater than or equal to zero.

With the new parameter g introduced, the most general definition of the function G(x) is:

$$G_{b,g}(x) = \frac{\ln\left[\frac{(g-1)b + (1-gb)b^x}{(1-b)}\right]}{\ln(gb)} \text{ where } b > 0 \cap b \neq 1 \cap bg \neq 1 \cap g > 1$$

The derivative can then be defined as with the same conditions on b and g:

$$G'(x) = \frac{\ln(b)(1-gb)}{\ln(gb)\,[(g-1)b^{1-x} + (1-gb)]}$$

Then follows:

$$G'(0) = \frac{\ln(b)(1-gb)}{\ln(gb)(1-b)} \text{ and } G'(1) = \frac{\ln(b)(1-gb)}{\ln(gb)\,g(1-b)}$$

And the distributions function:

$$F(x) = 1 - \frac{(1-b)}{(g-1)b^{1-x} + (1-gb)}$$

- Curve fitting

It is then possible to fit any exposure curve for any given pair of total loss probability and mean loss.

The parameter g is easily obtained as $g = \frac{1}{d}$

As for the parameter b, it can be obtained by solving iteratively the following equation:

$$\mu = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$$

It is also possible to fit an exposure curve for a given pair of mean loss and variance but this won't be detailed here.

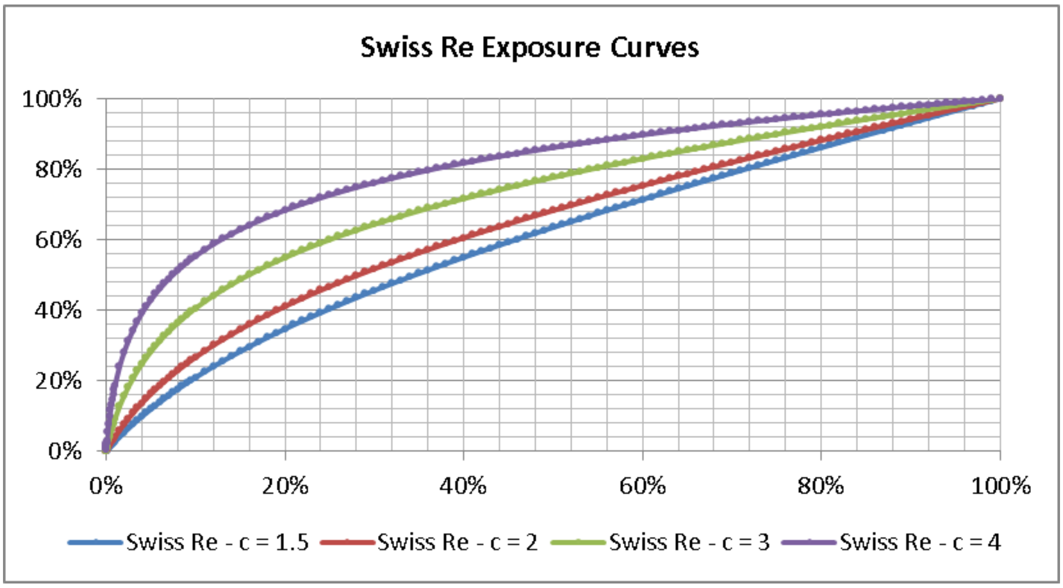
- Definition of a single parameter

In his paper Stefan Bernegger found that the exposure curves derived from the MBBEFD class of distributions could be defined using a single parameter noted c instead of the two parameters b and g.

$$b_c = e^{3.1-0.15(1+c)c} \ and \ \ g_c = e^{(0.78+0.12c)c}$$

- The link with the Swiss Re curves

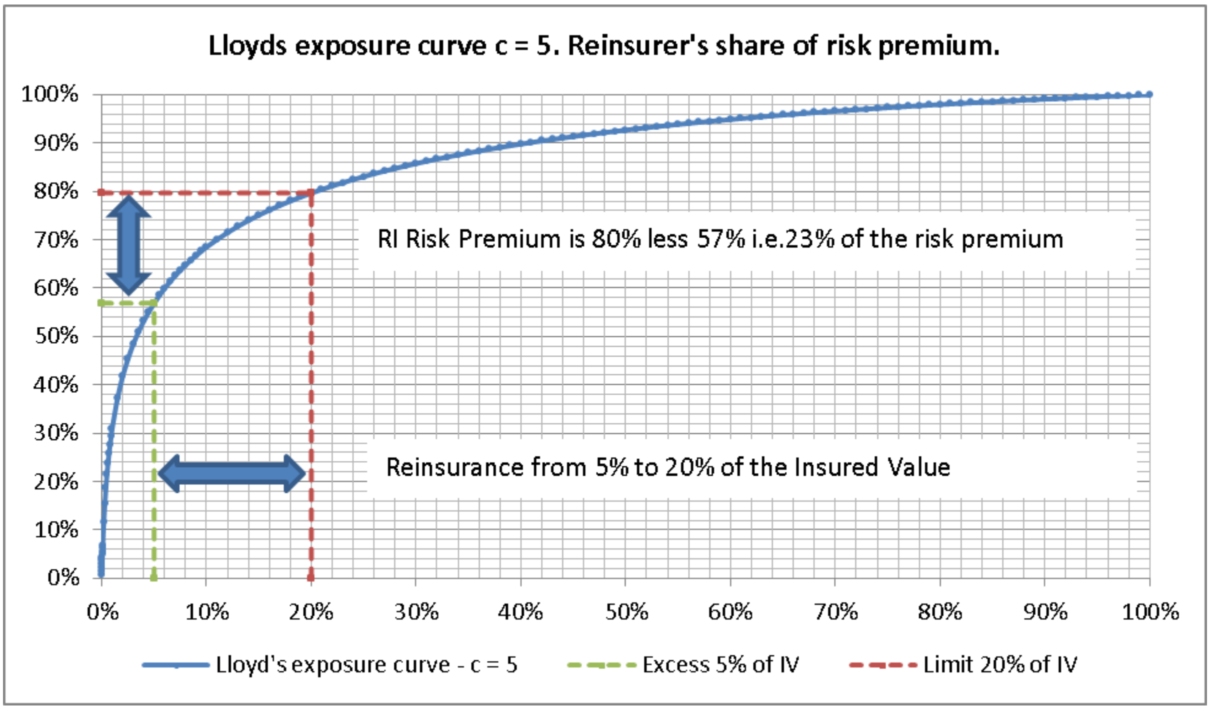
Therefore it was found that the 4 Swiss Re curves coincides very well with exposure curves with a c parameter of {1.5; 2; 3; 4} as shown in the graph below.



- The link with the Lloyd's exposure curve

The Lloyd's exposure curve turned out to coincide with an exposure curve with a parameter of 5.

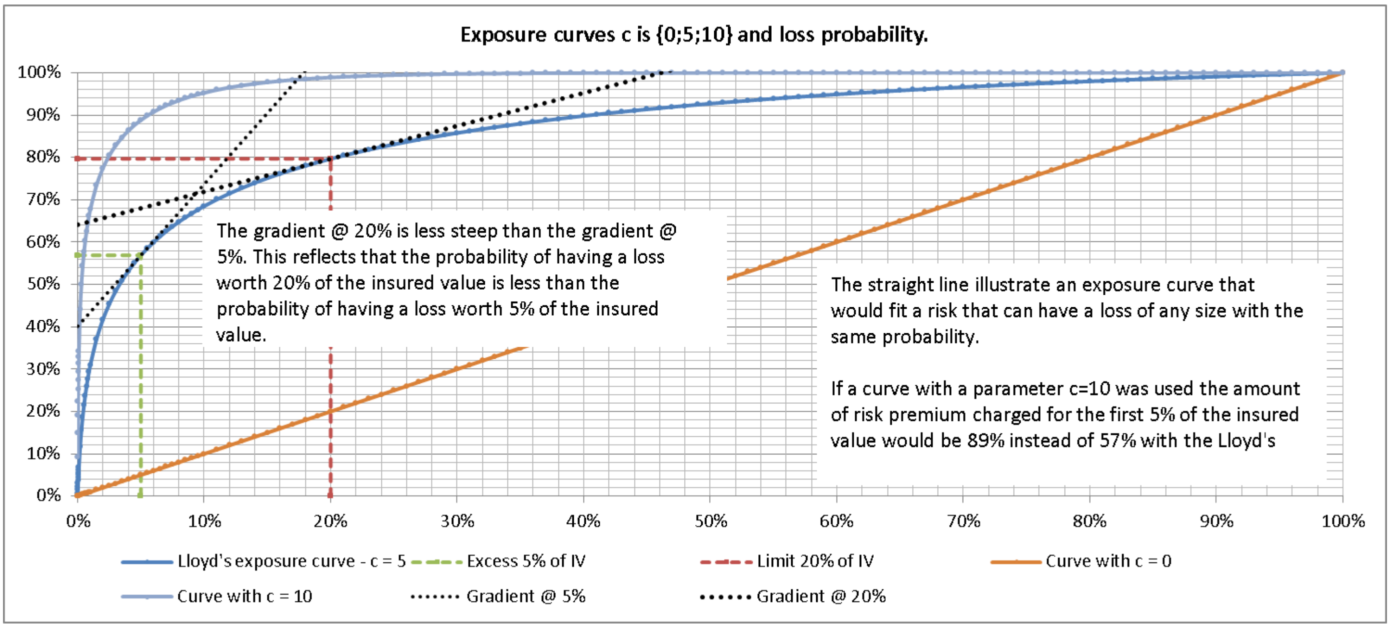
The graph below illustrates the Lloyd's exposure curve. It is an increasing and concave curve and defined on the interval [0, 1]. The graph below shows how the reinsurer's share of the risk premium is calculated based on the exposure curve. If the excess point represents 5% of the insured value and the ceiling (excess + limit) of the XoL cover represents 20% of the insured value then the reinsurer would get 23% of the risk premium. If another reinsurer was to take the portion of the risk beyond 20% of the insured value then he would only get 20% of the risk premium which is less than the reinsurer who took 23% of the risk premium for the first 5% to 20% of the risk. This reflects the fact that the probability of having a large loss is less than the probability of having attritional losses. This is the reason why the exposure curve is concave as its derivative is also a measure of the probability of a claim to occur at a given per cent of the insured value.



- Comments on the c parameter

The higher the c parameter is the more concave the curve is which means that the underlying risks are more likely to have attritional losses and very unlikely to have losses beyond a certain threshold.

In the extreme case where the c parameter is zero, the exposure curve is a straight line, which means that the probability of having a loss of any size is the same and that it is very likely to have a total loss therefore every share of coverage has a corresponding equal share of the risk premium.



<u>Application of exposure curves to risk profiles</u>

Let's assume that a layer of $5m in excess of $5m has to be priced. The risk profile presented earlier is used. We assume that the exposure curve is a Lloyd's curve i.e. its parameter c is 5 and that the underlying book is performing at 75%. The following table summarises the risk premium to the reinsurer for each bands.

Average SI	Risk Premium (75% LR)	Excess as % of Avg. SI	(Excess + Limit) as % of Avg. SI	Portion of IV covered	G(Excess)	G(Excess + Limit)	Portion of Risk Premium to the reinsurer	Risk Premium to the reinsurer
426,391	2,512,500	1173%	2345%	0%	100%	100%	0%	-
2,950,100	4,027,500	169%	339%	0%	100%	100%	0%	-
4,014,202	6,480,000	125%	249%	0%	100%	100%	0%	-
7,440,570	3,367,500	67%	134%	33%	96%	100%	4%	129,008
8,904,012	3,015,000	56%	112%	44%	94%	100%	6%	177,243
13,285,700	5,692,500	38%	75%	37%	89%	97%	8%	477,370
17,398,169	3,532,500	29%	57%	28%	85%	94%	9%	326,508
21,694,317	1,492,500	23%	46%	23%	82%	92%	10%	146,565
25,564,646	1,335,000	20%	39%	19%	79%	90%	10%	135,942
33,744,388	697,500	15%	30%	15%	75%	86%	11%	74,504
36,335,908	337,500	14%	28%	14%	74%	85%	11%	36,425
51,079,233	405,000	10%	20%	10%	68%	79%	11%	45,355
Grand Total	**32,895,000**							**1,548,921**

Overall the reinsurer would charge $1,548,921 for a $5m xs $5m cover given the risk profile above and the assumptions made regarding the underlying book's loss ratio and the exposure curve to use. This represents a LOL (Loss on Line) of 31% ($LOL = \frac{Loss\ cost}{Limit}$). This is a pure loss cost for unlimited reinstatements, in other words unlimited AAL. The section B.3.a. explains how the reinstatements can be taken into account once a loss cost or LOL (Loss on Line) has been selected.

The table below shows how the LOL could vary for different sets of loss ratio and exposure curve assumed.

LR / c	40%	50%	60%	70%	75%	80%	90%
1	60%	75%	90%	105%	112%	120%	135%
2	49%	62%	74%	86%	93%	99%	111%
3	38%	47%	56%	66%	70%	75%	84%
4	26%	33%	39%	46%	49%	52%	59%
4.5	21%	26%	32%	37%	39%	42%	47%
5	17%	21%	25%	29%	31%	33%	37%
5.5	13%	16%	19%	22%	24%	25%	29%
6	10%	12%	14%	17%	18%	19%	22%
7	5%	7%	8%	9%	10%	11%	12%
8	3%	4%	4%	5%	5%	6%	6%
9	2%	2%	2%	3%	3%	3%	3%
10	1%	1%	1%	1%	2%	2%	2%

- Fixed loss ratio, varying exposure curve

This table can be read in the following manner. If two reinsured, A and B, had the same underlying loss ratio, for example 75% but if the makeup of that loss ratio was different i.e. as shown in table below:

	A	B
Attritional	10%	65%
Large	65%	10%
Total	75%	75%

Let's assume that the large loss threshold is set at $5m, which is the reinsurance layer excess point. The reinsured A and B are actually looking to protect their book against these large losses. Given the loss ratios in the table above, the reinsured A will have a greater benefit from the reinsurance compared to reinsured B. Therefore the price charged to each of them should be different. In order to reflect this, the exposure curve parameter of the reinsured B will have to be higher than the one used for reinsured A. The choice of the right exposure is not an easy decision to make and is rather a subjective choice to make based on the expert knowledge of the reinsured and its market. However, in this case to illustrate our point, it can be said that for reinsured A a parameter of c=3 can be chosen producing a LOL of 70% and for reinsured B a parameter of c=7 producing a LOL of 10%

which is 7 times less that for reinsured A. This ratio is close to the large loss ratio of reinsured A being 6.5 times greater than the one of reinsured B.

- Fixed exposure curve, varying loss ratio

On other hand a situation where two reinsured A and B have a similar loss ratio structure i.e. for example 30% of losses are large (in excess of $5m) and 70% of losses are attritional. However, if reinsured A has a better overall loss ratio than reinsured B then the final LOL selection will be different. If a c parameter of 5 is chosen for both reinsured and reinsured A has a loss ratio of 40% and reinsured B a loss ratio of 80% then the difference in LOL would be of 17% for A vs. 33% for B.

- Conclusion

This shows that even if two reinsured have exactly the same risk profile, the quality of the underlying risks is crucial and this is what the loss ratio and exposure curve parameter are trying to capture.

For example the reinsured A may insure properties in Florida which is a state heavily exposed to US hurricanes from June to October whereas reinsured B may insure the exact same portfolio of properties but in Dubai where wind risk is much lower than in US Florida. Hence, reinsured A may suffer from large losses more often than reinsured B. Hence the exposure curve used for reinsured A will be tougher i.e. closer to the first diagonal (less concave) than the one used for reinsured B.

Then let's assume reinsured B also writes US Florida properties similar to reinsured A. But let's assume that the properties insured by B are much stronger, built with reinforced concrete, whereas properties insured by A are made from wood. Then the properties insured by A would be more vulnerable to the hurricane that the properties insured by A. Although the structures of the losses are likely to be the same, the overall loss ratio for B would be lower than the one of A.

When parameter selections are made discussions with the class underwriter helps to understand what the underlying risk covered is in order to come up with the best choice of parameters.

<u>Limitations</u>

The limitations of this method are:

- It is difficult to know which exposure curve to use as each exposure curve will allocate premium in different proportions.
- An implicit assumption in the exposure rating approach is that the same exposure curve applies regardless of the size of the insured value. For example, the likelihood of $10,000 loss on a $100,000 risk is equal to the likelihood of a $100,000 loss on a $1,000,000 risk. This assumption of scale independence might be correct on small risks but can be easily challenged for large industrial and commercial risks.
- It is difficult to estimate the ground up risk premium. Often the reinsured provides its ground up premium and its book is assumed to run at a certain loss ratio in order to estimate the risk premium.
- The reinsured provides "limits profiles" where risks are classified into bands of insured values. The classification has to be made on per location basis. If it is made based on total insured values for policies covering multiple locations then distortions will results.

c) Frequency-Severity model

<u>Introduction</u>

A third method for pricing per risk reinsurance contract is simulation based technique or alternatively called "Frequency-Severity" model. The advantage of frequency-severity model over the first two methods are:

- It can precisely take into account every features of the reinsurance contract. The terms of the contract can be applied to each and every loss simulated.
- The model is calibrated using the historical loss information and expert judgement adjustment can be made to reflect the understanding of the underlying risks.
- It produces a best estimate price but also the volatility around that price. Return periods for each XoL layer can be produced as well as series of statistical information.
- If the reinsured has no historical losses then standard parameters derived from similar books or market books can be used.
- It is an intermediary method between the experience and exposure rating. It requires some loss information in order to parameterise the frequency and severity distributions. Distributions can then be used to simulate losses in layers with no historical losses.

The table below summarises the distribution used for frequency and severity.

Frequency	Comment	Severity	Comment
Poisson		Log Normal	Suited for Large Losses
Negative Binomial	Greater variability	Pareto	Suited for Large/CAT losses
		Truncated Pareto	Suited for Large Losses

For frequency a Poisson distribution is used in general. The negative binomial could be used if more volatility is expected in the claims frequency which is often the case in reinsurance given the low number of historical claims.

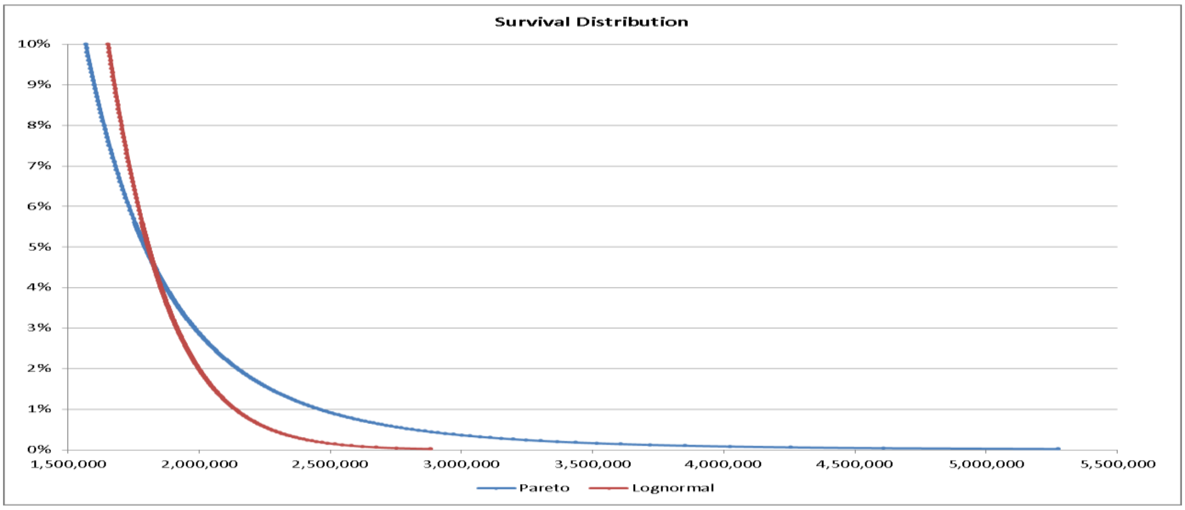
For severity the log normal is generally accepted as the best option to simulate single large losses. Pareto can also be used but it has a heavier tail than the lognormal and is more suited to simulate CAT type claims rather than per risk type claims.

In the Extreme Value Theory lognormal distribution belongs to the "Gumbel" class of distribution which is characterized for having intermediary tail .Pareto distribution belongs to the "Frechet" class of distribution which is characterised as having a heavy tail.

This can be illustrated comparing a lognormal and a Pareto distribution that have the same mean and standard deviation.

	Pareto	Lognormal
Mean	1,242,676	1,242,676
Std Dev	310,861	310,861
Scale	1,000,000	
Shape	5.12	
Mu		14
Sigma		0.25

It can be seen on the survival distribution that the tail of the Pareto distribution is heavier than he one of the Lognormal distribution.



<u>Frequency distributions</u>

The **Poisson distribution** is a discrete probability distribution defined as follow:

$$\boxed{P(N = k) = \frac{\lambda^k e^{-\lambda}}{k!}}$$

N is a random variable representing a number of loss that has been notified in a year.

The mean and the variance are defined as:

$$\boxed{Mean = Variance = \lambda}$$

The **negative binomial distribution** is a discrete probability distribution of the number of successes (denoted k) in a sequence of independent and identically distributed Bernoulli trials (with a success probability denoted p) before a specified (non-random) number of failures (denoted *r*) occur. It is defined as:

$$P(N = k) = \binom{k + r - 1}{k} (1 - p)^r p^k$$

The mean and variance are defined as:

$$Mean = \frac{pr}{1 - p} \; ; \; Variance = \frac{pr}{(1 - p)^2}$$

It can be noted that 0<p<1 hence Mean < Variance. This shows that using a negative binomial distribution allows for a greater variability in the simulated outcomes.

Severity distributions

The severity distributions defined below are used to simulate the loss amount of a notified claim. By definition a loss amount is greater than 0.

The **pareto distribution** can be easily defined using its cumulative density function (CDF) as:

$$F(x) = 1 - \left(\frac{\theta}{x}\right)^\alpha$$

The Pareto distribution has 2 parameters. A scale parameter noted θ and a shape parameter noted α. The scale parameter is the starting point of the distribution i.e. $x \geq \theta$.

The kth moments of the Pareto distribution can be defined as:

$$E(X^k) = \frac{\alpha\theta^k}{\alpha - k}$$

The mean of a Pareto distribution only exist if the shape parameter is strictly greater than 1. The variance exists only if the shape parameter is strictly greater than 2.

The Pareto distribution in theory can simulate any loss of any size (greater than the scale) given that it belongs to the class of heavy tailed distributions. In that aspect it is well suited to simulate CAT type losses. The Pareto distribution can also be used to simulate risk losses. However a small change has to be made to the distribution. An upper limit has to be set. It then becomes a truncated Pareto. The upper limit can be defined as the maximum line size of a portfolio or the limit set in a policy (i.e. the maximum loss a policy can have).

The **truncated Pareto distribution** can be defined using its CDF as:

$$F(x) = \frac{1 - \left(\frac{\theta}{x}\right)^\alpha}{1 - \left(\frac{\theta}{T}\right)^\alpha} \quad where\ \theta \leq x \leq T\ for\ \alpha \neq 0$$

T is the truncation point i.e. the upper limit of the distribution. The kth moments of the Pareto distribution can be defined as:

$$E(X^k) = \left(\frac{\alpha\theta^k}{\alpha - k}\right)\left(\frac{1 - \left(\frac{\theta}{T}\right)^{\alpha-k}}{1 - \left(\frac{\theta}{T}\right)^{\alpha}}\right) \; for\ \alpha \neq 0$$

The **lognormal distribution** can be defined as the distribution of a random variable X such that the logarithm of X has a normal distribution. It can be easily defined using its probability density function (pdf) as:

$$f(x) = \frac{1}{x\sigma\sqrt{2\Pi}} e^{\left(\frac{-(\ln(x)-\mu)^2}{2\sigma^2}\right)}$$

The lognormal distribution has 2 parameters μ and σ. The mean and variance are defined as:

$$Mean = e^{\left(\mu + \frac{\sigma^2}{2}\right)} \; ; \; Variance = (e^{\sigma^2} - 1)e^{(2\mu + \sigma^2)}$$

Parameterisation of distributions

For the frequency, it is often easier to adopt the Poisson distribution because the variance and the mean are equal. The Poisson parameter can be calculated using the historical loss information.

Let's assume that there is n years of experience. Let's write E_i the measure of exposure in year i. The exposure can be the amount of premium, a gross tonnage in marine insurance or the turnover of a company. Let's write N_i the number of reported losses in year i. The expected number of losses in year n+1, in other words an estimator of the Poisson parameter, can then be defined as:

$$\hat{\lambda} = N_{n+1} = \left(\frac{\sum_{i=1}^{i=n} N_i}{\sum_{i=1}^{i=n} E_i}\right) E_{n+1}$$

For the severity, historical loss information can be used to fit a Pareto or a lognormal distribution. The maximum likelihood method is used in order to derive the parameters of each distribution.

For the lognormal distribution it is complicated to write down an estimator of the parameters. Statistical software's such as R can be used.

For the Pareto distribution the shape parameter can be easily calculated. Let's write N the total number of losses across the whole loss history. Then we have:

$$\widehat{\alpha_{MLE}} = \frac{N}{\sum_{i=1}^{i=N} \ln\left(\frac{x_i}{\theta}\right)}$$

where x_i are individual losses.

The scale parameter is often defined based on personal judgement but it has to be defined below the excess point of the reinsurance contract. As shown on the formula above, the choice of the scale parameter has an impact on the shape parameter estimator. In general reinsured provides their loss information in excess of a threshold T which is below the reinsurance retention R. The scale parameter has to be in the following range $T \leq \theta \leq R$.

In the case of a truncated Pareto the estimator of the shape parameter is a bit more complicated and requires solving the following equation:

$$\widehat{\alpha_{MLE}} = \frac{N}{\sum_{i=1}^{i=N} \left(ln\left(\frac{x_i}{\theta}\right) - \left\{ \frac{N.ln\left(\frac{\theta}{T}\right).\left(\frac{\theta}{T}\right)^{\widehat{\alpha_{MLE}}}}{1-\left(\frac{\theta}{T}\right)^{\widehat{\alpha_{MLE}}}} \right\} \right)}$$

Statistical software like R are used in order to determine the shape parameter of a Truncated Pareto distribution.

<u>Monte Carlo simulation</u>

Once a distribution has been chosen for the frequency and the severity, losses can be simulated. The following process is followed.

1- Simulation of the number of losses noted N using the frequency distribution
2- For each loss, simulate a cost using the severity distribution
3- Apply the reinsurance contract terms to each loss
4- Aggregate the results to get the loss to the reinsurance contract
5- Repeat step 1 to 4 n times (for example 10000 times)
6- Calculate the average (and other statistics) to get the final loss to the contract (to get the variance, the return periods etc.)

2. Catastrophe coverage

Various methods exist to price cat covers although this is not trivial.

a) Return period method

A basic approach that is commonly used is to assume a return period for CAT for the considered layer. For example a 1 in 25 years mean that once every 25 years the layer will be completely exhausted. This approach is purely based on expert judgement. The expert judgement lies in the historical experience and knowledge of the market from the underwriter.

For example if a \$10m xs \$10m layer has to be priced and that the CAT element of the cover is assumed to have a price of 1 in 25 years, then the CAT price would be:

$$\boxed{CAT\ Price = \frac{1}{25} \times 10{,}000{,}000 = 400{,}000}$$

b) *Experience rating*

In a similar way to the per risk cover, experience rating could be used if CAT type claims exist in the loss history. However these types of claims are very sparse and produce very volatile results. For example let's assume that there is n years of historical experience and that there are in total N CAT losses noted C_j for the CAT loss number j. Let's write E_i the amount of exposure in year i, in general the exposure is measured as the amount of premium. Then a basic price for CAT exposure could be written as:

$$\boxed{CAT\ Price = \left(\frac{\sum_{j=1}^{j=N} C_j}{\sum_{i=1}^{i=n} E_i}\right) E_{n+1}}$$

The factor $A = \left(\frac{E_{n+1}}{\sum_{i=1}^{i=n} E_i}\right)$ can be seen as the weighted average number of years of experience. In most cases the historical experience won't be more than 15 years. However, within the available loss history it is possible to observe historical CAT losses that may not happen again for the next 3 decades. Hence, it is possible that the basic CAT price as calculated above is pessimistic and that it does not reflect the true exposure to CAT risk. In this case it is possible to readjust the price. For example if the factor A is 0.1 i.e. 1 in 10 years, it can be reassigned a value of 0.04 i.e. 1 in 25 years. These adjustments are purely based on personal judgement/expert judgement and understanding of the underlying risk.

c) *Exposure rating*

Introduction

Exposure rating is suited for per risk cover pricing. In ANV we have developed an approach that consist in using the exposure curve in order to have an approximate CAT price or rather a starting point to help actuaries and underwriter to come up with a reasonable price.

This method can be useful if there is no CAT losses in the historical data and if return periods are hard to define.

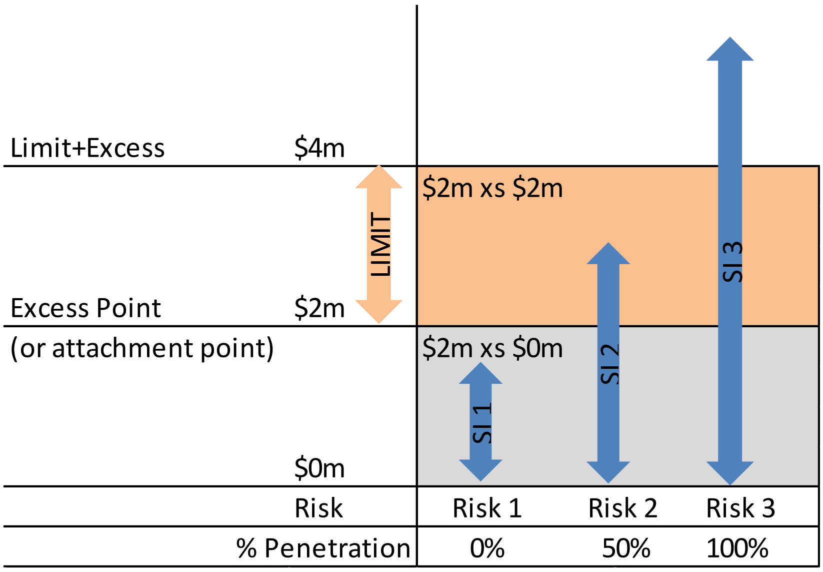
The idea is to use the total loss probability of each risk. In his paper "The Swiss Re exposure curve and the mbbefd distribution class" Stefan Bernegger shows that once an exposure curve has been selected a total loss probability can be derived. The total loss probability is then multiplied by the number of risks that falls into the layer. The resulting number is in fact a CAT ROL (Rate On Line). It means that if it is 1% then 1 in 100 years the full layer will be used. This approach makes sense as in the situation of a CAT all the risks that have an insured value falling into the layer will experience a total loss.

Illustration

In the exposure rating per risk cover section, the following parameters have been defined.

$$d = \frac{1}{g}$$ $$g_c = e^{(0.78+0.12c)c}$$

The total loss probability for a given exposure curve is basically the inverse of the g parameter. The g parameter is entirely defined by the selected c parameter. Let's assume there are n live risks in the reinsured's portfolio. Each risk has a sum insured noted SI_i for the risk number i. Some risks remain below the retention (or excess point) of the reinsurance contract. Some are partially going through the reinsurance contract. And some risks entirely go through the reinsurance contract. The diagram below illustrates this. The risk one has 0% penetration. The risk 2 has 50% penetration and the risk 3 has 100% penetration.



Let's write pr_i the penetration rate of risk I into the reinsurance layer. The penetration rate is calculated as:

$$pr_i = 100\% \; if \; SI_i \geq Limit + Excess$$

$$pr_i = 0\% \; if \; SI_i \leq Excess$$

$$pr_i = \frac{SI_i - Excess}{Limit} \; if \; Excess \leq SI_i \leq Excess + Limit$$

The total loss probability for the reinsurance layer, in other word the CAT rate On Line, can then be defined as:

$$CAT\ ROL = \left(\sum_{i=1}^{i=n} pr_i \right) \times d$$

For example, with the Lloyd's exposure curve (i.e. c=5) then the total loss probability for each risk is 0.1008%. Let's assume that the reinsurance layer is $2m xs $2m and that the reinsured's portfolio is made of 20 risks and each risk has an insured value of $10m. Therefore a total loss on each risk would produce a total loss to the reinsurance layer. The penetration rate is 100% for each risk. The CAT ROL is then 2.0156% = 20 * 0.1008% which is a 1 in 50 years return period. It can be noted that it is assumed that every risk will have a total loss and that the reinsurance contract will pay 20 times. The limitations of this approach are therefore listed below:

Limitations

The limitations of this approach are:

- The geographical spread of risks is not taken into account. It actually assumes that if a CAT happens all the risks in the book will experience a total loss from the CAT. For example in the case of a hurricane wiping Florida, it would assume that all the properties insured in the underlying book are there.
- It assumes that every risk will experience a total loss which may not be true as some risks may experience partial losses. In the case of a hurricane, some properties may have a location which is less exposed to the hurricane hence creating a lower destruction rate.
- It does not take into account the reinsurance contract terms in particular the AAL and the AAD.

Hence as long as the modeller understands the limitations of this approach and has a good understanding of the underlying risks (geographical spread and likelihood of being a total loss) the CAT ROL derived from the exposure curve can be used as a good starting point. This is again just an additional method to be used in conjunction with the other methods.

d) Frequency-Severity model

As discussed in the section per risk cover, a frequency-severity model can be used to price CAT covers. A Pareto curve can be used, instead of a truncated pareto, as it is heavy tailed and hence can produce some very large CAT claims. However, given the low number of CAT claims it might be hard to calibrate a Pareto curve. A standard approach would be to assume a standard shape parameter that is generally accepted within the market practises.

e) Event loss tables

An alternative to all these methods is to use outputs from CAT modelling firms such as RMS, AIR or Equicat. Models exist to simulate weather scenarios. The location of each risk is set on a map and the impact (destruction rate) of the weather scenarios on assets is then estimated. The output of the model is an event loss table. Each event has an ID with a probability of occurring; an average loss amount of the underlying book is calculated as well as a standard deviation. It is then possible to estimate the price of the CAT cover by simulating each event using a Poisson distribution (its mean is the probability for that specific event to occur) and a lognormal distribution using the average loss amount and its standard deviation. The simulated losses are passed to the layer and recoveries are calculated accordingly.

3. Adjustments to be made

a) Reinstatement premiums

Let's take the example given in the experience rating per risk covers.

The estimated loss to layer calculated above did not take into account the reinstatement premiums (RIPs) that the reinsured has to pay the reinsurer in order to reinstate the cover. There are two ways of dealing with this depending on whether you are in a lead or follower position.

- If the reinsurer is a lead, a quote has to be provided to the broker; hence the ROL is unknown and is precisely what has to be provided.

A probabilistic approach can be used in order to estimate the ROL. The estimated loss to layer calculated in sections above B.1.a i.e. $1,059,798 is divided by the limit of the reinsurance contract. This provides a Loss On Line (LOL).

$$LOL = \frac{Expected\ Loss\ to\ Layer}{Limit}$$

In this case the LOL is 53%. It means that on average roughly every other year the coverage will be entirely consumed. This can be seen as if there is 53% chance for a complete exhaustion of the layer to happen. As this is an expected value, let's assume the exhaustion of the layer follows a Poisson distribution which mean and variability is LOL.

Let's assume that there is only 1 reinstatement in place at 50%. As the reinsured will pay additional premium to the reinsurer in order to reinstate the cover then the ROL that will be charged has to be cheaper than the LOL. The ROL is the price to charge when the reinstatement is part of the contract. For the reinsurer its position should be neutral i.e. should be the same with or without reinstatement. Hence we can write the following equation where ROL is the unknown.

$$LOL = P(N = 0) * ROL + P(N > 0) * (1 + RIPperc) * ROL$$

Where N is the number of times the coverage is consumed. P(N=0) is the probability of having the coverage not consumed under a Poisson distribution.

$$P(N = k) = \frac{e^{-LOL} * LOL^k}{k!}$$

The formula above means that if there are no reinstatement the reinsured will pay LOL to the reinsurer. If there is 1 reinstatement @ RIPperc (in our example this is 50%), then the reinsured will pay either ROL if there are no losses i.e. the coverage is not consumed at all or (1+RIPperc) times

ROL (in our example 1.5 times ROL) if there are at least one loss that consumes the whole coverage. For the reinsurer both situations have to be equivalent. The formula above can be rewritten as:

$$ROL = \frac{LOL}{P(N=0) + P(N>0) * (1 + RIPperc)}$$

$$ROL = \frac{0.53}{e^{-0.53} * 0.53 + (1 - e^{-0.53} * 0.53) * 1.5} = 0.394$$

Therefore the price to charge for that cover with 1 reinstatement @ 50% on a per risk basis is 39.4% for every $USD of coverage.

This formula can be generalised to several reinstatements @ various percentages as follow:

$$LOL[r, LOL, RIPperc()] = \left(\sum_{k=0}^{r-1} P(N=k) \times ROL \times \left[1 + \sum_{j=0}^{k} RIPperc(j)\right]\right) + \left(P(N>r-1) \times ROL \times \left[1 + \sum_{j=0}^{r} RIPperc(j)\right]\right)$$

Where
r:number of reinstatement r>0
RIPperc():vector of Reinstatement percentages with a length of r and RIPperc(0)=0 by default

- If the reinsurer is a follow then the ROL is already determined and the question to answer for the reinsurer is the profitability of the contract.

Let's pursue the analysis with the example given in IV.B.1.a.

The contract was paying 20% ROL for $2m xs $2m. It means that the Minimum & Deposit premium is $400k.The following reinstatement premiums (RIP) have been calculated for each year. The average RIP payment as a percent of the loss to layer has been calculated as 6.9%.

Accident Year	Adjusted GNEPI ($USD)	Final Loss to Layer ($USD)	Burning Rate	Reinstatement premium	RIP as % of Loss to Layer
Y1	45,758,070	1,000,000	2.19%	100,000	10.00%
Y2	55,359,647	1,000,000	1.81%	100,000	10.00%
Y3	45,435,803	4,000,000	8.80%	200,000	5.00%
Y4	37,761,806	-	0.00%	-	0.00%
Y5	29,768,663	448,077	1.51%	44,808	10.00%
Y6	35,370,200	-	0.00%	-	0.00%
Total	**249,454,189**	**6,448,077**	**2.58%**	**444,808**	**6.90%**

	Estimated Loss to Layer ($USD)	ROL	M&D Premium ($USD)	Loss Ratio	Estimated RIP	Adj. LR for RIP
Renewal	1,059,798	20%	400,000	265%	73,108	224%

As the expected loss to layer is $1.1m for the renewed contract, the resulting estimated RIP is $73k assuming 6.9%. The loss ratio to the reinsurer is 265% and the adjusted loss ratio is 224% which is a reduction thanks to the additional premium income.

b) Brokerage and loading for profitability

Once the burn rate has been calculated i.e. the loss to the reinsurer for every $USD of premium written by the reinsured, adjustments have to be made for the brokerage fees as reinsurance contracts are always placed through the help of a broker like AON, Willis, JLT or Marsh to quote the biggest 4 ones in the world. In general on excess of loss contracts the brokerage is 10% of the reinsurance premium and sometimes 15%. This is quite high compared to quota share where brokerage fees range from 2.5% to 5%. This is due to the complexity of placing an excess of loss reinsurance programme.

Another adjustment that needs to be made is the profit margin. The reinsurer has to make money from the reinsurance contract sold. Depending on market conditions and the risk appetite of the reinsurer profit margins can range from 40% to 15%.

Hence the technical rate, i.e. the rate that needs to be charged in order to allow for brokerage fees and profit margin can be written as:

$$Technical\ Rate = \frac{Burn\ Rate}{(1 - Profit\ Margin) \times (1 - Brokerage\ Fee)}$$

C. Other considerations

This section attempts to highlight additional aspects that need to be considered when pricing an XoL layer.

1. The issue of free cover

Introduction to the issue

Let's take the same example as used in IV.B.1.a.

On the premium side the following was provided.

Calendar Year	GNEPI ($USD)	Rate Change	Inflation	RC. Factor	Inf. Factor	Adjusted GNEPI ($USD)
Y1	42,000,000	4%	3%	0.95	1.15	45,758,070
Y2	50,000,000	-2%	0%	0.96	1.15	55,359,647
Y3	41,000,000	-3%	3%	0.99	1.11	45,435,803
Y4	35,000,000	1%	2%	0.99	1.09	37,761,806
Y5	29,000,000	0%	5%	0.99	1.04	29,768,663

Y6	34,000,000	-4%	3%	1.03	1.01	35,370,200
Renewal	41,000,000	3%	1%			41,000,000

On the claims side the following was provided. Note that the large $25m claims is a CAT claim and will be excluded in this analysis.

Accident Year	Incurred Loss ($USD)	LDF	Inflation	Inf. Factor	Ultimate Adj. Loss ($USD)
Y1	5,700,000	1.00	3%	1.19	6,806,098
Y2	3,652,000	1.00	3%	1.16	4,233,669
Y3	4,543,000	1.01	3%	1.13	5,189,107
Y3	2,594,000	1.01	3%	1.13	2,962,920
Y3	3,304,000	1.01	3%	1.13	3,773,896
Y3	3,366,000	1.01	3%	1.13	3,844,714
Y5	25,000,000	1.06	3%	1.06	28,113,850
Y5	2,901,000	1.06	3%	1.06	3,249,139
Y6	1,694,000	1.26	3%	1.03	2,198,938
Y6	1,526,000	1.26	3%	1.03	1,980,861

Let's rate two layers A and B with the following characteristics

Treaty Terms	**Layer A**	**Layer B**
Excess	2,000,000	2,000,000
Limit	5,000,000	8,000,000
AAD	-	-
AAL	20,000,000	20,000,000
1st Reinstatement	0%	0%
ROL	20%	20%

Once the losses are passed through the layer A the following result is obtained:

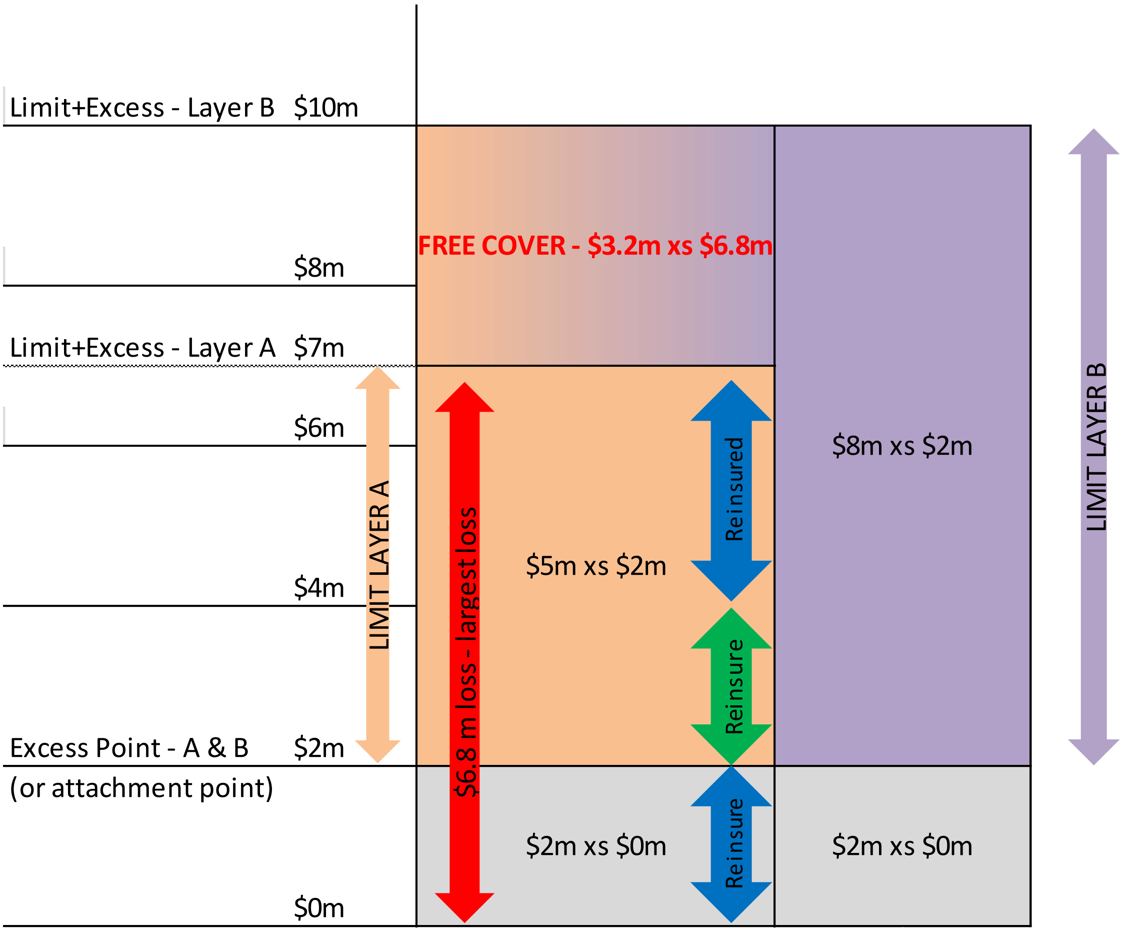
Summary by accident year and loss cost - Layer A			
Accident Year	Adjusted GNEPI ($USD)	Final Loss to Layer ($USD)	Burning Rate
Y1	45,758,070	4,806,098	10.50%
Y2	55,359,647	2,233,669	4.03%
Y3	45,435,803	7,770,637	17.10%
Y4	37,761,806	-	0.00%
Y5	29,768,663	1,448,077	4.86%
Y6	35,370,200	-	0.00%
Total	**249,454,189**	**16,258,482**	**6.52%**

So the final burn rate is equal to 6.52% of the total subject premium.

On other hand with the layer B, the following results are obtained:

Summary by accident year and loss cost - Layer B			
Accident Year	Adjusted GNEPI ($USD)	Final Loss to Layer ($USD)	Burning Rate
Y1	45,758,070	4,806,098	10.50%
Y2	55,359,647	2,233,669	4.03%
Y3	45,435,803	7,770,637	17.10%
Y4	37,761,806	-	0.00%
Y5	29,768,663	1,448,077	4.86%
Y6	35,370,200	-	0.00%
Total	**249,454,189**	**16,258,482**	**6.52%**

The results are basically the same. This is due to the fact that the largest adjusted loss in the claims history is $6.8m. It means that no losses penetrate the coverage beyond that point. Hence the coverage given by the layer B from $6.8 up to $10m is basically a free cover as no losses penetrate into that zone to produce a burning cost. The diagram below illustrates this.



A solution

A solution to solve the issue of free cover is to use exposure rating as a proxy. Exposure rating could be used to price the following layers: \$5m xs \$2m and \$3m xs \$7m. The price difference between these two layers can be used as a proxy to price the \$3m xs \$7m layer for experience rating as summarised in the table below. The price for the \$3m xs \$7m in exposure rating is half of the \$5m xs \$2m layer's price. Therefore it is assumed that the \$3m xs \$7m layer in experience rating is half of the 6.52% cost established for the \$5m xs \$2m layer. The final total price is therefore 9.78% instead of the initial 6.52%.

Burn Rate	**Experience rating**	**Exposure rating**
\$5m xs \$2m	6.52%	8.00%
\$3m xs \$7m	***3.26%***	4.00%
\$8m xs \$2m	**9.78%**	**12.00%**

However, one can argue that the issue of free cover is not an issue in fact but rather reflects, assuming that the reinsured has sufficient loss history, that the underlying book is actually performing well. Therefore some credibility should be given to that good performance. Although that cover should not be priced at zero as the reinsurer by providing this coverage has to put up some capital against it. Hence the "correct" price should be somewhere between a minimum which is the cost of capital for the reinsurer to provide that cover and the experience rating price calculated above thanks to the help of the exposure rating.

The following section proposes one method to calculate a credibility factor. This method is rather basic but its simplicity makes it easy to use. More sophisticated methods exist but have not been taken into account in this work.

2. Credibility between experience vs. exposure rating

a) Introduction

Credibility factor is used in insurance to reward (penalize) risks that are better (worse) than the average risk in a portfolio. In order to achieve this, a credibility factor has to be calculated. The resulting premium for a risk is a weighted average between an a priori risk premium that would represent an average risk premium for the portfolio and a risk premium solely based on the insured own loss history.

$$\boxed{\widehat{\mu_\alpha} = \alpha \times \bar{X} + (1-\alpha) \times \mu_0}$$

$\bar{X}$: The average risk premium of the insured based on its own loss history
μ_0 : The average risk premium for the portfolio
α : The credibility factor
μ: The exact risk premium that should be charged
$\widehat{\mu_\alpha}$: The estimated risk premium based on credibility factor α

There are mainly two approaches to calculate credibility factors. The classical approach and the Bayesian approach.

The Bayesian approach is the best one as it produces an estimator with the lowest variance, in other words the error, in the estimation of the exact premium compared to any other approach.

$$E[(\widehat{\mu_B} - \mu)^2] \leq E[(\widehat{\mu_Z} - \mu)^2]$$

$\widehat{\mu_B}$: The estimated risk premium based on Bayesian credibility factor
$\widehat{\mu_Z}$: The estimated risk premium based on classical credibility approach
E: The expected value

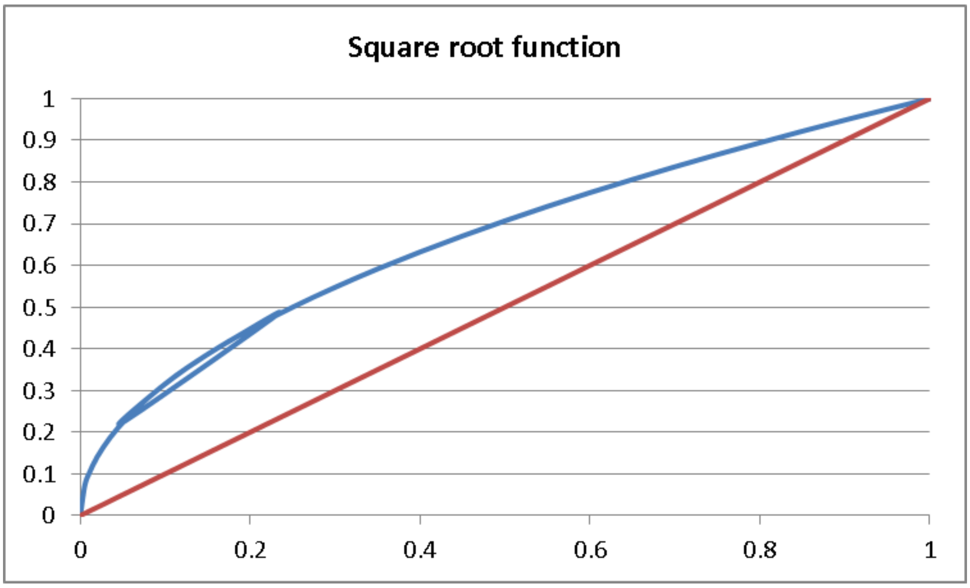
b) Classical credibility

A classical approach to credibility is to use a simple formula to derive the credibility factor.

$$\alpha = \sqrt{\frac{n}{F}}$$

Where F is the expected a priori value and n is the observed value. The square root function, is concave between 0 and 1, hence it enables to give greater credibility to the experience as soon as some experience is observed against the expectations.

For example if F represents the expected claim frequency says 10. If a risk in a portfolio has on average over the last 10 years 5 claims then the ratio would be 50%. In order to give more credibility to the experience the square root function is used and the resulting credibility is 70%.



This approach is easy and straightforward to put in practice. However, it is not the best credibility approach. The risk premium calculated using this approach is an estimator of the true exact risk premium. The variance (or error) to the exact risk premium can be reduced using another approach, the Bayesian approach.

c) Bayesian Credibility

Bayesian approach

The exact risk premium for a risk in a portfolio can be defined as:

$$\mu(\theta) = E[X_{n+1}\,|\theta]$$

X is a random variable representing a loss for a risk in the portfolio. X is following a distribution that is defined on a set of parameters θ . Therefore the exact risk premium to charge in year n+1 is the expected value of the random variable X.

On other hand, the portfolio is made of various groups of risks. The θ set of parameters is used to reflect the difference between these groups of risks. The θ set of parameters is assumed to be a random variable and to follow a distribution which CDF can be noted U.

In that case, the average risk premium for the portfolio can defined as:

$$\mu_0 = \int_\theta \mu(\theta)dU(\theta) = E[X_{n+1}]$$

The integral above is simply a weighted average of each exact risk premium for each risk. The weights are the probability of having a certain θ parameters.

The Bayesian premium can then be defined as:

$$\mu_B(\theta) = E[\mu(\theta)|X]$$

The Bayesian risk premium means that it is conditional to the historical loss experience of the risk considered. X in the equation above represents the set of historical losses from year 1 to n.

In order to calculate the Bayesian premium stated above, one needs to know the underlying distribution of the random variables X and θ. In practice this is almost impossible to know. On other hand, even if these distributions were known, an analytical result for the Bayesian premium might not exist. Even if it does it might be too complicated to implement.

The idea of credibility is to weight the average experience against individual experience in a linear formula using a credibility factor, as shown in the introduction, without the need of defining distributions and solely based on historical loss information.

A linear Bayesian approach

The Buhlmann model is a Bayesian linear approach to derive a credibility weighted risk premium. If a portfolio is made of I risks and that the portfolio has n years of history.

The formula is:

$$\widehat{\mu_B(\theta)} = \alpha\bar{X}_i + (1-\alpha)\bar{X}$$

Where:

$\alpha = \frac{n}{n+\frac{\sigma^2}{\tau^2}}$: Is the credibility factor

$\bar{X} = \frac{1}{n \times I}\left[\sum_{i=1}^{I}\sum_{j=1}^{n} X_{ij}\right]$: is the natural estimator of μ_0 the average risk premium for the portfolio

$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}$: is the average risk premium of the risk I solely based on its own experience

X_{ij}: Is a loss in year j for the risk i

The estimators of σ and τ are unbiased and convergent, defined as:

$$\widehat{\sigma^2} = \frac{1}{I(n-1)}\sum_{i=1}^{I}\sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2$$

$$\widehat{\tau^2} = \frac{1}{I-1}\sum_{i=1}^{I}(\bar{X}_i - \bar{X})^2 - \frac{\widehat{\sigma^2}}{n}$$

σ: It represents the internal variability of an individual risk over years. τ: It represents the overall variability in the portfolio between risks.

Some comments can be made regarding the credibility factor.

- The higher the number of years of experience n, the higher is the credibility factor. This makes sense as the more data is available the more credible is the experience of each individual risk.
- The smaller the variability of a risk over years, the bigger the credibility factor. If a risk has every year an average loss that doesn't vary much then trust can be put into experience rating.
- The bigger the variability in the portfolio across risks, the bigger the credibility. If each risk is very different from each other then it is better to rely on each risk's experience.
- The Buhlmann credibility factor is simple to execute, it is linear and Bayesian, however it is not as good as a pure Bayesian credibility factor. The estimation error in the risk premium is higher in this case but still better than the classical credibility approach.

The Buhlmann and Straub model could be introduced as well but the only difference with the Buhlmann model is that weights of each risk are taken into account. In an insurance context, that weight could be the amount of exposure that each risk represents.

d) *Bayesian vs. Classical approach*

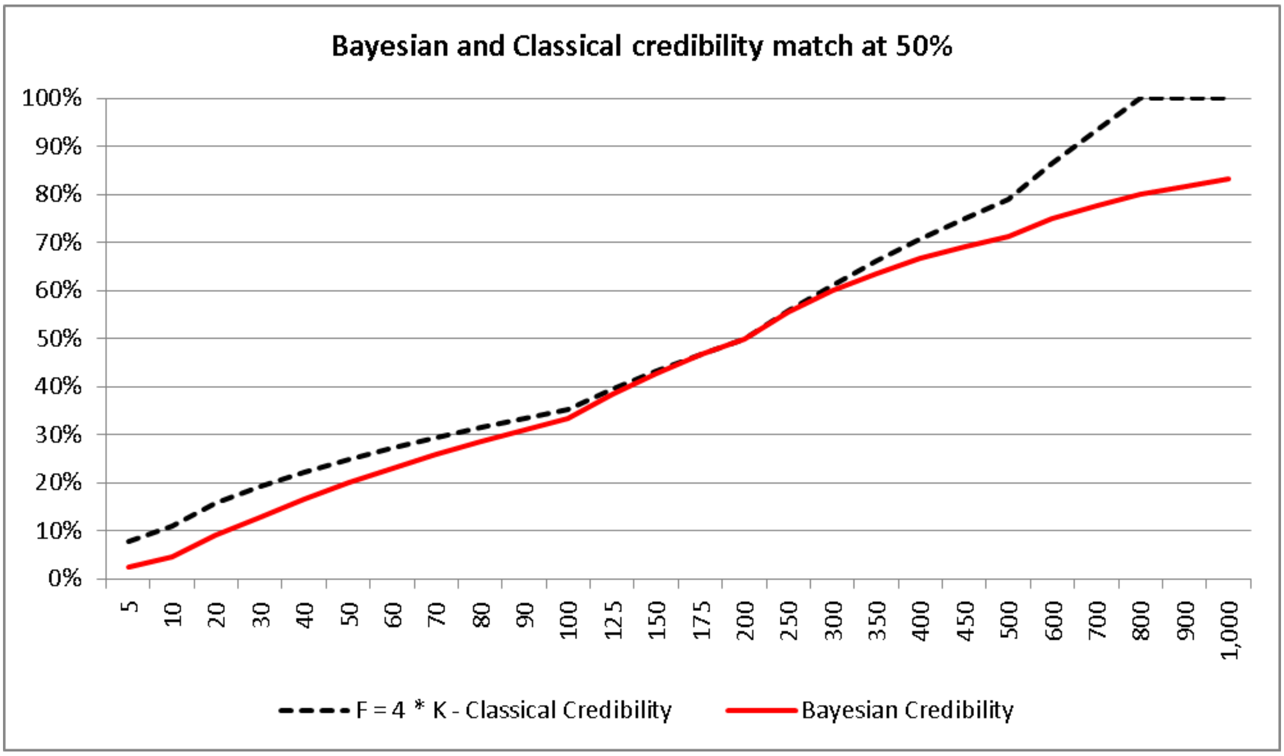
In his paper "An actuarial note on credibility parameters" Howard C. Mahler shows that it is possible to use the classical approach as a good approximation of the Bayesian approach.

A Bayesian credibility factor can be written as $Z_B = \frac{n}{n+k}$ where $k = \frac{\sigma^2}{\tau^2}$ as explained in the section above. The classical approach can be written as $Z_C = \sqrt{\frac{n}{F}}$. The number n can represent the exposure amount i.e. number of years of experience or the number of claims observed.

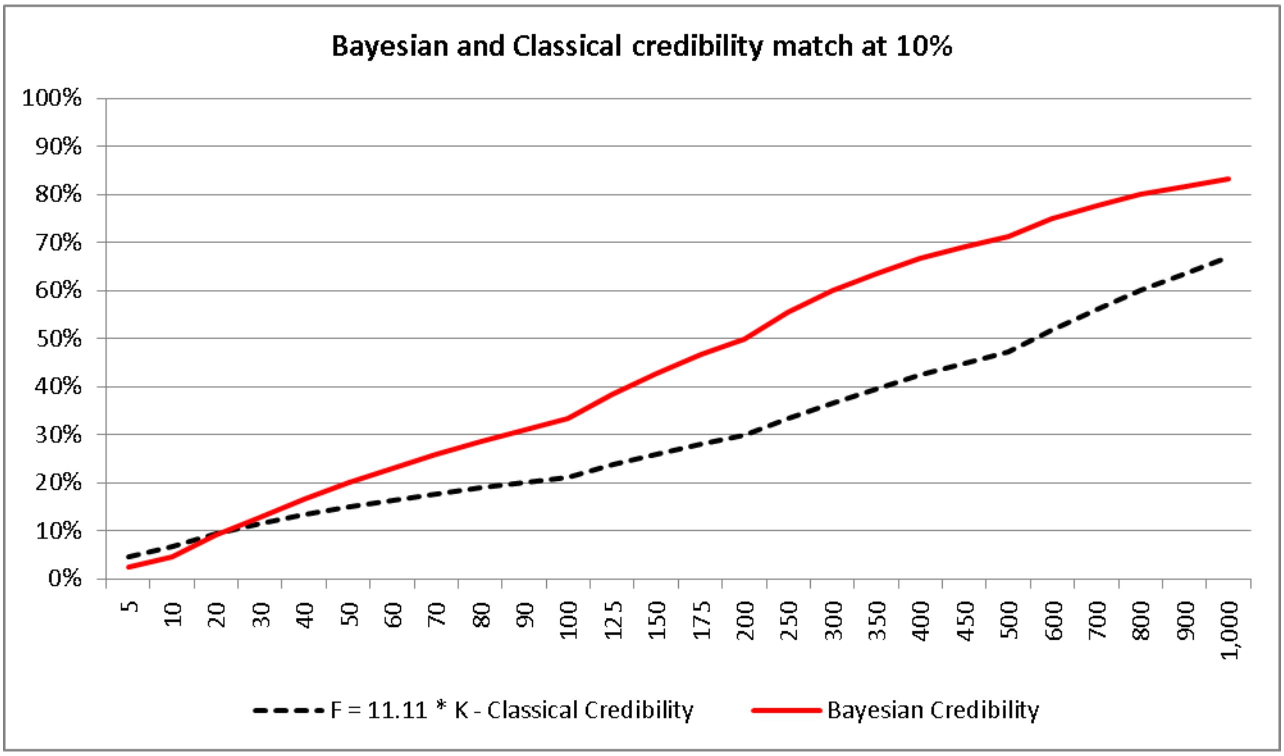
The author attempts to find a relation between the factor k and the factor F that would reduce the difference between the two credibility factors. Let's find the relation between k and F that would give $Z_B = Z_C = Z$. After a few calculation this leads to $k = FZ(1 - Z)$

Let's define $R = \frac{F}{k}$. Then we have $\frac{1}{R} = Z(1 - Z)$.

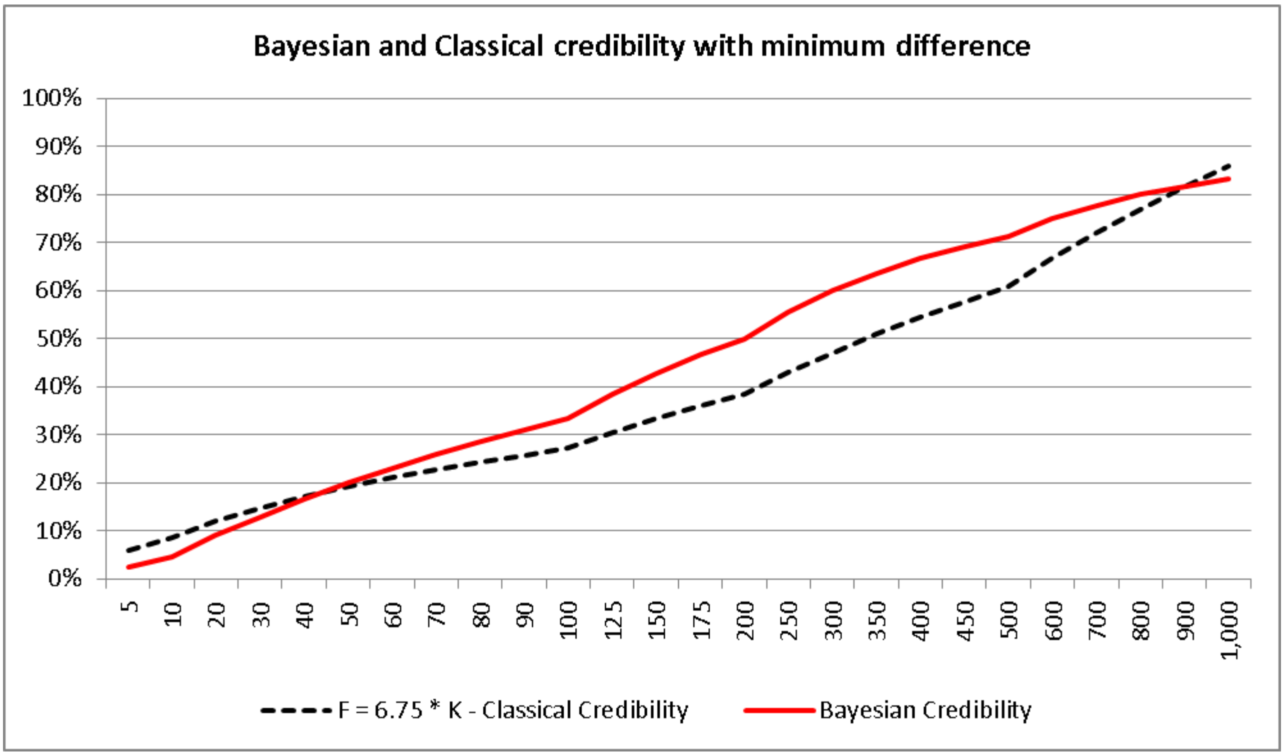
If one want the two credibility curves to match at Z=0.5 then the value for R is 4. Let's assume that there is enough historical data and that the factor k has been calculated using the Buhlmann credibility as explained in the previous section. Then in order to have the classical credibility to match the Bayesian credibility at 50% one need to take F to be equal to 4 times k.



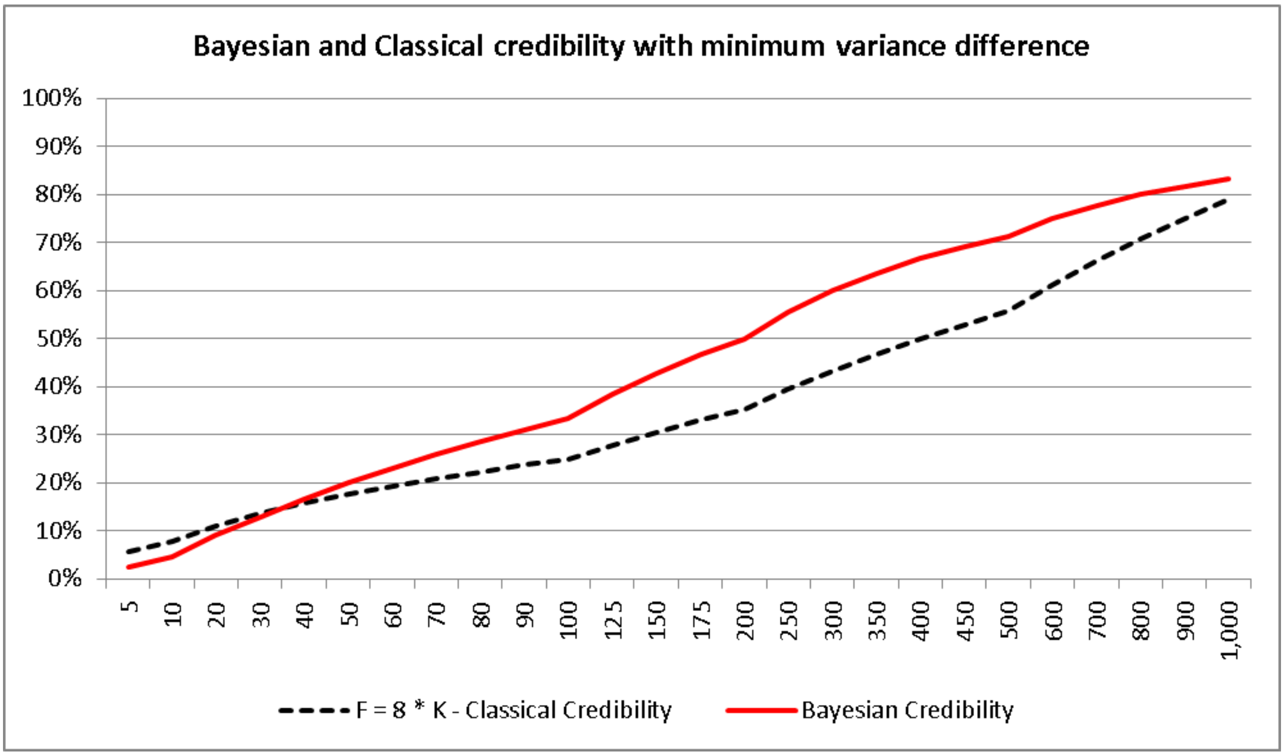
If one want the two curves to match for Z=0.1 then the R factor is around 11.11.



However, one is interested in having the two curves as close as possible over the whole range. Hence one criterion could be to choose the factor R such that the maximum difference between the two curves is reduced to a minimum. In his paper, the author finds that for a value of R=6.75 this is achieved.



As explained in the section about the Bayesian approach, it provides the best credibility factor in terms of variance or error. Hence any other approach chosen can only increase the variance i.e. the error from the true exact value. Therefore another criterion could be to choose the R factor such that the increase in variance is minimal. The author founds that this is achieved for a value of R=8.



Overall the credibility theory is used as a guidance to establish a final risk premium in order to take into account the insured's own experience. The work above shows that the differences between the classical and the Bayesian approach are not that significant. In the real world applications this level of accuracy can be more than acceptable as other considerations such as the market rates, the overall portfolio strategy, the aggregation of risks, the cost of capital have to be taken into account when writing policies.

e) Application to reinsurance

Introduction

In reinsurance, it is hard to adopt a Bayesian approach for credibility as the amount of data given the nature of the product is very small. Only a few claims in ex cess of a certain threshold exist. For that reason the classical credibility approach has been used and implemented in the reinsurance rater that ANV uses.

$$Z_c = \sqrt{\frac{n}{F}}$$

In the sections above the credibility was about a risk with its own experience against the overall portfolio experience. In reinsurance the credibility weighting is between exposure rating and experience rating.

The method

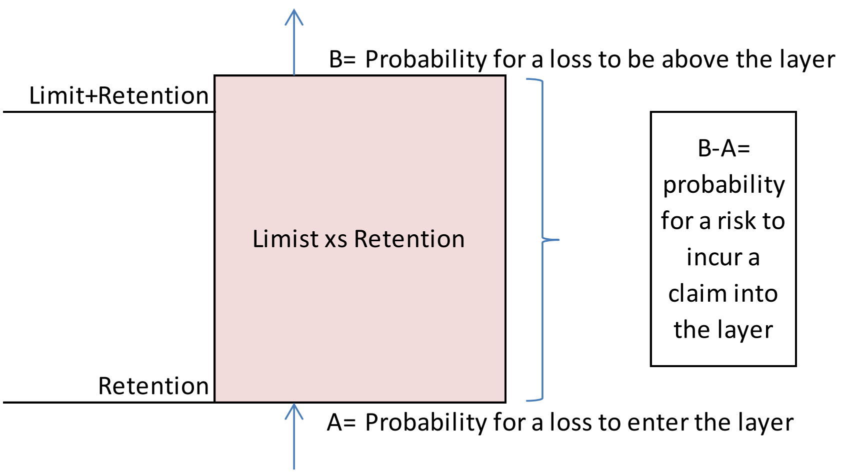
In classical credibility the factor F would be the expected claim count or average claim of the overall portfolio. In reinsurance the factor F would be the claim count expected using exposure rating. The factor n would be the average on levelled claim count from experience.

The latter is relatively easy to calculate. See section IV.B.1.a where claims are on levelled for inflation, premium for rate changes. The number of claims above the retention is calculated and divided by the on levelled historical premium in order to have an average frequency. The renewing premium is then applied to the frequency in order to estimate the claims count for the renewing exposure.

To calculate the parameter F, the exposure curve is used. We use the exposure curves to calculate the probability of having a claim to the layer. The derivative of the exposure curve actually provides the survival distribution of the underlying claims (see section C.2.b.2).

$1 - F(x) = \frac{G'(x)}{G'(0)}$ It represents the survival distribution of the random variable x i.e. the probability of having a loss in excess of x. It appears that it is equal to the derivative of the exposure curve at the point x divided by the derivative of the exposure curve at its origin. In this case the probability A and B shown on the diagram below are calculated as follow:

$$B = \frac{G'(x_2)}{G'(0)} \text{ and } A = \frac{G'(x_1)}{G'(0)} \text{ where } x_2 = \frac{Limit + Retention}{Insured\ Value}\ ; x_1 = \frac{Retention}{Insured\ Value}$$



The difference between B and A provides the probability for a risk in the portfolio to fall into the reinsurance layer. The expected number of claims expected to fall into that reinsurance layer is then obtained by multiplying that expected probability by the number of risks in the portfolio. The resulting number is the factor F.

This approach is applied individually for each band in the risk profile.

Limitations of classical credibility

- In the situation where the reinsured does not have a single claim for 10 years, the method would give 100% credibility to the exposure rating and 0% to the experience rating. However, even if a credibility factor for the clean experience should be given on working layers, this method would always suggest giving more weight to the exposure rating on top layers which is the reasonable selection to make.
  Consideration therefore needs to be given to the expected return period (market Rate on Line) and the number of historical years and claims provided.

- The method is purely based on the claims frequency. The severity of the claims is not taken into account, which might be seen as a limitation.

It has to be noted that the limitations of this method are well understood by the Actuarial and Underwriting Teams. The Underwriter has the ability to select a different credibility factor when he judges that the credibility factor suggested by the Pricing Tool is not appropriate.

V. Applications on a Cargo book

The theory explained in the section IV has been implemented into an Excel based rating tool. The Excel rater does experience rating and exposure rating. The rating engine is using the software R in order to do simulation and curve fitting. Rexcel is used in order to have R and Excel to work together.

In the next sections, one numerical applications will be made on marine cargo data. The data stems from ANV's clients. The rating engine is connected to an Access database. Hence every time a client's data is used in order to price XoL reinsurance layers every information is recorded into the database. The most important information for modelling are:

- The clients' income
- The clients' risk profile (exposure table)
- The clients' historical losses

Due to the privacy of the data, the data of multiple clients was aggregated making it impossible to find out what the original data for each client was.

A. Data

The data is made of 22 clients that ANV is reinsuring.

Loss Data

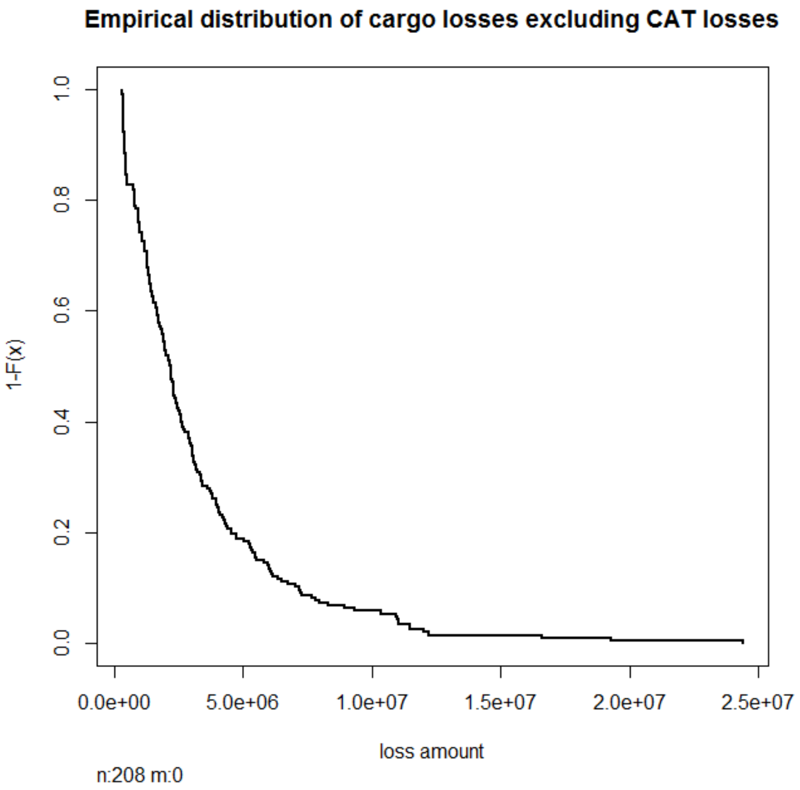
There are 211 losses attaching to year of accounts that spans from 2005 to 2014. The losses are in excess of $200k. The table below summarise the data.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
294 900	977 100	2 196 000	4 669 000	4 062 000	201 000 000

There are 3 losses that have been categorized as CAT losses. These are:

YOA	Incurred	On Levelled
2011	185 651 748	200 955 423
2014	55 874 000	71 826 362
2009	53 138 599	59 842 693

The remaining 208 losses have been classified as risk losses. The losses have been developed to ultimate using Lloyd's benchmark development pattern. An inflation of 2% has been applied across all the years to bring the losses to today's terms. The empirical distribution of the losses can be viewed on the plot below.



Exposure Data

The exposure data of the 22 clients has been aggregated in the following bands. This represents all the live exposures that the clients have in their portfolio. For example there are 21 risks that have an insured value in between $300m and $400m. The total premium collected from these risks amount to $11 994 376.

Cargo			
LB	**UB**	**Premiums**	**Risk Count**
-	100 000	5 262 829	3 586
100 001	250 000	49 873 086	9 075
250 001	500 000	177 097 323	29 248
500 001	750 000	8 324 734	1 200
750 001	1 000 000	5 818 036	930
1 000 001	1 500 000	30 108 216	1 995
1 500 001	2 000 000	90 356 030	4 198
2 000 001	3 000 000	209 243 751	32 161
3 000 001	4 000 000	175 393 955	5 661
4 000 001	5 000 000	23 403 440	1 013
5 000 001	6 000 000	7 339 903	178
6 000 001	7 000 000	86 049 350	1 874
7 000 001	8 000 000	87 353 415	73 055
8 000 001	9 000 000	96 119 919	1 600
9 000 001	10 000 000	6 031 295	160
10 000 001	12 500 000	92 099 777	1 958
12 500 001	15 000 000	27 065 892	459
15 000 001	20 000 000	89 326 998	1 999
20 000 001	25 000 000	49 965 362	717
25 000 001	30 000 000	25 972 136	305
30 000 001	40 000 000	37 412 815	562
40 000 001	50 000 000	8 466 170	78
50 000 001	75 000 000	17 328 015	99
75 000 001	100 000 000	11 767 059	8
100 000 001	200 000 000	23 973 200	41
200 000 001	300 000 000	22 401 204	62
300 000 001	400 000 000	11 994 376	21

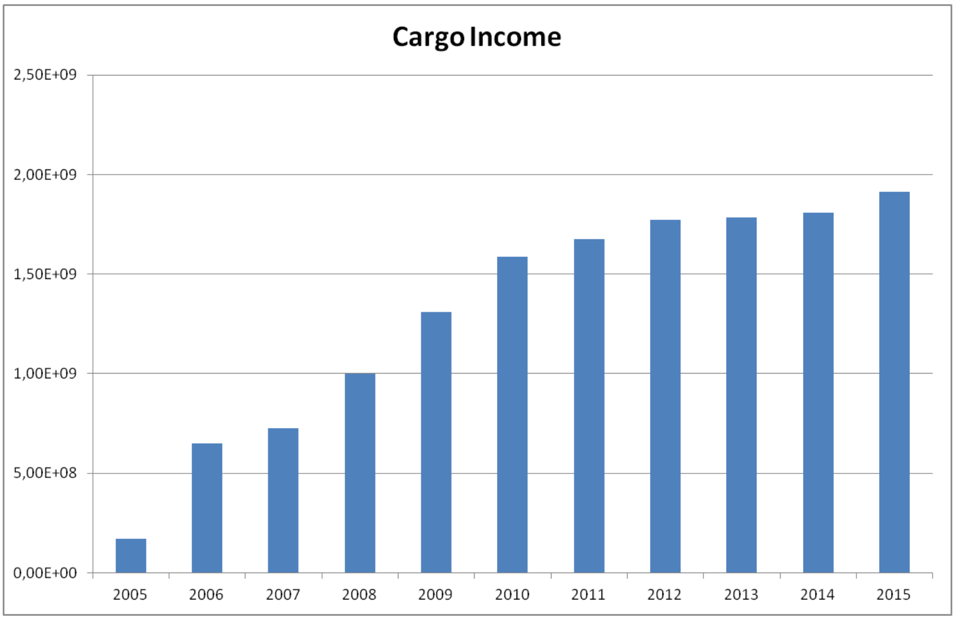
400 000 001	500 000 000	-	-
500 000 001	**1 000 000 000**	**-**	**-**
1 000 000 001	2 000 000 000	-	-

Income Data

The aggregated premium income for the 22 clients from Cargo insurance for years of account from 2005 to 2015 are as follow.

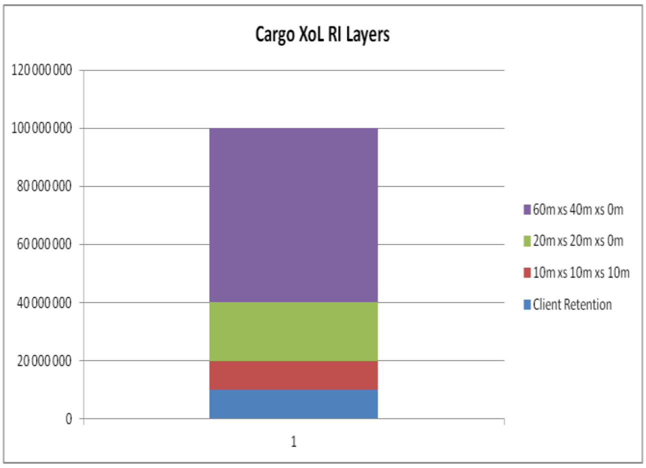
YOA	Cargo
2005	162 895 449
2006	623 863 846
2007	711 924 331
2008	1 009 610 760
2009	1 312 668 825
2010	1 587 196 120
2011	1 677 618 335
2012	1 773 742 338
2013	1 783 621 923
2014	1 808 200 347
2015	1 911 728 923

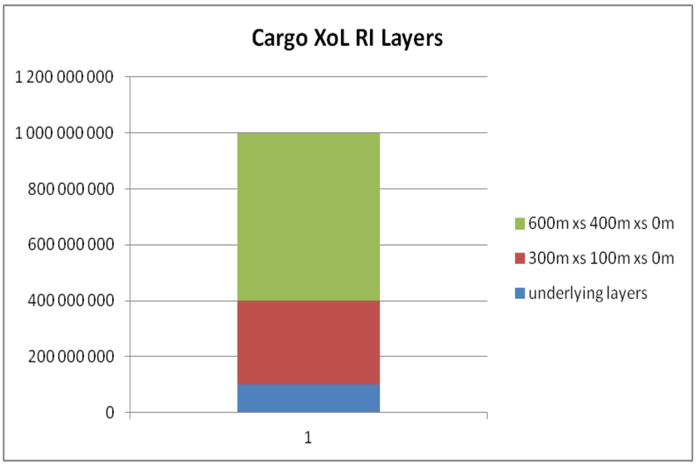
This can be summarised in the graph below. As it can be seen, the amount of business written has increased significantly since 2005. The premium has been on levelled for rate changes using the Lloyd's benchmark rate change for cargo business. No inflation adjustment has been made. This means that the underlying coverage have not changed. The primary insurer have not increased their lines i.e. the sum insured have remained stable.



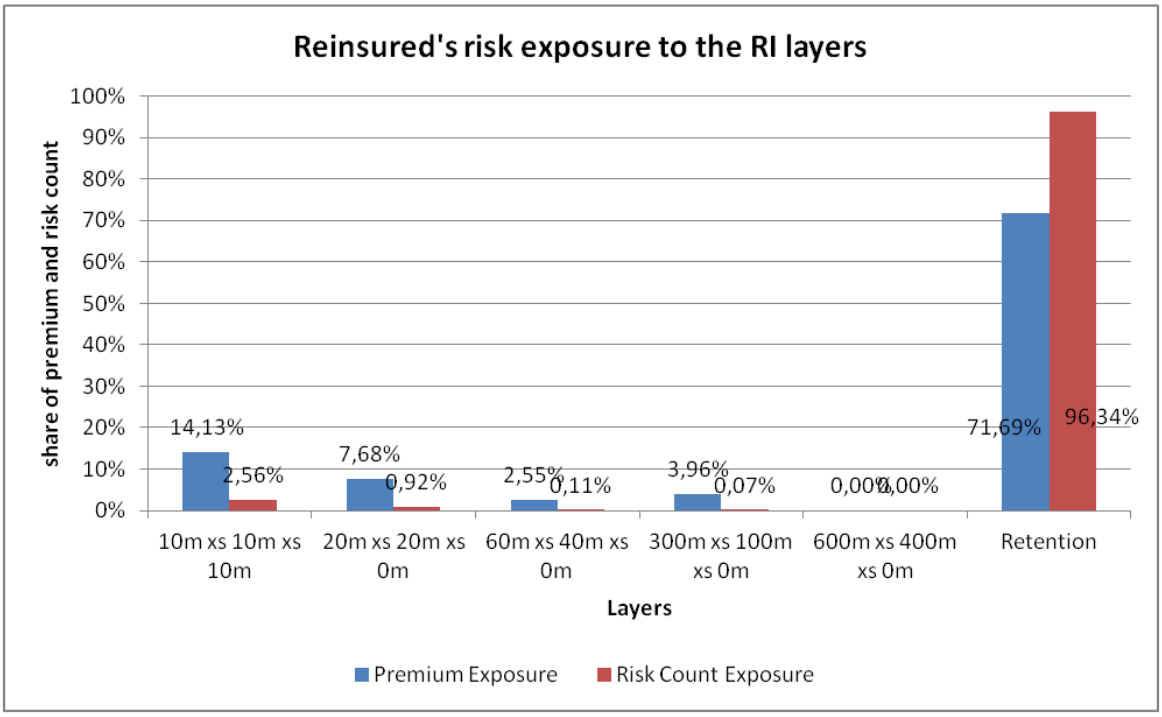
B. The reinsurance programme

Let's assume the following reinsurance programme has to be priced. There are 5 layers sitting on top of each other. The reinsured's retention is $10m. The highest layer is $600m xs $400m. This layer is a CAT layer as the highest sum insured is $400m. The first layer has an AAD of $10m. This is to protect the reinsurer as the first layer has a lot of loss activity. This is a working layer, historically 12 risk losses have touched the layer in addition to the 3 CAT losses. The first layer has 3 reinstatements at 100%, this is again because it is a working layer so it is very likely to be fully used by risk or CAT losses, hence the need to the reinsured to reinstate that layer several times. The second layer has 2 reinstatements. The third one has 1 reinstatement and the last two layers have none. The last two layers have a lower probability of being touched hence there is no need to have reinstatement on them.





Based on this programme the reinsured exposure is split as follow. It can be noticed that most of the premium exposure i.e.72% is below the retention point i.e. $10m hence the reinsurer is not exposed to those risks if a risk type loss occurs. If a CAT type loss occurs the reinsurer would be exposed to those risks. In terms of risk count, 96% of the risks are below the retention point. There are only a very few risks i.e. 0.07% of the total exposed to the $300m xs $100m layer, yet they represent almost 4% of the premium collected by the reinsured. The graph below clearly shows the motivation for an insurer to buy XoL reinsurance. Only a few risks have insured values beyond $10m. The insurer would like to cap its exposure to $10m yet writing these risks as they are paying a significant amount of premium. On other hand the insurer wants to protect itself against any potential CAT loss. In this case risks with any insured value may aggregate if a CAT happens and the cost to the insurer can reach the $billion mark. The total exposure of the insurer on average is around $850 billion. That is why the last layer has been bought.



If historical losses were put through the RI programme, the recoveries shown in the table below would have been made. It appears clearly that almost every year the first layer is producing recoveries. This is called a working layer. This is the reason why an AAD of $10m has been put in place in order to reduce the losses to the reinsurer. 2014 has $21m recoveries, this suggest to the reinsured that reinstatement should be purchased. The CAT loss of $200m that happened in 2011 goes through every layer and produce $100m recovery on the $300m xs $100m layer.

The issue of free cover can be seen in the table below. For the 4th layer where $300m of limit is provided, only $100m has been used in the last 10 years. If a pure experience rating approach was taken, the limit provided between $200m and $400m would be priced at nil i.e. would be free. The same thing can be said on the last layer where there is no losses. However the price for this layer can't be nil as the reinsurer provides its capacity and has to put capital due to solvency 2 regulations in order to provide this cover.

Historical Loss To Layers

YOA	10m xs 10m xs 10m	20m xs 20m xs 0m	60m xs 40m xs 0m	300m xs 100m xs 0m	600m xs 400m xs 0m
2005	10 958 783	4 433 328	-	-	-
2006	1 469 889	-	-	-	-
2007	-	-	-	-	-
2008	6 615 809	-	-	-	-
2009	10 000 000	20 000 000	19 842 693	-	-
2010	-	-	-	-	-
2011	10 331 899	20 000 000	60 000 000	100 955 423	-
2012	1 989 047	-	-	-	-
2013	4 649 369	-	-	-	-
2014	21 318 316	20 000 000	31 826 362	-	-

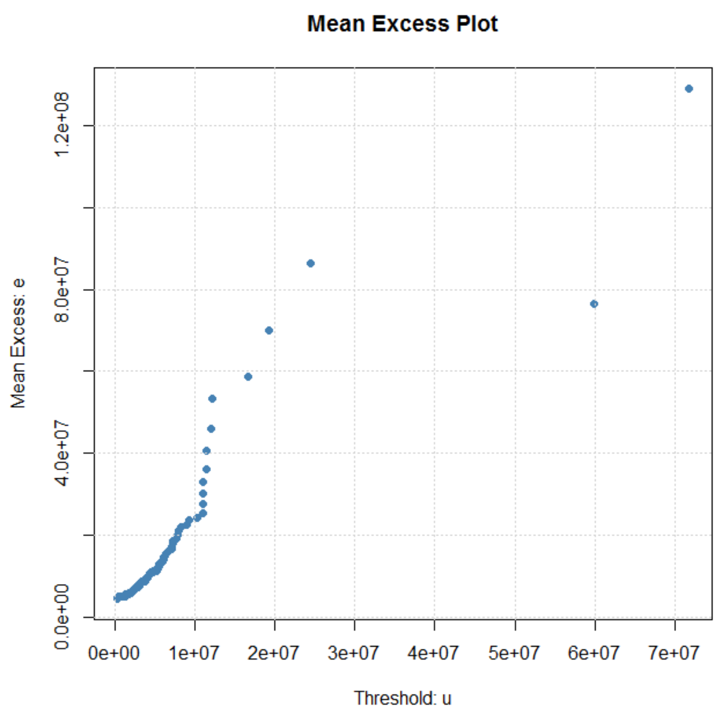
Historical Loss Count To Layers					
YOA	10m xs 10m xs 10m	20m xs 20m xs 0m	60m xs 40m xs 0m	300m xs 100m xs 0m	600m xs 400m xs 0m
2005	2	1	0	0	0
2006	1	0	0	0	0
2007	0	0	0	0	0
2008	1	0	0	0	0
2009	1	1	1	0	0
2010	0	0	0	0	0
2011	2	1	1	1	0
2012	1	0	0	0	0
2013	3	0	0	0	0
2014	4	1	1	0	0

C. Model assumptions

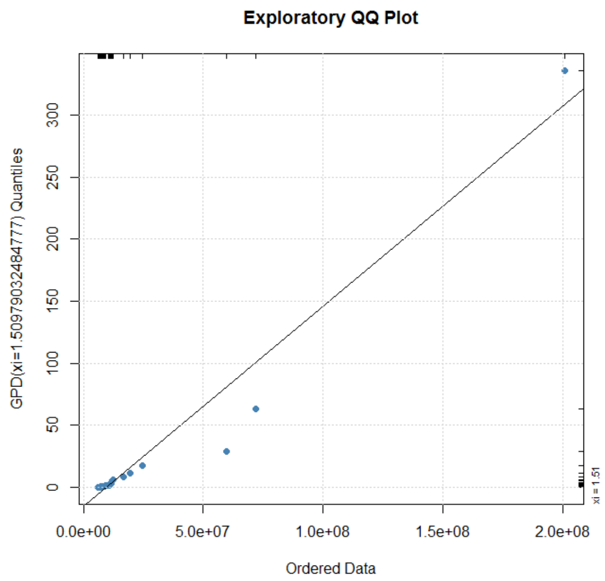
1. The selection of Pareto distribution

One may wonder why a Pareto curve is suited to simulate Large/CAT losses. The following tests have been made in order to assess the suitability of a Pareto curve.

In extreme value theory the mean excess plot is a good way to know if the data is distributed according to a Pareto distribution. The mean excess plot has been built using R. The result below suggests that the data can be modelled using a Pareto distribution as it looks like a straight line.



The QQ plot can also be shown. It has been built using R. The data points seems to be aligned on the straight line which confirm the appropriateness of a Pareto curve.



In the next section statistical test such as Kolmogorov-Smirnov are shown to support this view.

2. Scale parameter of the Pareto distribution

There are in reality 2 frequency severity models. The first one is for the risk type losses and the second one for CAT type losses. The frequency distribution used is always Poisson. The severity distribution used for risk type losses is lognormal distribution. The lognormal distribution fits that purpose as it is an intermediary heavy tailed distribution enabling to simulate some large losses but also some attritional losses. The Pareto distribution is used to simulate Large/CAT type losses. This fits that purpose as Pareto is a heavy tailed distribution.

A cut off point has to be chosen, in other word the scale parameter of the Pareto distribution. This represents the point from which the Pareto distribution starts and hence Large/CAT type losses will be simulated. The Pareto distribution will be fitted based on all losses beyond that threshold.

The lognormal distribution will be fitted to all losses excluding the losses flagged as CAT, there are 3 of them in this example.

It is not easy to find out the threshold, however several criteria can be taken into account.

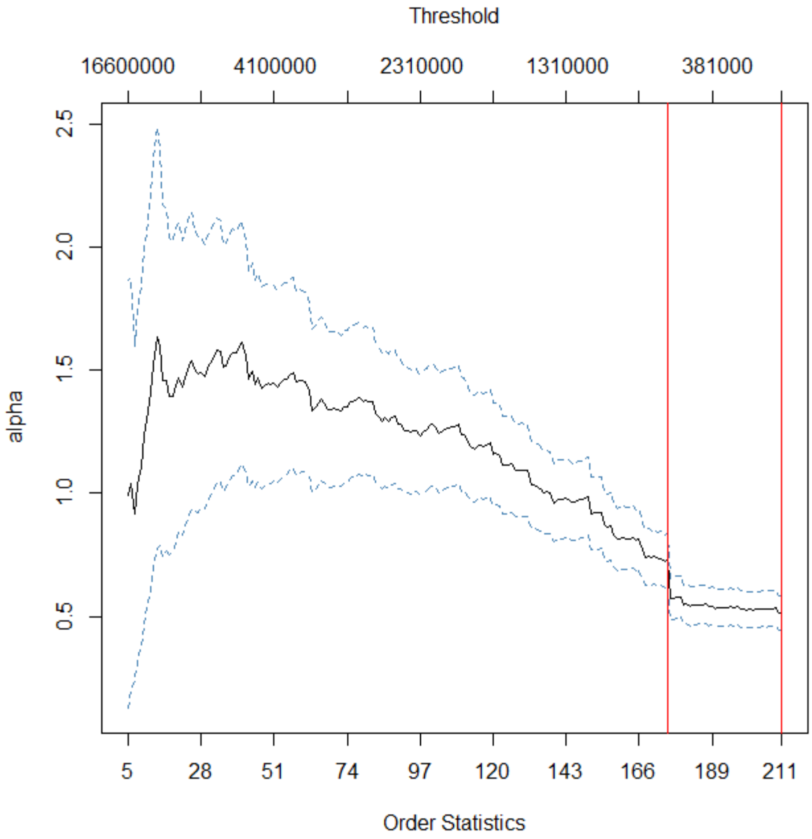
- The first criteria is a qualitative one, the threshold has to be chosen so that there are enough data on both sides of the threshold so that the curve fitting can be done.
- The second criteria is also qualitative and depends on the starting point of the reinsurance programme. As XoL RI is to protect against Large/CAT claims, it is better to start the Pareto distribution before the start point of the RI programme i.e. $10m in this case.
- The third criteria is quantitative and relies on the Hill estimator of the shape parameter seen in the extreme value theory. The Pareto distribution has the particularity of having a stable

shape parameter whatever the threshold chosen is. Therefore if the Hill estimator is plotted the threshold can be determined as the point where the estimator is stable.
The Hill estimator is:

$$\alpha_{k,n}^{Hill} = \frac{1}{k}\sum_{i=1}^{k} \ln\left(X_{(i)}\right) - \ln\left(X_{(k+1)}\right)$$

If \$10m was chosen as the threshold then there would be 15 losses above to fit a Pareto curve. This seems to be not enough, therefore the threshold has to be lower than \$10m.

The hill estimator has been plotted below using the function *hillPlot* in the package *fExtremes* in R and it shows that the estimator is gaining some stability beyond the 175th largest loss (see in between the red lines on the graph below). The 175th largest loss is about \$5.5m. This means that if that threshold is chosen there will be 31 losses to fit a Pareto curve, this is the double of the number of losses beyond \$10m. Therefore the threshold is chosen to be \$6m.



3. Curve fitting and simulation

Once the threshold has been chosen Lognormal and Pareto curves can be fitted. The fitting of lognormal curve is done within R using the function *fitdist*, it is using maximum likelihood approach.

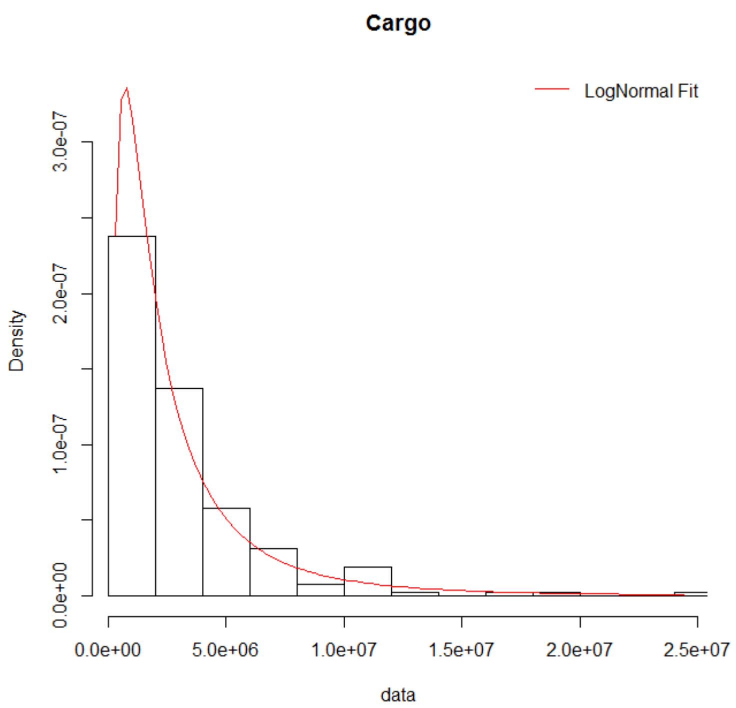
For the Pareto curve, the maximum likelihood is used to estimate the shape parameter. This can be done easily within Excel as the estimator of the shape parameter has a simple analytical expression. A truncated Pareto distribution with an upper limit set to the maximum line size of the reinsured i.e. $400m is also fitted.

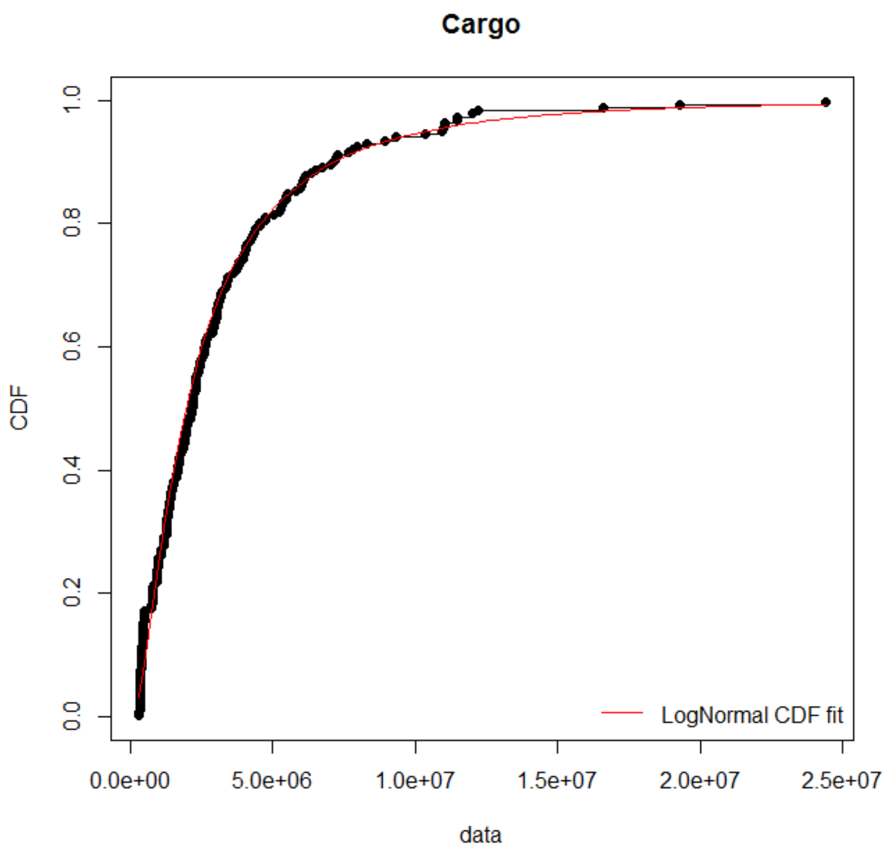
The parameters for both curves are as follow:

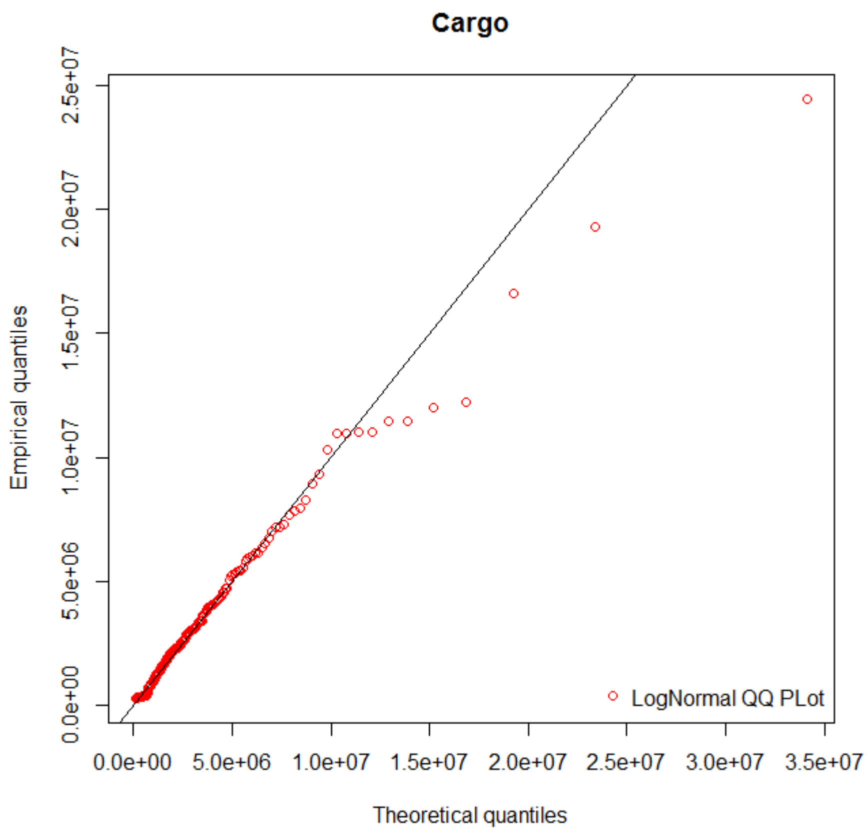
	Scale/Mean	Shape/Std Dev	Frequency
Pareto	6 000 000	1,51	4,76
Lognormal	14,48	1,02	4.76

After adjustment for growth in premium income, it has been estimated that the number of expected losses beyond the $6m threshold is 4.76. Poisson distribution is used to simulate the number of losses in each scenario.

The next 3 graphs show how the lognormal curve fits to the data excluding the 3 Cat losses. The first one shows the probability density function (pdf) fit. The second one shows the cumulative distribution function (cdf) of the fitted lognormal and the empirical cdf. The third graph shows a q-q plot to validate the choice of a lognormal curve for irks type losses.

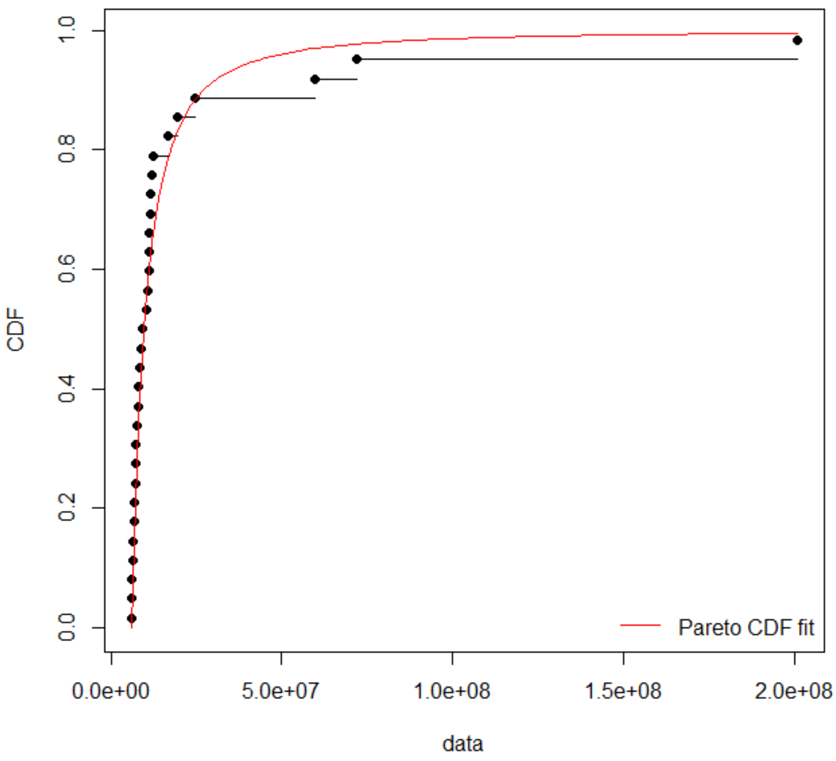



Cargo
CDF
1.0
0.8
0.6
0.4
0.2
0.0
LogNormal CDF fit
0.0e+00
5.0e+06
1.0e+07
1.5e+07
2.0e+07
2.5e+07
data

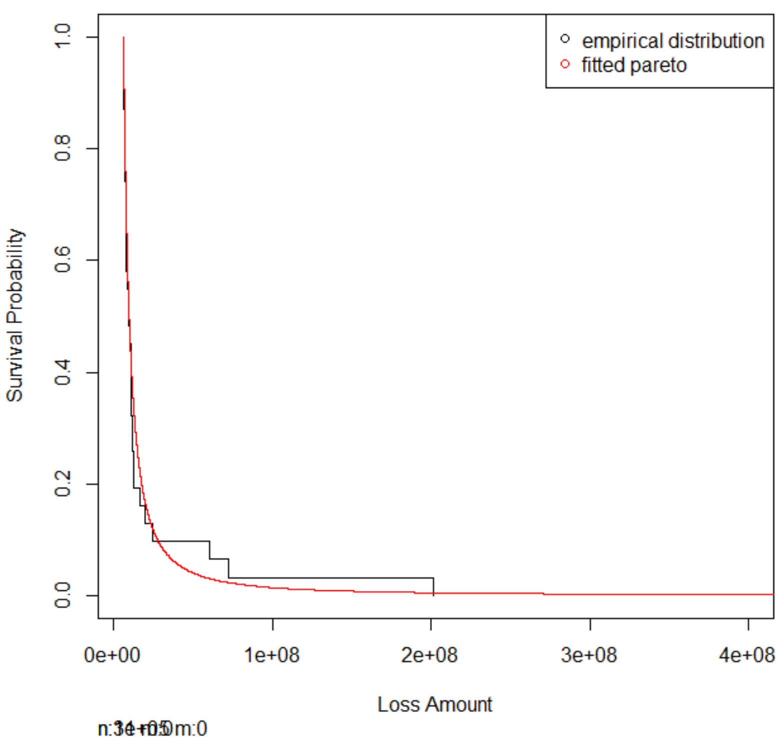

Cargo
Empirical quantiles
2.5e+07
2.0e+07
1.5e+07
1.0e+07
5.0e+06
0.0e+00
LogNormal QQ PLot
0.0e+00
5.0e+06
1.0e+07
1.5e+07
2.0e+07
2.5e+07
3.0e+07
3.5e+07
Theoretical quantiles

The following graphs show the fitting of the Pareto curve to the losses beyond the $6m threshold.

Cargo



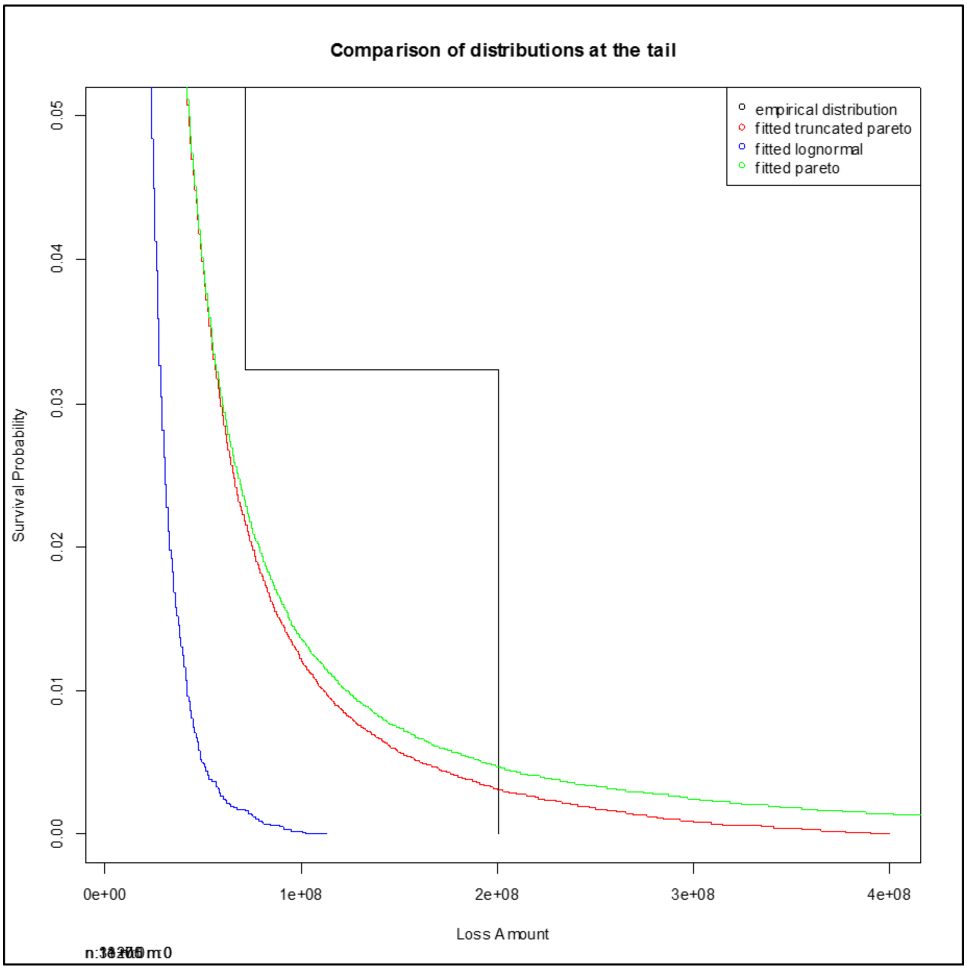
Fitting of pareto to empirical distribution



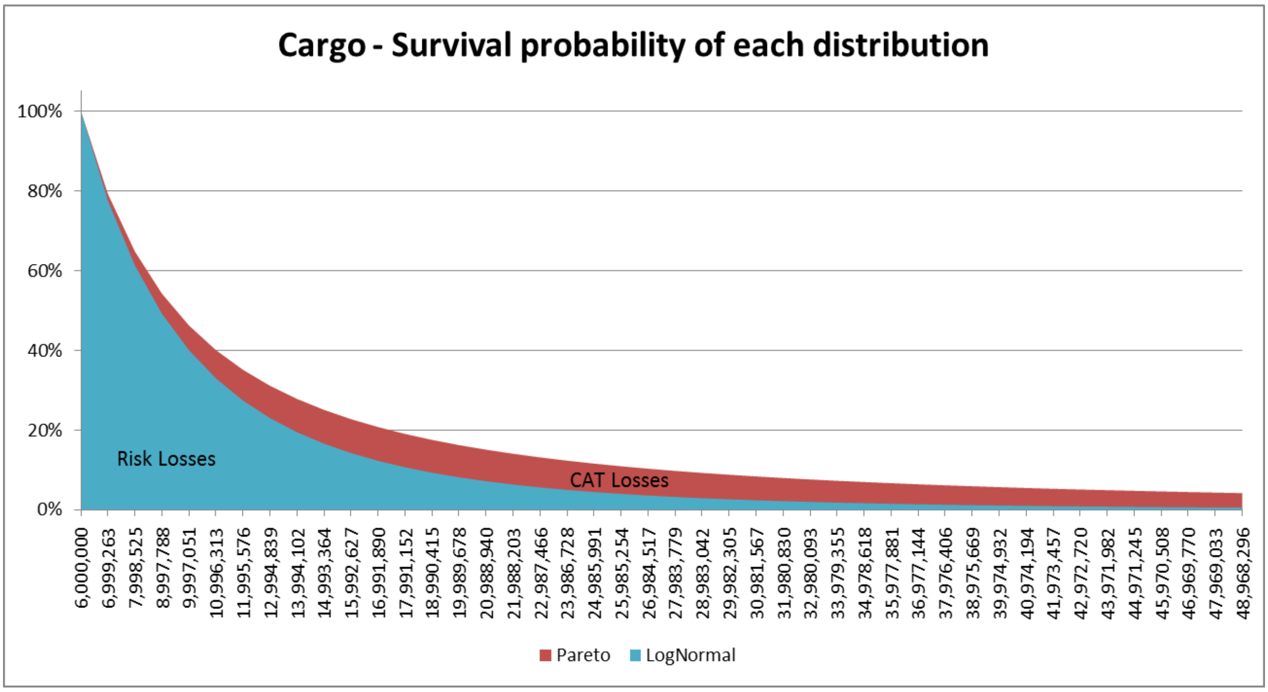
The Kolmogorov –Smirnov test has been executed in R to check whether statistically these fittings can be validated. The results confirmed the selections. The p-value for the Pareto test is at 45% which is well above 5%, if 5% was the threshold chosen to reject the distribution. The p-value for the lognormal test is at 11% which is good enough to keep that distribution.

```
> lg.test<-ks.test(classlosses$UltimateLoss[classlosses$UltimateLoss<CATthreshold],"plnorm",fit$estimate[1],fit$estimate[2])
Warning message:
In ks.test(classlosses$UltimateLoss[classlosses$UltimateLoss < CATthreshold],  :
  ties should not be present for the Kolmogorov-Smirnov test
> lg.test

        One-sample Kolmogorov-Smirnov test

data:  classlosses$UltimateLoss[classlosses$UltimateLoss < CATthreshold]
D = 0.082323, p-value = 0.1193
alternative hypothesis: two-sided

> y<- z[z>Lower[1]]
> lg.test<-ks.test(y,"ppareto",Lower[1],AlphaT[1])
> lg.test

        One-sample Kolmogorov-Smirnov test

data:  y
D = 0.14895, p-value = 0.4538
alternative hypothesis: two-sided
```

If a closer look is given to the tail of the empirical distribution i.e. beyond the 95% quantile. It can be clearly seen that the lognormal distribution is not heavy enough in the tail to produce Large/CAT losses beyond $100m. The Truncated Pareto distribution is heavier but dies off at $400m due to the upper limit. The Pareto distribution is heavy enough to produce CAT type losses beyond the $400m point.



Once a Lognormal and Pareto distributions have been fitted some assumptions have to be made regarding the claims modelling. In this model, it has been assumed that the Pareto distribution is the best representation of claims beyond the $6m threshold. This can be confirmed by the KS-Test and the various graphs plotted above. However this would simulate all possible losses i.e. CAT and Risk losses. As shown above, there are 4.76 claims expected beyond the $6m threshold. Therefore the Pareto distribution with 4.76 expected claims is chosen to simulate the total losses to the layers. In order to get the risk and the cat component, the fitted lognormal distribution is used. The expected 4.76 claims is simulated assuming it is distributed following the tail (beyond the $6m threshold) of the lognormal distribution. The resulting claims represent the risk losses. Those losses are subtracted from the losses simulated by the Pareto curve in order to have the CAT losses. This can be summarised in the graph below. The blue area represent the risk type losses and the red area the CAT component. This is to say that the Pareto distribution embeds both risk and cat losses and that lognormal distribution is assumed in order to estimate the risk element.

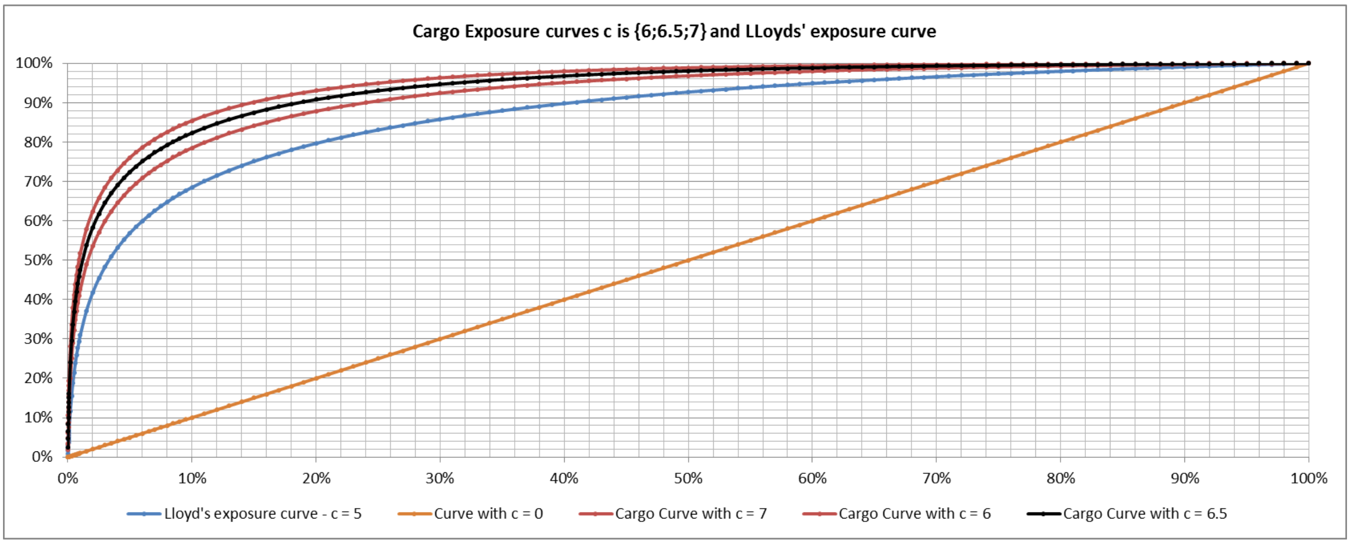


4. Other assumptions

This section relates to assumptions used in the exposure and experience rating.

<u>Exposure rating</u>

For cargo the underlying exposure curve is assumed to have a parameter of c=6.5. It is possible to shift that curve upwards or downwards by 0.5 i.e. the curve can be based on a parameter of 6 or 7. This is selected by the user and depends on the propensity of the underlying book to produce large losses. The exposure curve used in Lloyds in general is one with a parameter of c=5. This means that compared to the Lloyd's one the cargo book should have lower propensity to produce large losses. This can be seen on the plot below.

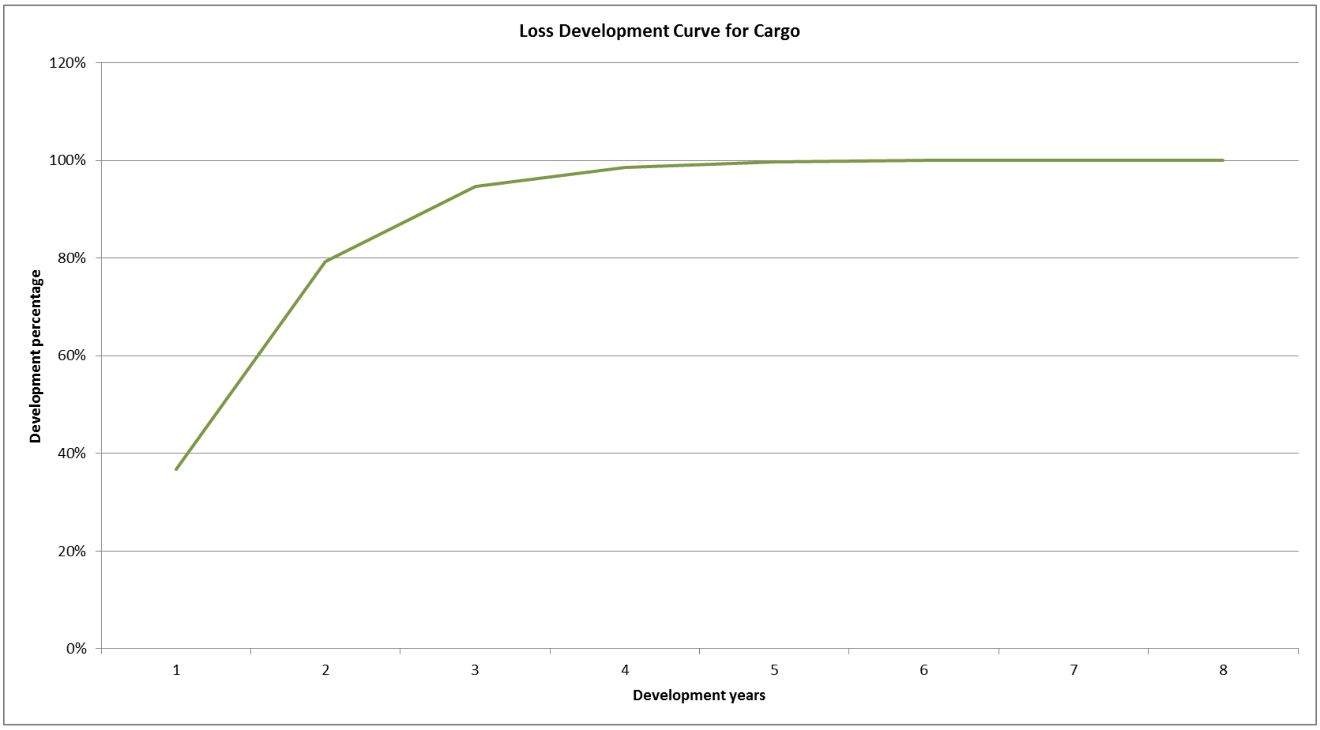


Another crucial assumption is the loss ratio for the underlying book. The market benchmark suggest that 75% is a reasonable starting point. If the book is deemed better another loss ratio can be used. In this numerical application 75% is used.

<u>Experience rating</u>

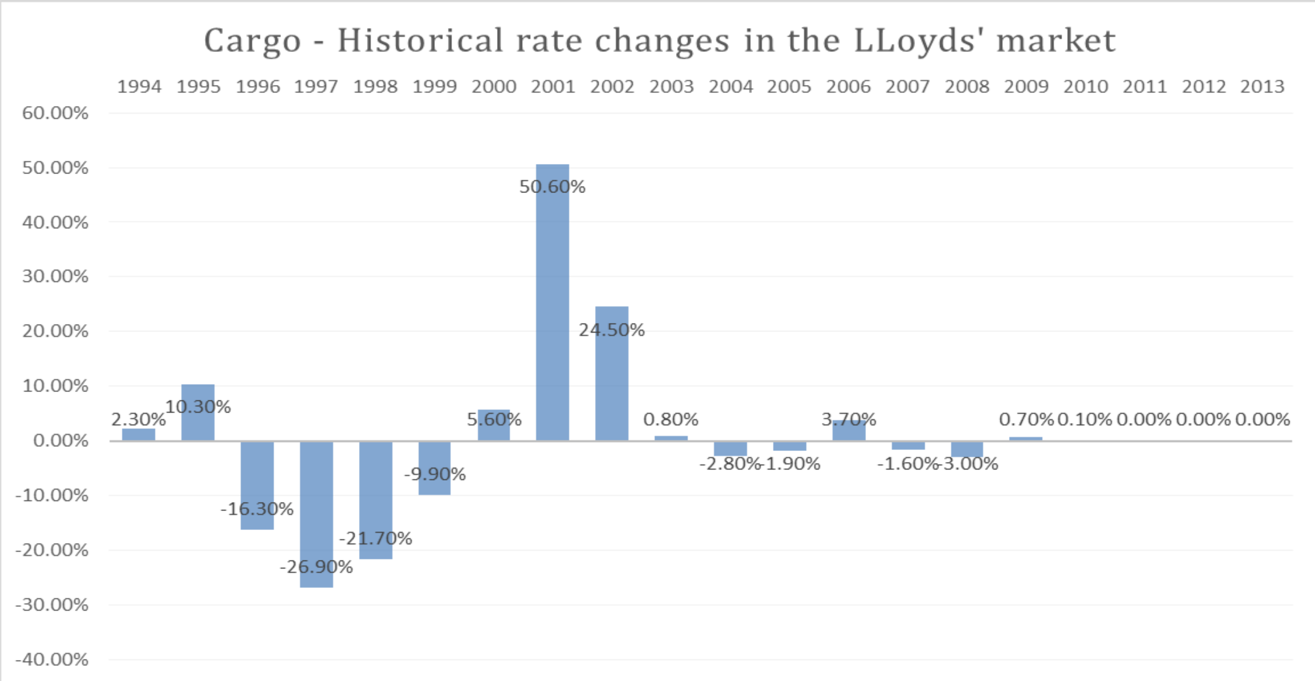
In terms of claims and premium on levelling the following assumptions have been used.

Loss development factor has been derived from market data. As shown on the graph below, if a claim is open, the following pattern is applied to develop it to ultimate.



For inflation a starting point is to apply 3% on average across all years. In this example an inflation of 2% has been assumed in order to bring historical losses to today's terms.

Inflation is assumed to be nil for premium, however historical rate change has been applied to on level premium to today's market condition. The rate change for cargo is provided by the market. Rate changes for 2012 and 2013 are not available at this stage but given the recent stability in rate changes it has been assumed to be nil for those years too.



D. Results

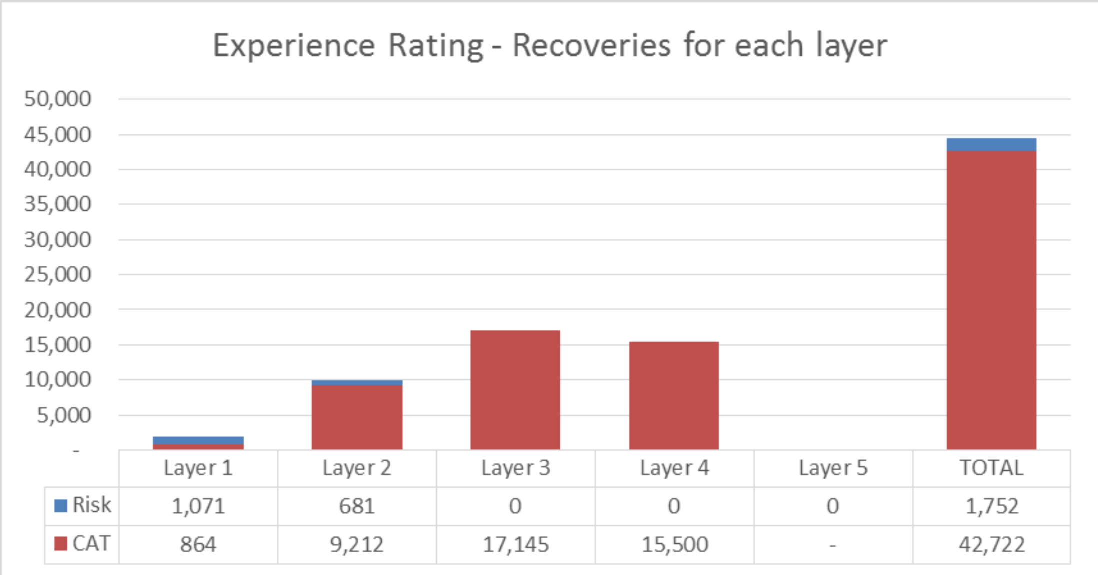
The results provided by the different method would be shown first. Sensitivity tests on assumptions will be carried as well. Then credibility results will be used in order to get to the final results that reflects the best the risk the reinsured has and gives credibility or not to its historical experience.

The brokerage fee used across all the layers is 10% and the margin required has been set at 25%.

1. Experience rating

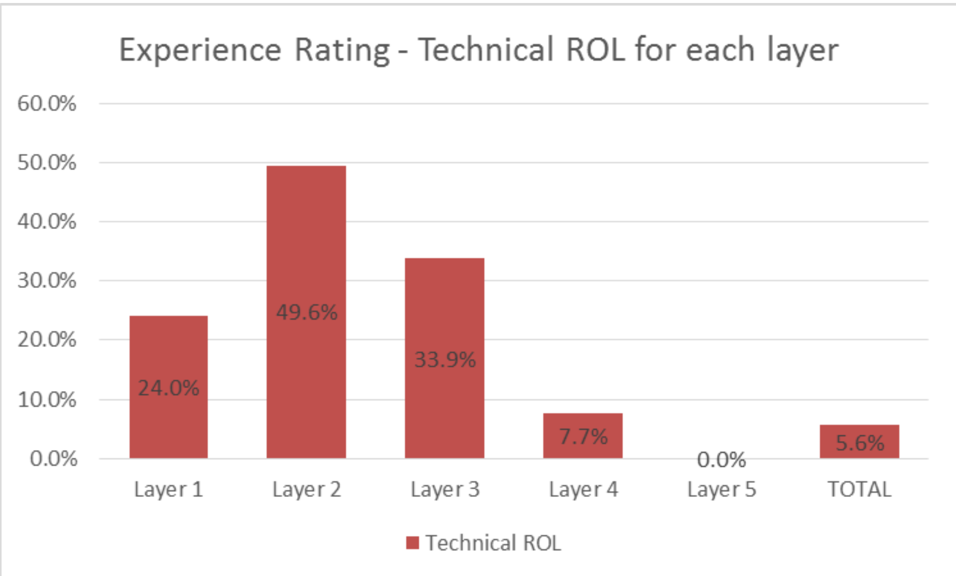
Burning cost

The graph below shows the amount of recoveries calculated for each layer and split between risk and cat. It can be noticed that beyond layer 2 only the CAT component is driving the price, although there are risks that have insured values that can fall into these layers. Therefore it can be said that if the experience rating was used the risk component beyond layer 2 would be free to the reinsured. However, this does not tell us much in order to compare the layers between them and to have a sense of the market price. Therefore the LOL (Loss on Line) adjusted for reinstatement premium, brokerage and margin is rather used.



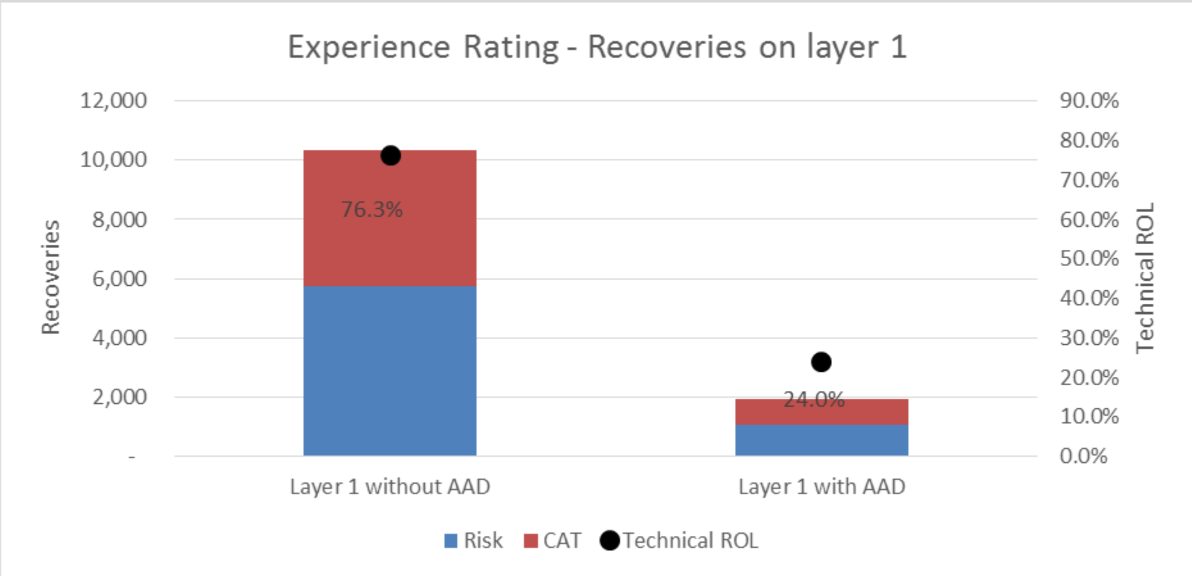
<u>Technical ROL</u>

The technical ROL (Rate on Line) to charge, if experience rating was used on its own, for each layer should be as shown in the graph below. Based on experience rating, the layer 5 is free which is non sense as there is the cost of capital for the reinsurer to provide this cover. The second layer is showing an expensive technical ROL of 49.6% which looks very hard to achieve in the market place where ROL of 35% are considered to be the highest a client would accept to pay. A solution could be to introduce an AAD or to accept that the layer 2 will be unprofitable and spread the loss on the remaining layers by increasing their ROL such as on layer 4 and layer 5. Layer 3 looks also expensive at 33.9%. Layer 1 has an acceptable price given it is a working layer highly exposed to potential losses. The layer 4 looks acceptable. Over whole the reinsurance programme should be priced at 5.6% ROL for the reinsurer in order to achieve an expected 75% loss ratio after brokerage fee.



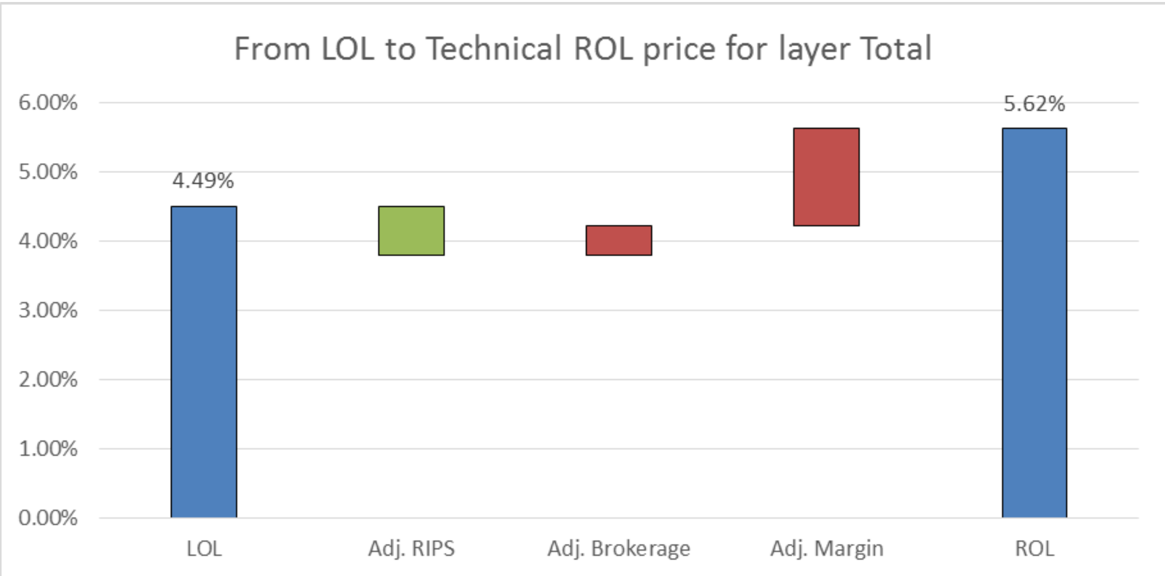
Benefit of the AAD

The graph below shows the benefit of having an AAD on the first layer. If the AAD was not in place the technical ROL would have been of 76.3%. It can be also noticed that the AAD reduced both risk and CAT type losses.



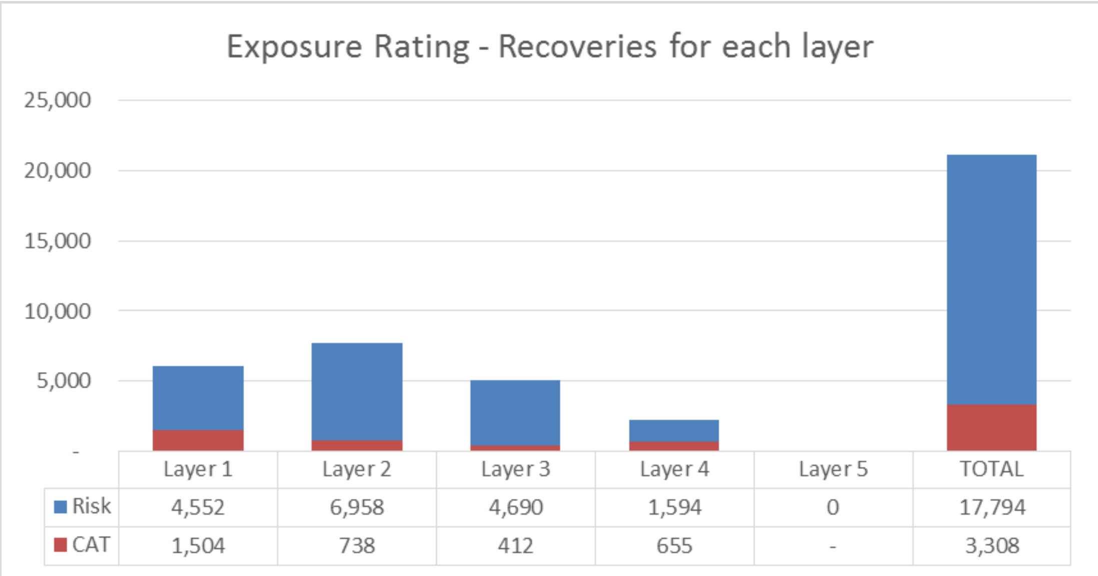
From LOL to ROL

The waterfall graph below shows how the final technical ROL is calculated from the initial LOL which is the pure burn rate for the whole reinsurance programme. Similar graphs can be produced per layer. The RIPs has a beneficial impact for the reinsured. It reduced the LOL by 0.7% point. The reinsurer has then to build in an allowance of %10 for brokerage and 25% margin for profit.

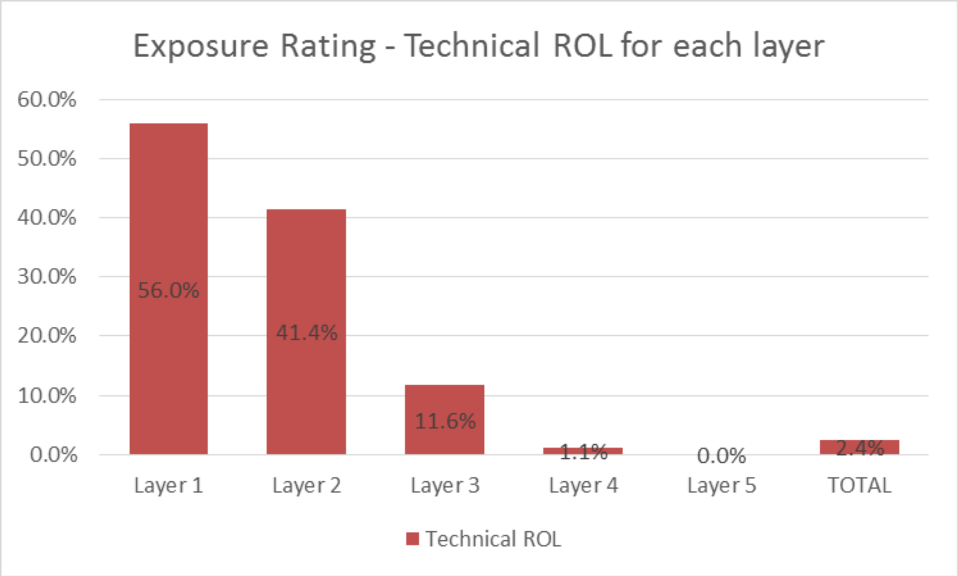


2. Exposure rating

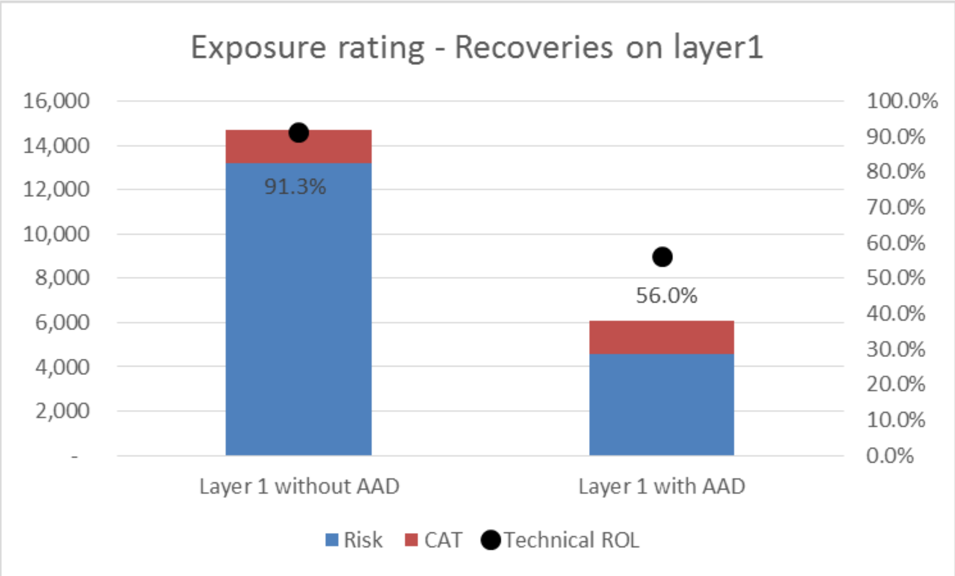
If the exposure rating was selected the following recoveries would be expected to be made on each layer based on the current risk profile of the reinsured. As expected most of the recoveries are risk based as the exposure rating is primarily fit to price risk based reinsurance contracts. The difference with the experience is very striking as most of the recoveries were CAT driven in the experience rating. As expected there are no recoveries in layer 5 as that layer is in excess of $400m and the maximum line size is $400m. This is where the need for frequency severity type modelling is needed in order to estimate the potential CAT that could trigger losses in that layer. The historical experience only spans over 10 years, it is possible that a one in 25 years of 1 in 50 years or 1 in 100 years CAT has an aggregated cost of more than $1bn and therefore the layer would produce recoveries.



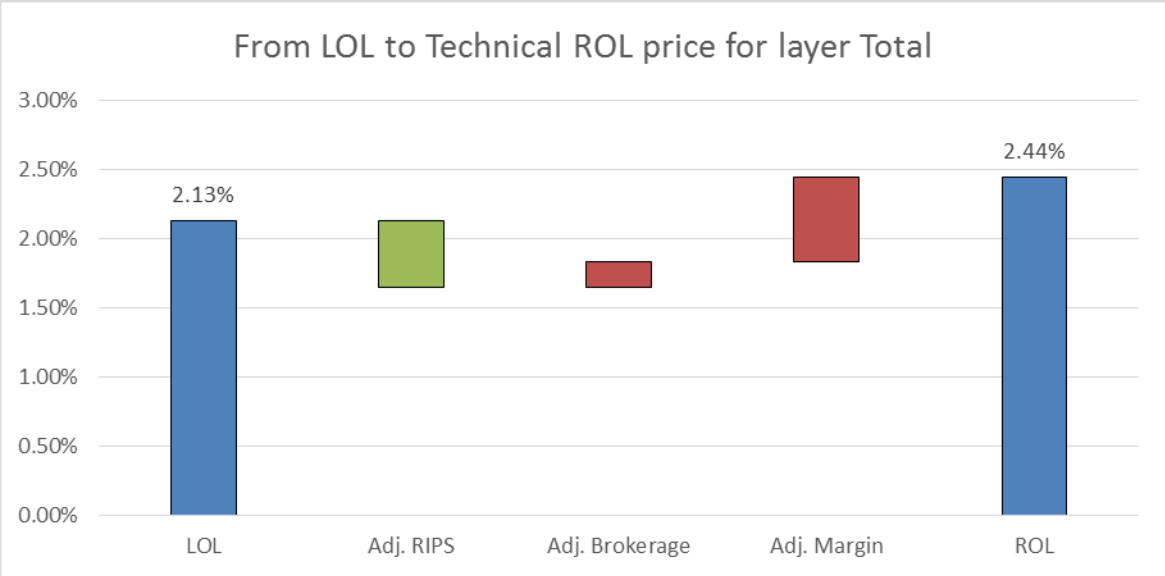
In terms of technical price the following would be charged. Overall the technical ROL is estimated to be 2.4% which is almost half of the technical ROL derived from experience rating. This is mainly due to the CAT losses that exist in the experience and the lack of the exposure rating in estimating CAT losses. The first two layers are very expensive under the exposure rating. This can make sense as most of the exposure is on the first two layers. Section A shows that about 14% of the clients exposure fall in the first layer and 7% in the second layer which is significant given that 72% of client's exposure is retained i.e. below the $10m retention. The 3rd and 4th layer are cheaper under an exposure rating method than under an experience rating method this is due to the CAT losses that exist in the experience and goes through these layers.



The benefit of the AAD on the first layer has also been estimated for exposure rating. If the AAD was not in place the technical ROL would have been of 91.3% instead of 56%.



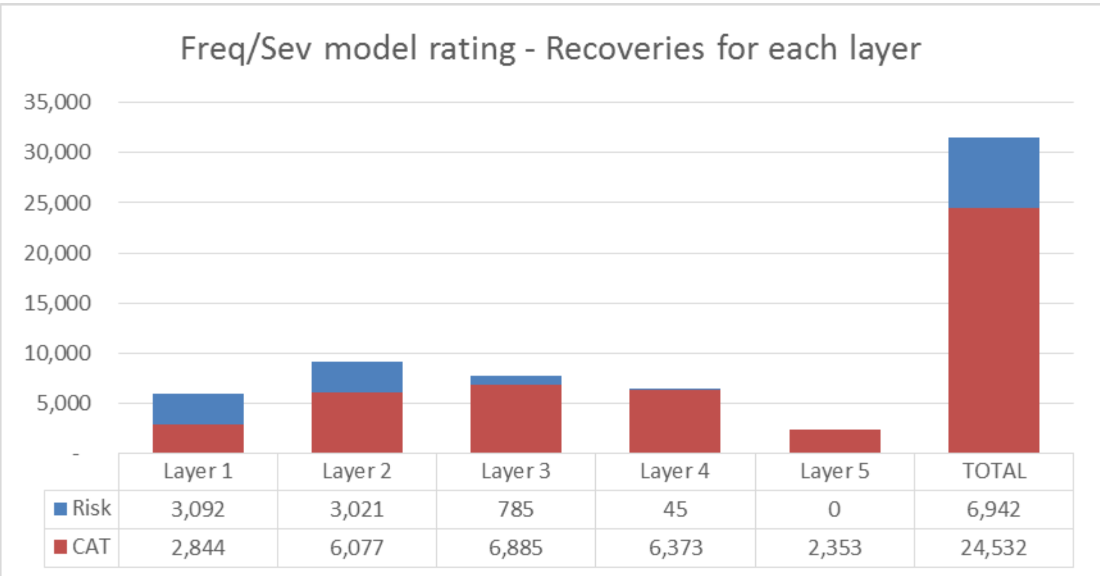
The graph below shows how the technical ROL is derived.



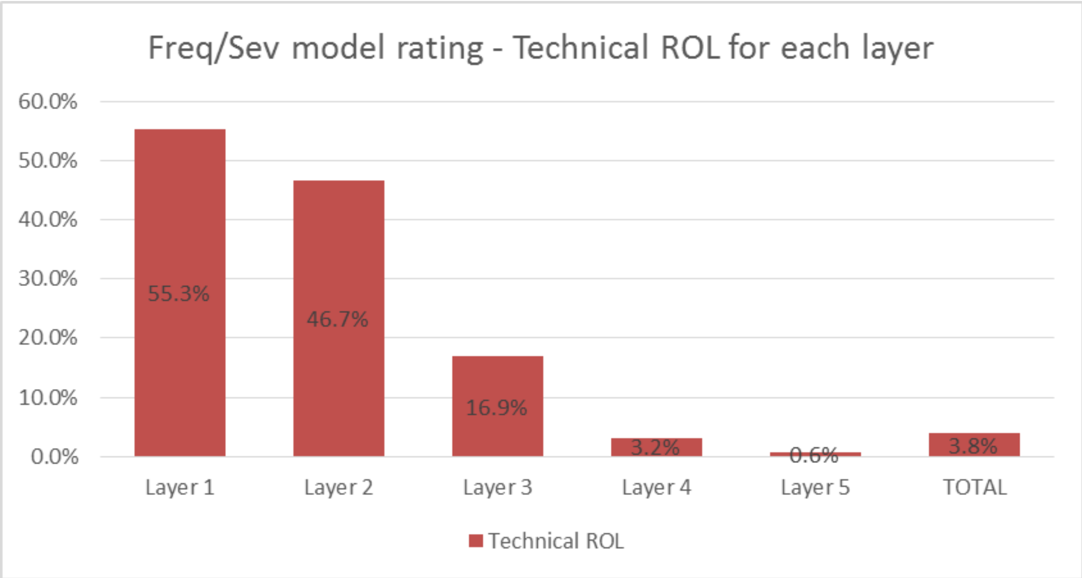
3. Frequency severity

The frequency severity model produce results that rather make sense in terms of CAT vs. risk split. The recoveries originating from risk losses are almost nil beyond layer 3. This make sense as only a very few risk have insured value beyond $100m. In fact there are 124 risks against 172244 risks in total. Unlike experience and exposure rating, frequency severity model produces recoveries in the 5th layer. This is thanks to the use of a Pareto distribution which belongs to heavy tailed distributions.

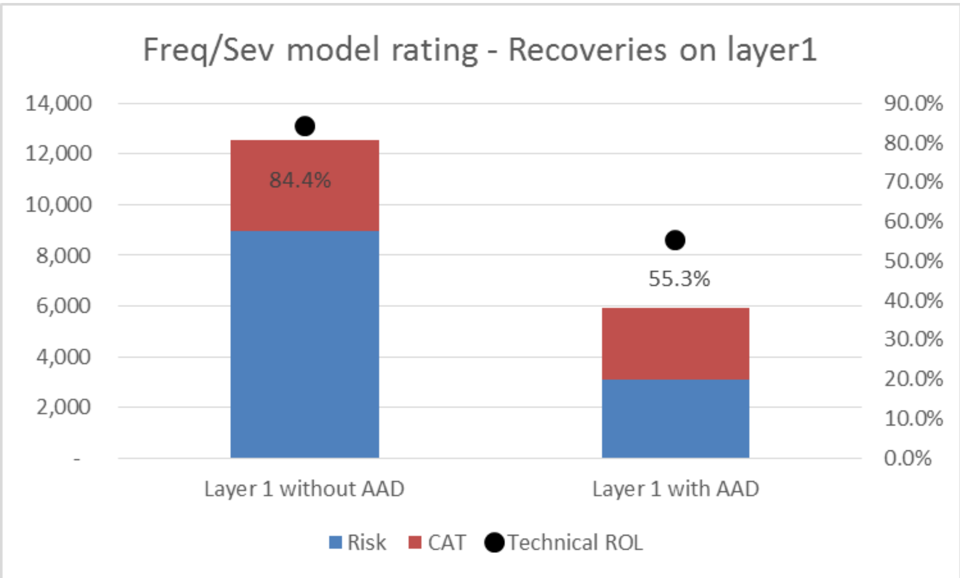
In aggregate the experience rating is dominated by CAT losses (95% of recoveries) while the exposure rating is dominated by risk type losses (85% of recoveries). The frequency severity model produces results where CAT represents 78% of recoveries. In that sense the results presented by that model make more sense as XoL is mainly used to cover reinsured against CAT type losses.



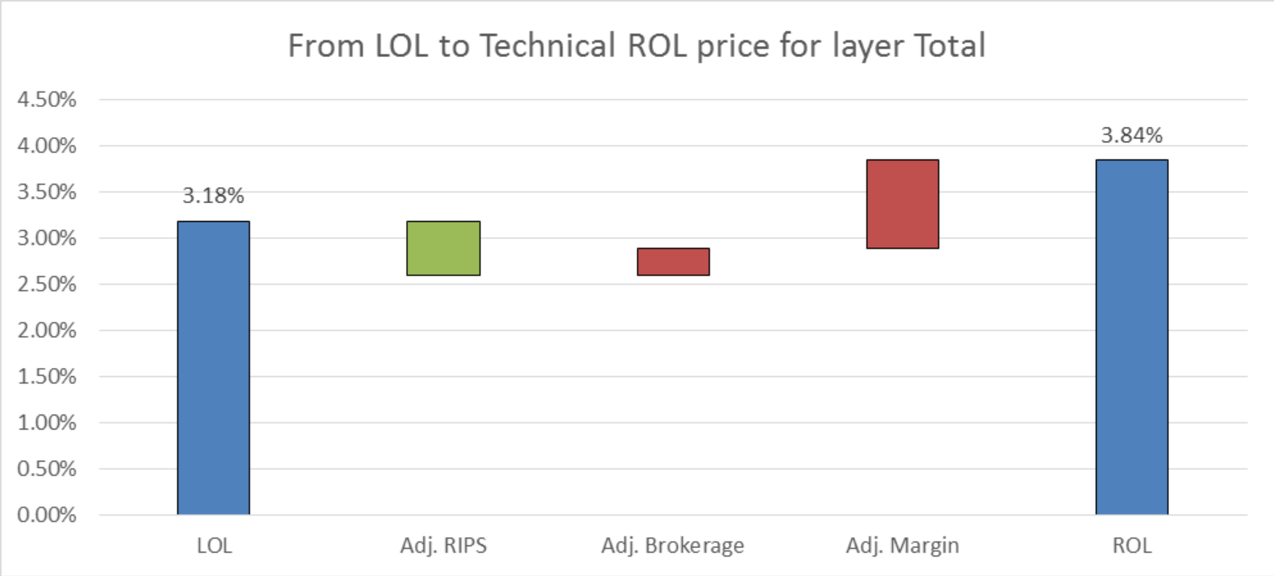
In terms of technical ROL the first two layers are more expensive than what the exposure or experience rating would suggest. Therefore under this model it is expected on average that the first two layers will be loss making for the reinsurer as such technical ROL can't be achieved in the market place. On aggregate the ROL is at 3.8%. This is somewhat in between the exposure and the experience rating.



If the AAD was not in place on the first layer the technical ROL would have been of 84.4% against 55.3% with the AAD. The AAD help reduce the price by almost a third.

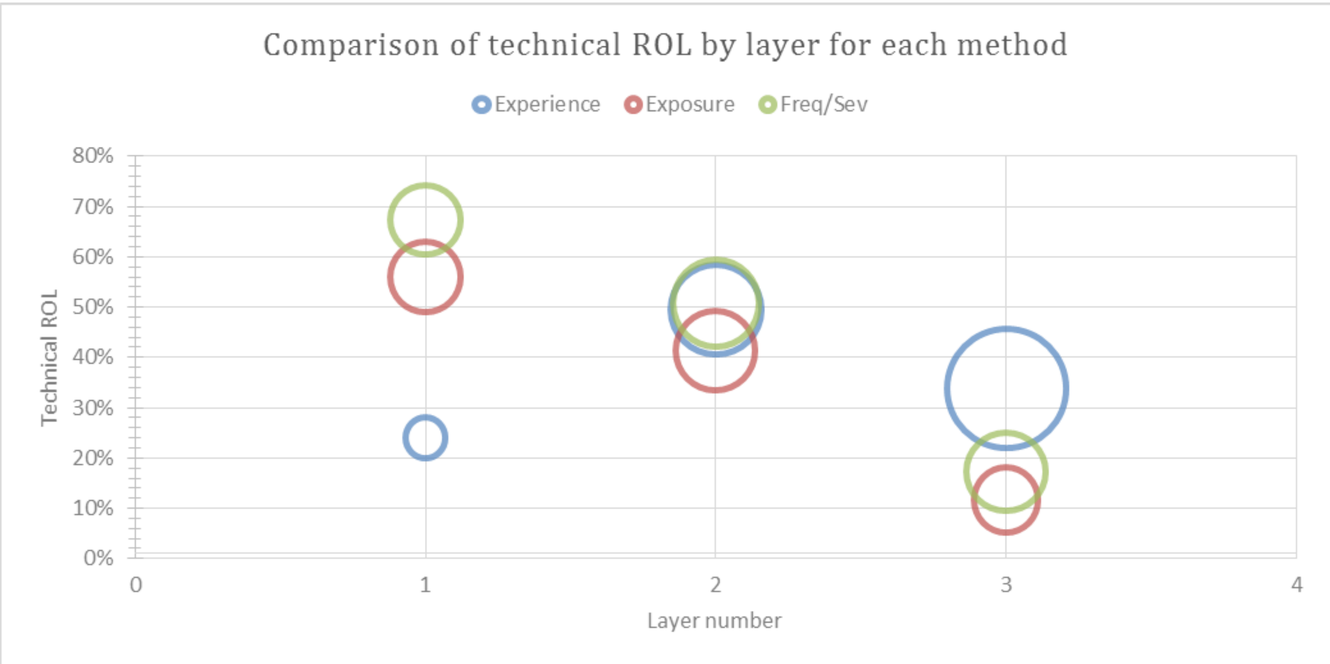


Finally the graph below shows how the final technical ROL is obtained.



4. Summary

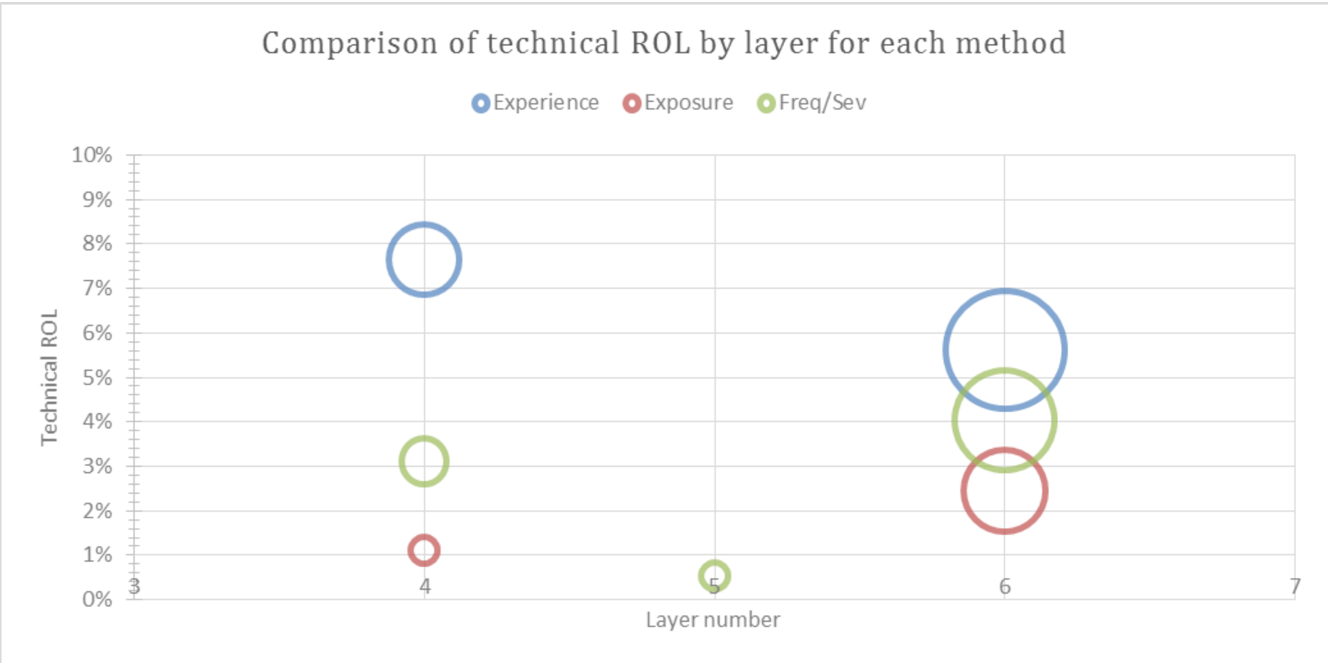
The bubble plot below shows for each layer on the horizontal axis the corresponding technical ROL that each method gives on the vertical axes. The size of the bubble is determined by the amount of recoveries made by the reinsured. For example for the 1st layer the frequency severity model provides the highest ROL, it is then followed by the exposure rating and then the experience rating. The experience rating is very small compared to the other two. This gives an idea of how far from each other are the methods for each layer. On the second layer we can see that experience and frequency severity model are almost perfectly overlapping with exposure rating not too far away. On the 3rd layer we can see that the experience rating is providing the highest ROL while frequency severity model and exposure rating are not too far from each other.



For layer 4 all methods are providing very divergent results. Experience rating is the highest one due to a CAT loss. On layer 5 only the frequency severity model produce some recoveries. In aggregate (see layer 6) it can be seen that the frequency severity model is in between the experience and the exposure rating. The FS model takes into account all the range of possible outcomes by simulating 10000 scenarios. The model strongly relies on the historical losses. Those are in a significantly relevant amount i.e. 211 losses over the last 10 years to provide robust fitting for Lognormal and Pareto curves. The exposure rating is the lowest as it struggles to provide a strong view on the CAT component. The experience rating is the highest as the historical data contains one significant large loss of $200m. That loss happened in the last 10 years. However looking at the distribution of simulated losses, a CAT loss of $200m correspond to the 99.52552% percentile of the distribution. It means that a $200m loss would happen once every 210 years. However there is only 10 years of data and it is rather unlucky to have experienced a 1 in 210 years event in the last 10 years. This explains why the experience rating is the worst.

The table below summarise the return period for a given loss amount. For example every other year there will be a loss greater than $10m and every 6 years a loss will be greater than $20m.

Loss Amount($)	**10,000,000**	**20,000,000**	**40,000,000**	**100,000,000**	**400,000,000**	**1,000,000,000**
Return Period (years)	2	6	18	72	551	2,155

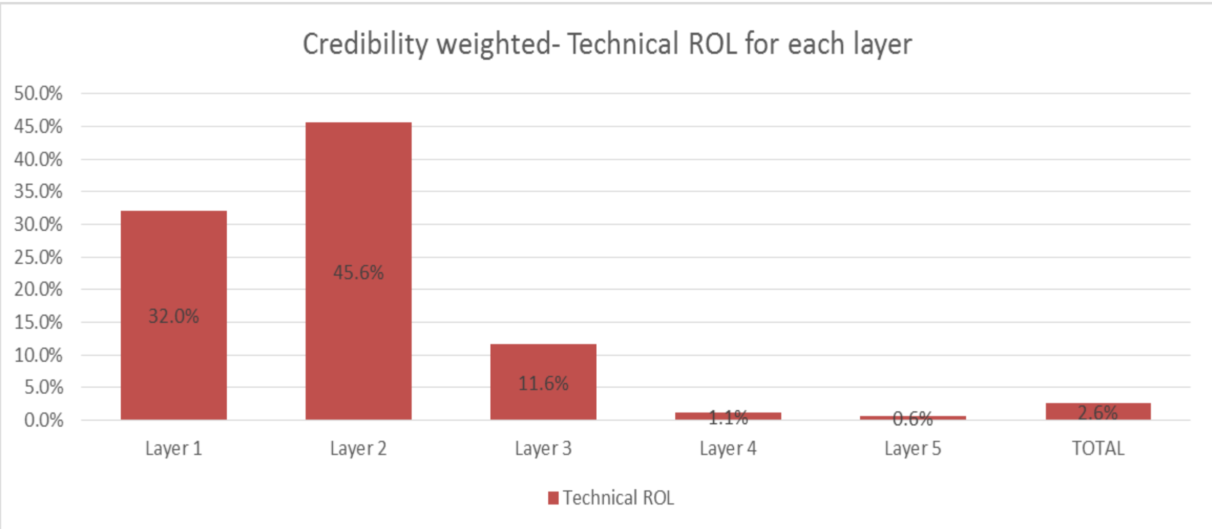


5. Credibility

The credibility theory presented in the sections above has been implemented in the rater. In this example for layer 1 the credibility theory suggest to give 80% weight to the experience and 20% weight to the exposure. On the layer 2, the credibility theory suggests to give 50% weight to th experience and 50% weight to the exposure. On the layer 3, 4 and 5, the suggestion is to use 100% weight on the exposure rating. However, as the layer 5 is a CAT layer, 100% weight is given to the frequency severity model. If these weights were applied, the overall final technical ROL obtained would have been 2.6% which is slightly above the overall exposure rating ROL.

Risk Weights	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Experience	80%	50%	0%	0%	0%
Exposure	20%	50%	100%	100%	100%
FS Model	0%	0%	0%	0%	0%

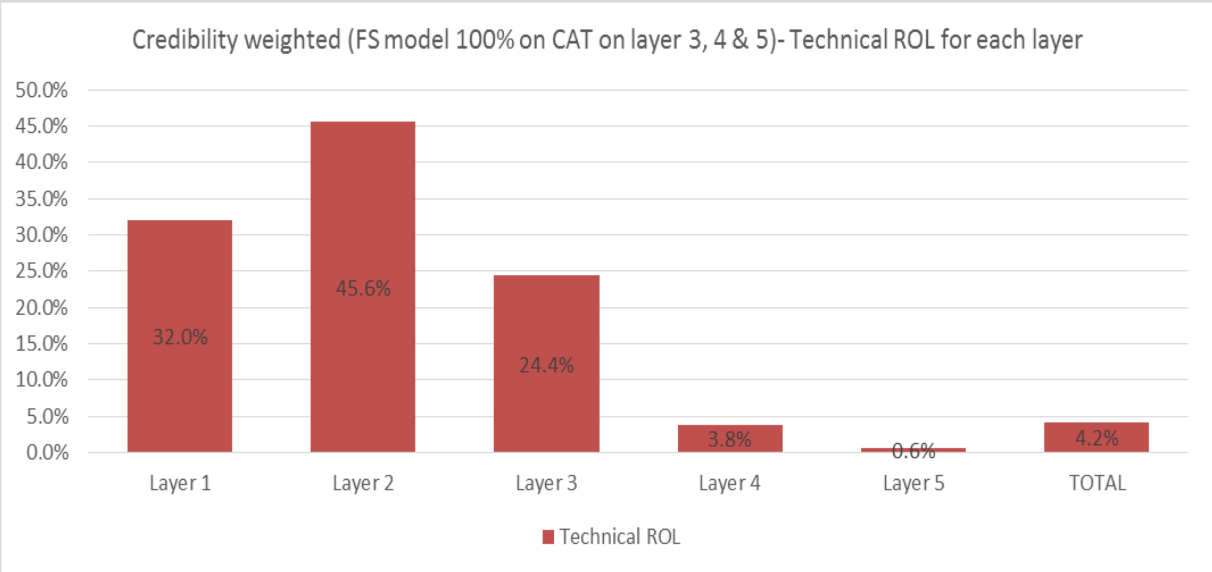
CAT Weights	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Experience	80%	50%	0%	0%	0%
Exposure	20%	50%	100%	100%	100%
FS Model	0%	0%	0%	0%	0%



However, as explained above the exposure rating is not really good at estimating the CAT component. Therefore on the CAT side if 100% weight was given to the frequency severity model on layer 3, 4 and 5 then the overall technical ROL would be 4.2% which is close to what the FS model provide on its own.

Risk Weights	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Experience	80%	50%	0%	0%	0%
Exposure	20%	50%	100%	100%	100%
FS Model	0%	0%	0%	0%	0%

CAT Weights	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
Experience	80%	50%	0%	0%	0%
Exposure	20%	50%	0%	0%	0%
FS Model	0%	0%	100%	100%	100%



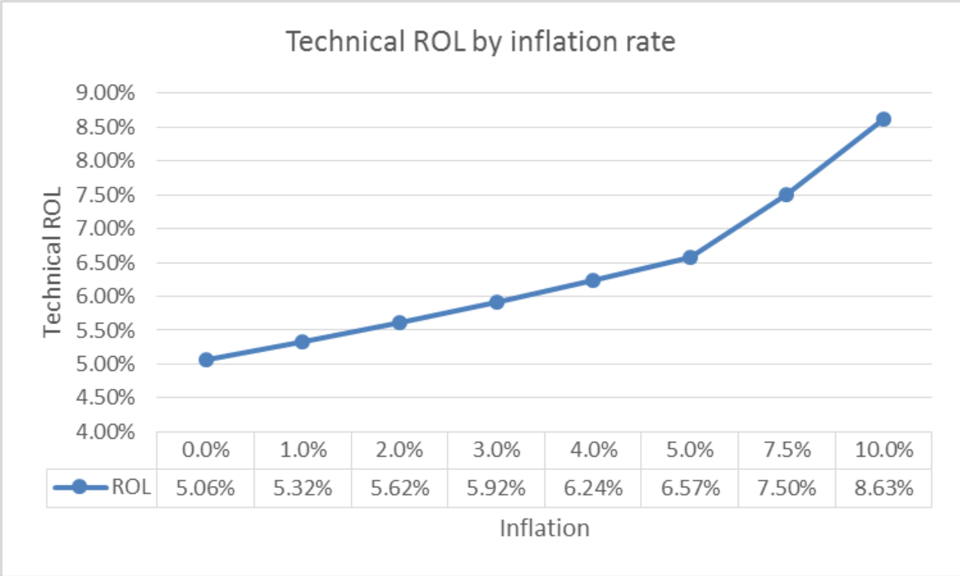
E. Sensitivity tests

In this section we will look at how much the overall technical ROL vary when assumptions are changed. On experience rating we will play on the inflation factor. On the exposure rating we will play on the exposure curve selection and the underlying business loss ratio. On frequency severity model we will look at the impact of the inflation rate on the fitting and as a result on the technical ROL.

<u>Experience rating</u>

The inflation rate has a significant impact on the final overall technical ROL as the graph below shows. The base case chosen was 2% providing an overall technical ROL of 5.6%.

On average 1% increase in inflation increase the technical ROL by 0.36% point. Overall the sum insured by the reinsurer across the 5 layers is $990m hence an increase by 0.36% means an additional $3.5m that needs to be charged to the reinsured for the whole programme. If the experience rating based on a 2% inflation assumption was chosen to derive the technical ROL then the overall premium for the programme would have been $55.6m (5.6% * $990m). As the technical ROL has been calculated in order to produce a 75% loss ratio, the margin would be $13.9m (0.25% * $55.6m). Hence a mistake of 1% in the inflation assumption would mean that the margin would be eroded by 25%.

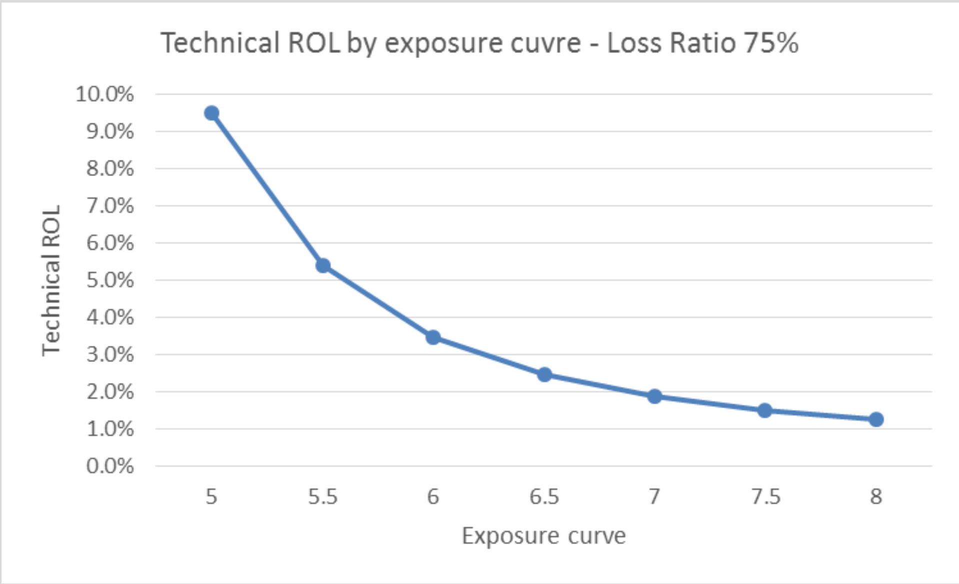


Exposure rating

The table below shows how the technical ROL would move if the exposure curve and the underlying loss ratio assumed for the reinsured's book were changing. The base case was an exposure curve at c=6.5 and a loss ratio of 75%. In general the impact of a 0.5 point change in exposure curve has a significant impact compared to a 5% point change in the loss ratio. From 6.5 to 6 at 75% loss ratio the Technical ROL would increase by 1% point. If the technical ROL chosen was based on 6.5 and 75% LR the overall premium collected by the reinsurer would have been $24.2m (2.4% * $990m). The margin would represent $6.1m (25% * $24.2m). Hence if the wrong exposure curve was chosen i.e. a 6 instead of 6.5 then the impact would be of $10m which means that the $6m margin would be blown up totally. Therefore compared to an error in the estimation of the inflation rate, an error in estimating the right parameter for the exposure curve has a greater impact for the reinsurer.

If the exposure curve chosen was correct but the loss ratio assumed for the underlying book was 90% instead of 75% then the impact would be in terms of premium of $3.2m that should have been charged in addition. This means that about half of the original margin would have been eaten up. The impact of an error in estimating the loss ratio is less important than an error in picking the exposure curve but yet remains more important than an error in estimating the inflation rate.

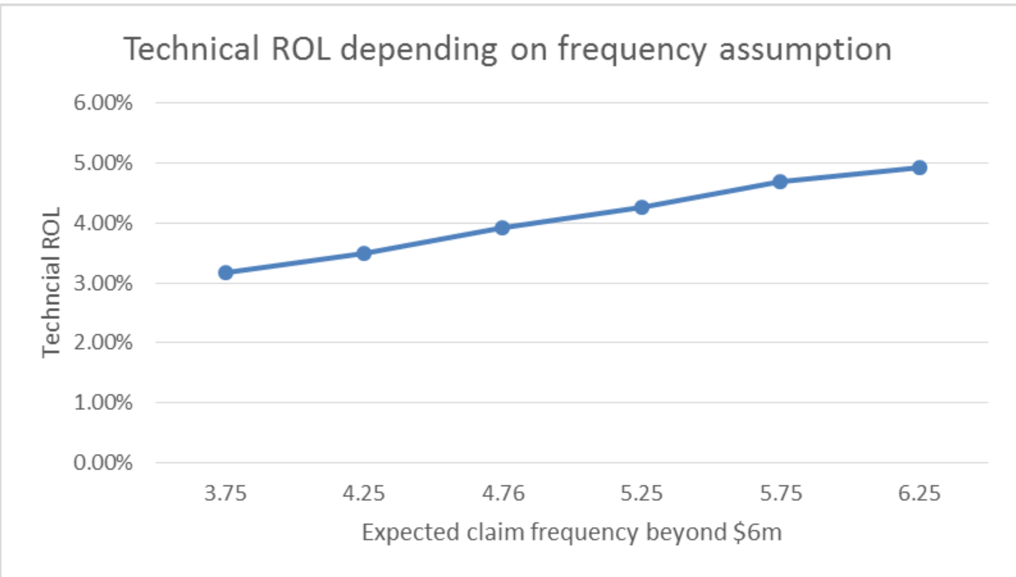
LR \ C	5	5.5	6	6.5	7	7.5	8
60%	9.0%	5.0%	3.1%	2.1%	1.6%	1.2%	1.0%
65%	9.2%	5.1%	3.2%	2.2%	1.7%	1.3%	1.1%
70%	9.3%	5.3%	3.3%	2.3%	1.8%	1.4%	1.2%
75%	9.5%	5.4%	3.5%	2.4%	1.9%	1.5%	1.2%
80%	9.7%	5.5%	3.6%	2.6%	2.0%	1.6%	1.3%
85%	9.8%	5.7%	3.7%	2.7%	2.1%	1.7%	1.4%
90%	10.0%	5.8%	3.8%	2.8%	2.1%	1.7%	1.5%



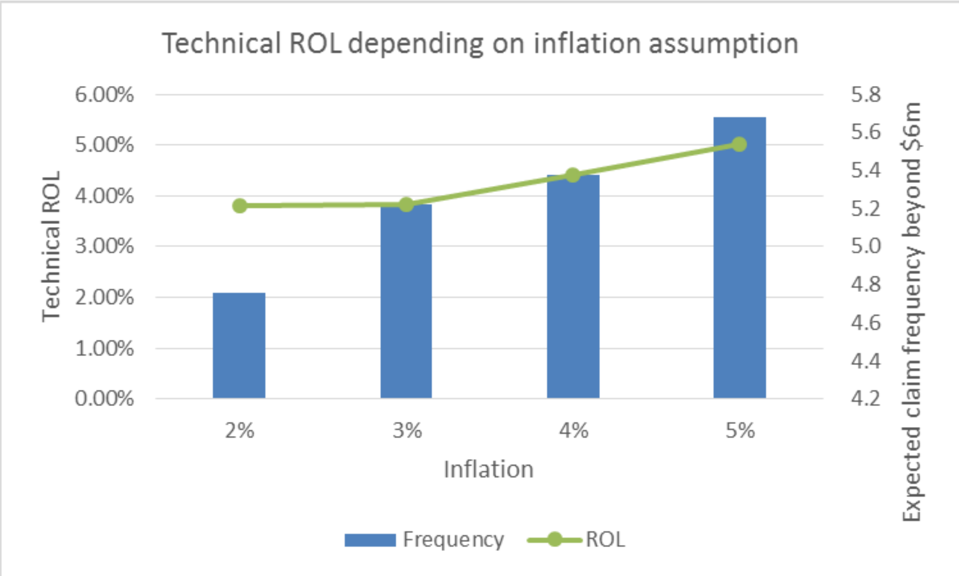
Another interesting point to note is that if the experience rating was assumed to be the right answer, then the assumptions to use in exposure rating would rather be {5.5;80%} instead of {6.5;75%} in order to produce similar technical ROL. In order to match the results provided by the frequency severity model the parameters {6; 90%} or {5.5;75%} could have been used.

Frequency severity

In the frequency severity model we first assess the impact of an error in the expected claim count beyond the $6m threshold. The current assumption based on historical losses and exposure information is that 4.76 claims are expected. If an error of 0.5 claim was made in that estimation it would increase the technical ROL by roughly 0.35%. In terms of premium, if 3.9% overall technical ROL was charged to the reinsured $38.9m of premium would have been collected. The margin built in that amount represents $9.7m. An error of 0.35% represents $3.3m. This shows that the margin in place can absorb such an error.



The graph below shows the sensitivity of the frequency severity model to the inflation rate. As the inflation rate increase some claims go above the $6m threshold and therefore the expected claim frequency increase. As a result this increase the technical ROL. However there are other dynamics such as the fitted Pareto and lognormal curves. This explains why the impact is minimal if the inflation rate was chosen at 2% or 3%. The difference between 3% and 4% means an increase in expected frequency of 0.15 claims. In terms of ROL the increase is of 0.6% which is significant compared to the previous sensitivity test.



F. Curve parametrisation

As explained at the beginning of this numerical application, the data collected from 22 clients has been aggregated. This aggregation of data provided a large amount of data to work on. This helped to fit Pareto and lognormal curves. It is then possible to use these parameters to price the cargo book of a client with no loss history or very few losses so that it makes it impossible to make reasonable fits. If the layer to reinsurer is beyond $6m, a Pareto curve with a shape parameter of 1.5 can be assumed. On the same note and as explained in the sections regarding sensitivity tests, the frequency severity model can be used to calibrate the exposure curves. In our example it shows that for cargo an exposure curve with a parameter 6.5 is rather light and a parameter of 5.5 with an assumed underlying loss ratio of 75% would be more suited.

VI. Conclusion

In a first part the Lloyds market has been presented. It is nowadays a unique market in the world with a genuine history and a particular way of doing business. ANV operates within that market and write in particular specialty risks such as property reinsurance. The features and terms of reinsurance contract are various. Excess of loss reinsurance requires a high level of technical knowledge and understanding of the underlying risks and contract terms. The aim of this work was to build a pricing tool that can take into consideration the excess of loss contract terms, the clients' premium, loss and risk profile information in order to come up with a view on the profitability of the contract for ANV. The 3 ways to price excess of loss have been embedded within the tool and they complement each other. The pricing tool is used on a daily basis by the underwriting team and each quotes and/or bound contract are recorded in a central database. The collection of data from multiple clients helps to parameterise the exposure curve and frequency severity model and provides benchmarks for clients with poor or no loss history.

VII. Appendix

A. Lloyd's market glossary

Policyholders

Policyholders request insurance cover. Businesses, organisations, other insurers and individuals from around the world want to protect themselves against risks that could affect them. They approach a broker and explain their individual needs.

Local Brokers

Any insurance broker can access the expertise and resources of Lloyd's by making contact with an accredited Lloyd's broker.

Lloyd's Brokers

Accredited Lloyd's brokers place risks in the Lloyd's market on behalf of clients. These brokers use their specialist knowledge to negotiate competitive terms and conditions for clients. Currently there are over 180 firms of brokers working at Lloyd's, many of whom specialise in particular risk categories.

Coverholders

Coverholders place the risks. They are companies authorised by a managing agent to enter into contracts of insurance and/or issue insurance documentation, on behalf of the members of a syndicate.

Service Companies

Service companies place risk and are approved coverholders that Lloyd's has classified as a 'service company' by reason of it being a wholly owned subsidiary of either a managing agent or its holding company.

Syndicates

A Lloyd's syndicate is made up of one or more members that join together as a group to accept insurance risks. They operate on an ongoing basis, although they are technically annual ventures. Members have the right but not an obligation to participate in syndicates for the following year.

In practice, most syndicates are usually supported by the same capital providers for several years. The stability of the core capital providers mean syndicates function like permanent insurance operations, under the Lloyd’s umbrella.

Syndicates tailor solutions to respond to the specific risks of the client base and compete for business, thus offering choice, flexibility and continuing innovation. Syndicates cover either all or a portion of the risk and are staffed by underwriters, the insurance professionals on whose expertise and judgement the market depends.

Managing Agents

A managing agent is a company set up to manage one or more syndicates, on behalf of the members who provide the capital. The managing agent employs the underwriting staff and handles the day-to-day running of a syndicate’s infrastructure and operations.

Often a single corporate group will manage and fund a syndicate, thereby aligning the management and capital provision. For other syndicates, a number of different members – which can include both private capital and corporate groups – not connected with the managing agent provide the capital (these are known as ‘unaligned’ syndicates).

New syndicates are often established under a ‘turnkey’ model, where an existing managing agent establishes and manages the syndicate on behalf of a third party capital provider. After a period of time, the capital provider may seek regulatory approval to establish their own managing agent.

Members of Lloyd's

Members provide the capital to support the syndicates’ underwriting. Members include some of the world’s major insurance groups and companies listed on the London Stock Exchange, as well as individuals and limited partnerships. Corporate members provide most of the capital for the Lloyd’s market.

The Corporation of Lloyd's

The Corporation of Lloyd’s oversees and supports the market, and promotes Lloyd’s around the world. This includes determining the capital that members must provide to support their proposed underwriting, working with the management of underperforming syndicates to improve performance, undertaking financial and regulatory reporting for the Lloyd’s market, managing and developing Lloyd’s global network of licenses and the Lloyd’s brand.

B. R Code

1. Fit lognormal distribution and plot it

To fit

The function below in R, fits a lognormal distribution on two intervals.

```
library(fitdistrplus)
 #create function call to generate the results
 Fitlnorm <- function (claims,start=1,end=1e9,min=1)
{
        #claims is column of claims data

        #only use claims above min value before generating census fit
        #census fit is to claims between start and end., with census below start and above end

        claims<-claims[!is.na(claims) & claims>=min]
   xmin<-start
   xmax<-end
   left<-claims
   right<-claims
   left[left<xmin]<-NA
   right[right<xmin]<-xmin
   right[right>=xmax]<-NA
   left[left>=xmax]<-xmax
   data<-data.frame(left,right)
   f1n<-fitdistcens(data, "lnorm")
   Output<-data.frame(mu=0,sigma=0)
   Output$mu[1]<-f1n$estimate[1]
   Output$sigma[1]<-f1n$estimate[2]
   return (f1n)
}
```

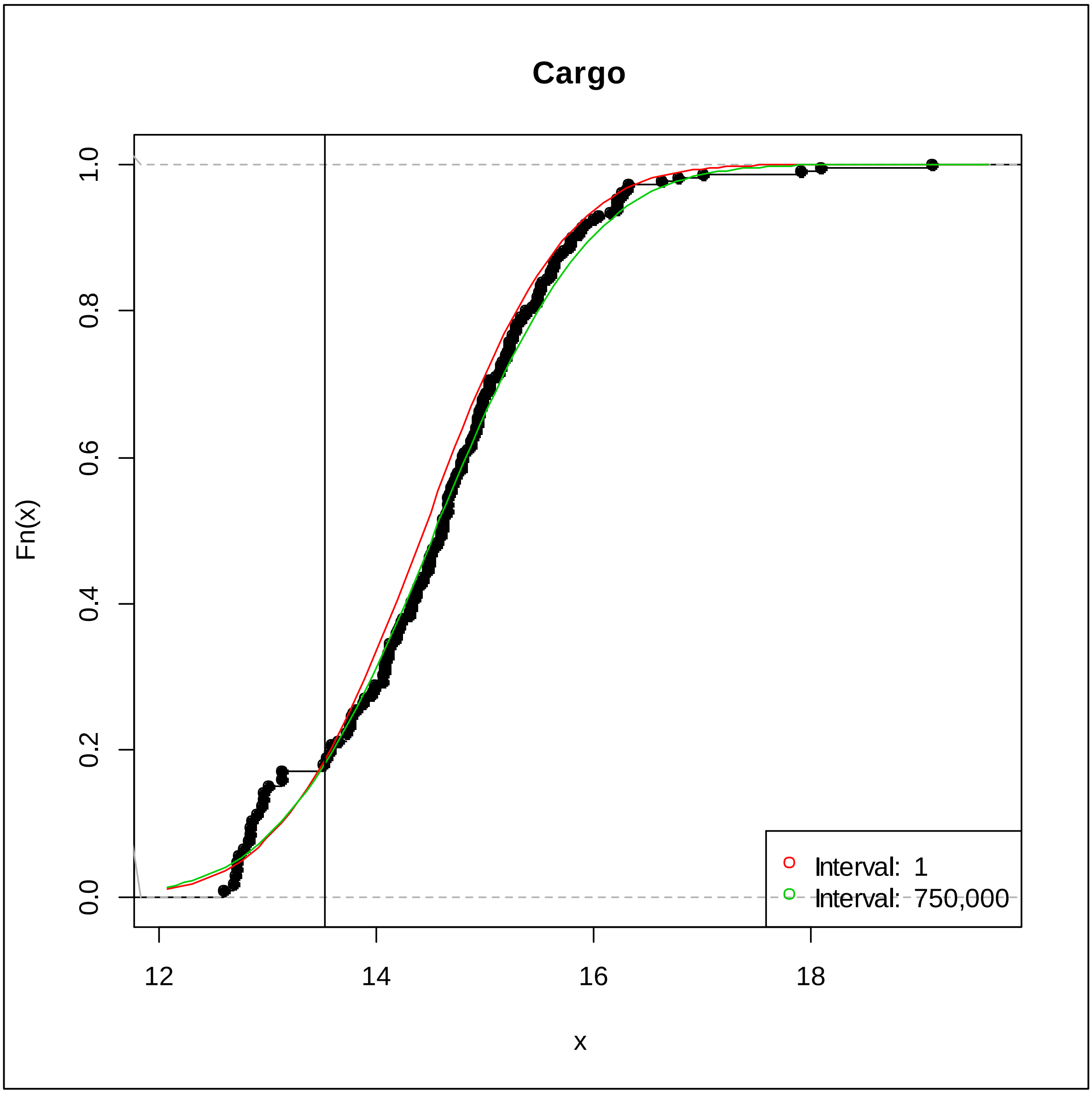
To plot

The following R code plots the fitted lognormal cumulative distribution against the empirical cumulative distribution.

```
fittedplot<-plot(ecdf(log(classlosses$UltimateLoss)), main=ChartName)
x <- seq(from=0, to=20, by=.1)

n <- dim(claim.banding)[1]
legend.data <- data.frame(col=rep(0,n),
        legend=array("a",dim=c(n)))
```

```
for (i in 1:n) {
	fittedplot<-curve(pnorm(x, claim.fitting.out$mu[i],
claim.fitting.out$sigma[i]),add=T,col=(i+1))
	legend.data$col[i] <- i+1
	legend.data$legend[i] <- paste("Interval: ", format(claim.banding$low[i], scientific=FALSE,
big.mark = ",", big.interval = 3))

	fittedplot<-abline (v=log(claim.banding$low[i]))
}
legend("bottomright", pch=1, col=legend.data$col, legend.data$legend)3
```

The output of this code is a graph as shown below that shows the lognormal fit to cargo losses in the first interval (Red line) from [1;750000] and on the second interval (green line)[750000;200000000].



2. Run simulation & calculate recoveries & reinstatement premiums

The R code below is used to simulate the losses and calculate the corresponding recoveries for Risk losses and CAT losses separately. It also calculates the corresponding reinstatement premium. The code enables the user to choose either a Lognormal or Pareto distribution to simulate CAT losses. The Risk losses are simulated using a Lognormal distribution.

```
library(VGAM)
library(MASS)
#Matrix of 15 columns, as up to 15 different layers can be priced. It records the recoveries after application of excess and limit.
RecoveriesPerSim<<-matrix(0,ncol=15,nrow=NbSims)
#AExp function applies the AAD and the AAL to calculate the final recoveries. AExp also calculates the amount of reinstatement
premium(RIP).
AExp <- function (x,AggClaimLimit,AggClaimDeductible,PerClaimLimit,RIPperc,sROL,j){
#x = vector of recoveries after application of excess and limit.
#AggClaimLimit = AAL.
#AggClaimDeductible = AAD.
#PerClaimLimit = Limit.
#PIPperc = Matrix of 5 * 15. This is the percentage at which the reinstatement happens. Up to 5 reinstatement on every layer, up to 15
layers.
#sROL = Vector of 15 rows. This is the Rate On Line for each reinsurance layer.
#j = This specifies which layer is being priced. Up to 15 layers.
 cl<-pmin(pmax(x-AggClaimDeductible,0),AggClaimLimit)
 RecoveriesPerSim[,j]<-cl
 RIP<-rep(0,length(cl))
 v1<-cl
 v2<-pmax(v1-PerClaimLimit,0)
 v3<-pmax(v2-PerClaimLimit,0)
 v4<-pmax(v3-PerClaimLimit,0)
 v5<-pmax(v4-PerClaimLimit,0)
 v6<-pmax(v5-PerClaimLimit,0)
 v7<-pmax(v6-PerClaimLimit,0)
 v<-data.frame(v1,v2,v3,v4,v5,v6,v7)
 w<-data.frame(w1=v1-v2,w2=v2-v3,w3=v3-v4,w4<-v4-v5,w5<-v5-v6,w6<-v6-v7)
 for (i in 1:length(RIPperc)){RIP<-RIP+ w[,i]*RIPperc[i]*sROL}
 AExp<-rep(0,3)
 AExp[1]<-mean(cl)
 AExp[2]<-sd(cl)
 AExp[3]<-mean(RIP)
 return(AExp)
}
#PerClaimToLayer applies the excess and the limit to every loss, then it calls AExp function to apply AAD and AAL and calculates RIP.
PerClaimToLayer <- function (x,PerClaimLimit,PerClaimExcess,AggClaimLimit,AggClaimDeductible,RIPperc,sROL,j){
#x = vector of simulated losses.
#AggClaimLimit = AAL.
#AggClaimDeductible = AAD.
#PerClaimLimit = Limit.
#PerClaimExcess = Excess.
#PIPperc = Matrix of 5 * 15. This is the percentage at which the reinstatement happens. Up to 5 reinstatement on every layer, up to 15
layers.
#sROL = Vector of 15 rows. This is the Rate On Line for each reinsurance layer.
#j = This specifies which layer is being priced. Up to 15 layers.
 x<-colSums(pmin(pmax(x-PerClaimExcess,0),PerClaimLimit),na.rm=T)
 PerClaimToLayer<-AExp(x,AggClaimLimit,AggClaimDeductible,PerClaimLimit,RIPperc,sROL,j)
 return(PerClaimToLayer)
}
#SimX simulates the CAT losses using either a lognormal or pareto distribution depending on the user's decision.
SimX<-function(nclmP,m,odf,nyr,Distrib,AlphaT,Lower,Upper){
#nclmP   = avg no of claims per year in the second interval.
#m       = Matrix that contains the interval information and the parameters of the lognormal distribution on both intervals.
```

```
#odf     = overdispersion factor for NegBin. Fixed at 2 to have a Poisson distribution in fact.
#nyr     = number of simulation. Set to 10 000 by default.
#Distrib = Indicator. 0 means use of Lognormal distribution. 1 means use of Pareto distribution.
#AlphaT = Shape parameter for the Pareto distribution.
#Lower =  Scale parameter for the Pareto distribution.
#Upper =  Upper bound for the Truncated Pareto distribution.

  m<-with(m,m[!lo==hi,])
  global_mcP<<-nclmP
  global_odf<<-odf
  global_m<<-m
  global_Distrib<<-Distrib
  nodes <- list(year = nyr)
#Lognormal distribution to simulate losses.
  if (global_Distrib<1){
    mf <- expression(year = rnegbin(mu=global_mcP,theta=global_mcP/(global_odf-1)))
    ms <- expression(year=rlnorm(meanlog=global_m$mu[2],sdlog=global_m$sigma[2]))
    pf <- simul(nodes, mf, ms)
    rm(list=grep("glob", ls(1), value=T), envir=globalenv())
    sf<-severity(pf, by = "year")
    return(sf$main)
  }
#OR Truncated Pae=reto distribution to simulate losses.
  if (global_Distrib>0){
    mf <- expression(year = rnegbin(mu=global_mcP,theta=global_mcP/(global_odf-1)))
    ms<- expression(year=rtruncpareto(lower=Lower,upper=Upper,shape=AlphaT))
    pf <- simul(nodes, mf, ms)
    rm(list=grep("glob", ls(1), value=T), envir=globalenv())
    sf<-severity(pf, by = "year")   #see severity claims by year
    return(sf$main)
  }
}
#SimXRisk simulates the risk losses using lognormal.
SimXRisk<-function(nclm,m,odf,nyr)    {
#nclm   = avg no of claims per year in the first interval.
#m      = Matrix that contains the interval information and the parameters of the lognormal distribution on both intervals.
#odf    = overdispersion factor for NegBin. Fixed at 2 to have a Poisson distribution in fact.
#nyr    = number of simulation. Set to 10 000 by default.
  m<-with(m,m[!lo==hi,])
  global_mc<<-nclm
  global_odf<<-odf
  global_m<<-m
  nodes <- list(year = nyr)
  mf <- expression(year = rnegbin(mu=global_mc,theta=(global_mc)/(global_odf-1)))
  ms <- expression(year=rlnorm(meanlog=global_m$mu[1],sdlog=global_m$sigma[1]))
  pf <- simul(nodes, mf, ms)
  rm(list=grep("glob", ls(1), value=T), envir=globalenv())
  sf<-severity(pf, by = "year")
  return(sf$main)
}
#Record the parameters for every layer and every class into global vectors.
nclm<<-c(nclm1,nclm2,nclm3,nclm4,nclm5)
nclmP<<-c(nclmP1,nclmP2,nclmP3,nclmP4,nclmP5)
Distrib<<-c(Distrib1,Distrib2,Distrib3,Distrib4,Distrib5)
AlphaT<<-c(AlphaT1,AlphaT2,AlphaT3,AlphaT4,AlphaT5)
Lower<<-c(Lower1,Lower2,Lower3,Lower4,Lower5)
Upper<<-c(Upper1,Upper2,Upper3,Upper4,Upper5)
PerClaimLimit<<-
c(PerClaimLimit1,PerClaimLimit2,PerClaimLimit3,PerClaimLimit4,PerClaimLimit5,PerClaimLimit6,PerClaimLimit7,PerClaimLimit8,PerClaimLi
mit9,PerClaimLimit10,PerClaimLimit11,PerClaimLimit12,PerClaimLimit13,PerClaimLimit14,PerClaimLimit15)
```

```
PerClaimExcess<<-
c(PerClaimExcess1,PerClaimExcess2,PerClaimExcess3,PerClaimExcess4,PerClaimExcess5,PerClaimExcess6,PerClaimExcess7,PerClaimExcess8
,PerClaimExcess9,PerClaimExcess10,PerClaimExcess11,PerClaimExcess12,PerClaimExcess13,PerClaimExcess14,PerClaimExcess15)
AggClaimLimit<<-
c(AggClaimLimit1,AggClaimLimit2,AggClaimLimit3,AggClaimLimit4,AggClaimLimit5,AggClaimLimit6,AggClaimLimit7,AggClaimLimit8,AggCl
aimLimit9,AggClaimLimit10,AggClaimLimit11,AggClaimLimit12,AggClaimLimit13,AggClaimLimit14,AggClaimLimit15)
AggClaimDeductible<<-
c(AggClaimDeductible1,AggClaimDeductible2,AggClaimDeductible3,AggClaimDeductible4,AggClaimDeductible5,AggClaimDeductible6,Agg
ClaimDeductible7,AggClaimDeductible8,AggClaimDeductible9,AggClaimDeductible10,AggClaimDeductible11,AggClaimDeductible12,AggCl
aimDeductible13,AggClaimDeductible14,AggClaimDeductible15)
sROL<<-as.numeric(c(sROL1,sROL2,sROL3,sROL4,sROL5,sROL6,sROL7,sROL8,sROL9,sROL10,sROL11,sROL12,sROL13,sROL14,sROL15))
RIPperc<<-
data.frame(RIPperc1,RIPperc2,RIPperc3,RIPperc4,RIPperc5,RIPperc6,RIPperc7,RIPperc8,RIPperc9,RIPperc10,RIPperc11,RIPperc12,RIPperc13
,RIPperc14,RIPperc15)
#Master function to simulate Risk and CAT losses for every class (up to 5) and calculate the recoveries on each layer (up to 15).
MasterSimX<-function(InclusionMatrix,LayerToPrice,NbSims){
#InclusionMatrix = 5 * 15 matrix with 0 or 1 to indicate which classes are included in which layers.
#LayerToPrice = Vector of 15 rows with 0 or 1 to indicate which layer has to be prioced.
#Final matrix to collect recoveries from the Risk losses and the CAT losses.
output<<-
data.frame(Recoveries=rep(0,15),sdRecoveries=rep(0,15),RIP=rep(0,15),RecoveriesRisk=rep(0,15),sdRecoveriesRisk=rep(0,15),RIPRisk=rep(0
,15))
 for (j in 1:15){
  if(LayerToPrice[j,1]>0){
   Snclm<-nclm * InclusionMatrix[j,]
   SnclmP<-nclmP * InclusionMatrix[j,]
   X<-matrix(0,ncol=NbSims,nrow=2)
   Y<<-matrix(0,ncol=NbSims,nrow=2)
   if(as.numeric(InclusionMatrix[j,1])>0){
    X1<-SimX(as.numeric(SnclmP[1]),m1m,2,NbSims,Distrib[1],AlphaT[1],Lower[1],Upper[1])
    colnames(X)<-colnames(X1)
    X<-rbind(X,X1,deparse.level=0)
    Y1<-SimXRisk(as.numeric(Snclm[1]),as.numeric(SnclmP[1]),m1m,2,NbSims,Distrib[1],AlphaT[1],Lower[1],Upper[1])
    colnames(Y)<-colnames(Y1)
    Y<-rbind(Y,Y1,deparse.level=0)
    X<-rbind(X,Y1,deparse.level=0)
    }
   if(as.numeric(InclusionMatrix[j,2])>0){
    X2<-SimX(as.numeric(SnclmP[2]),m2m,2,NbSims,Distrib[2],AlphaT[2],Lower[2],Upper[2])
    colnames(X)<-colnames(X2)
    X<-rbind(X,X2,deparse.level=0)
    Y2<-SimXRisk(as.numeric(Snclm[2]),m2m,2,NbSims)
    colnames(Y)<-colnames(Y2)
    Y<-rbind(Y,Y2,deparse.level=0)
    X<-rbind(X,Y2,deparse.level=0)
    }
   if(as.numeric(InclusionMatrix[j,3])>0){
    X3<-SimX(as.numeric(SnclmP[3]),m3m,2,NbSims,Distrib[3],AlphaT[3],Lower[3],Upper[3])
    colnames(X)<-colnames(X3)
    X<-rbind(X,X3,deparse.level=0)
    Y3<-SimXRisk(as.numeric(Snclm[3]),m3m,2,NbSims)
    colnames(Y)<-colnames(Y3)
    Y<-rbind(Y,Y3,deparse.level=0)
    X<-rbind(X,Y3,deparse.level=0)
    }
   if(as.numeric(InclusionMatrix[j,4])>0){
    X4<-SimX(as.numeric(SnclmP[4]),m4m,2,NbSims,Distrib[4],AlphaT[4],Lower[4],Upper[4])
    colnames(X)<-colnames(X4)
    X<-rbind(X,X4,deparse.level=0)
    Y4<-SimXRisk(as.numeric(Snclm[4]),m4m,2,NbSims)
    colnames(Y)<-colnames(Y4)
    Y<-rbind(Y,Y4,deparse.level=0)
```

```
      X<-rbind(X,Y4,deparse.level=0)
      }
     if(as.numeric(InclusionMatrix[j,5])>0){
      X5<-SimX(as.numeric(SnclmP[5]),m5m,2,NbSims,Distrib[5],AlphaT[5],Lower[5],Upper[5])
      colnames(X)<-colnames(X5)
      X<-rbind(X,X5,deparse.level=0)
      Y5<-SimXRisk(as.numeric(Snclm[5]),m5m,2,NbSims)
      colnames(Y)<-colnames(Y5)
      Y<-rbind(Y,Y5,deparse.level=0)
      X<-rbind(X,Y5,deparse.level=0)
      }
      output[j,1:3]<-PerClaimToLayer(X,PerClaimLimit[j],PerClaimExcess[j],AggClaimLimit[j],AggClaimDeductible[j],RIPperc[,j],sROL[j],j)
      output[j,4:6]<-PerClaimToLayer(Y,PerClaimLimit[j],PerClaimExcess[j],AggClaimLimit[j],AggClaimDeductible[j],RIPperc[,j],sROL[j],j)
    }
   }
  return (output)
}
out<-MasterSimX(InclusionMatrix,LayerToPrice,NbSims)
Recoveries<-t(out$Recoveries)
RIP<-t(out$RIP)
RecoveriesRisk<-t(out$RecoveriesRisk)
RIPRisk<-t(out$RIPRisk)
```

3. Plot survival distributions and other

```
#Plot lognormal and pareto on same graphs
PL<-par(mfrow = c(1, 2), cex = 0.7)
z<-classlosses$UltimateLoss
g<-Ecdf(z,what="1-F",xlim=c(0,Lower[1]),xlab="Loss Amount",ylab="Survival Probability",main="Fitting of lognormal to
empirical distribution")
l<-rlnorm(100000,meanlog=m1m$mu[1],sdlog=m1m$sigma[1])
g<-Ecdf(l,what="1-F",add=T,col="blue")
legend.data2 <- data.frame(col=rep(0,2),legend=array("a",dim=c(2)))
legend.data2$col[2] <-"blue"
legend.data2$legend[2]<-"fitted lognormal"
legend.data2$col[1] <-"black"
legend.data2$legend[1]<-"empirical distribution"
legend("topright", pch=1, col=legend.data2$col, legend.data2$legend)
#g<-abline(v=Lower[1])
x<-z[z>Lower[1]]
f<-Ecdf(x,what="1-F",xlim=c(Lower[1],Upper[1]),xlab="Loss Amount",ylab="Survival Probability",main="Fitting of pareto to
empirical distribution")
y<-rtruncpareto(100000,lower=Lower[1],upper=Upper[1],shape=AlphaT[1])
f<-Ecdf(y,what="1-F",add=T,col="red")
legend.data2 <- data.frame(col=rep(0,2),legend=array("a",dim=c(2)))
legend.data2$col[2] <-"red"
legend.data2$legend[2]<-"fitted truncated pareto"
legend.data2$col[1] <-"black"
legend.data2$legend[1]<-"empirical distribution"
legend("topright", pch=1, col=legend.data2$col, legend.data2$legend)

#Comparing lognormal vs pareto at the tail
z<-classlosses$UltimateLoss
PL<-par(mfrow = c(1, 1), cex = 0.7)
x<-z[z>Lower[1]]
```

```
f<-Ecdf(x,what="1-F",ylim=c(0,0.05),xlim=c(Lower[1],Upper[1]),xlab="Loss Amount",ylab="Survival
Probability",main="Comparison of distributions at the tail")
y=rtruncpareto(100000,lower=Lower[1],upper=Upper[1],shape=AlphaT[1])
f<-Ecdf(y,what="1-F",add=T,col="red")
p=rpareto(100000,scale=Lower[1],shape=AlphaT[1])
f<-Ecdf(p,what="1-F",add=T,col="green")
l<-rlnorm(100000,meanlog=m1m$mu[1],sdlog=m1m$sigma[1])
f<-Ecdf(l[l>Lower[1]],what="1-F",add=T,col="blue")
legend.data2 <- data.frame(col=rep(0,4),legend=array("a",dim=c(4)))
legend.data2$col[2] <-"red"
legend.data2$legend[2]<-"fitted truncated pareto"
legend.data2$col[1] <-"black"
legend.data2$legend[1]<-"empirical distribution"
legend.data2$col[3] <-"blue"
legend.data2$legend[3]<-"fitted lognormal"
legend.data2$col[4] <-"green"
legend.data2$legend[4]<-"fitted pareto"
legend("topright", pch=1, col=legend.data2$col, legend.data2$legend)

#Plot mean excess, hill, QQ plot, empirical
PL<-par(mfrow = c(2, 2), cex = 0.7)
z<-classlosses$UltimateLoss
Ecdf(z,what="1-F",main="Empirical distribution")
mePlot(z)
qqparetoPlot(z[z>Lower[1]],xi=AlphaT[1])
hillPlot(z,start=5,ci=0)
```

C. The Rating Engine

1. Intro and assumptions

On the front sheet of the rater, the user specifies key policy information. He then has to choose which line of business to price. In this case Cargo. A series of assumptions has then to be made such as the loss ratio of the underlying business. In this example 75%. This assumption is used in the exposure rating. The loss ratios are based on the Lloyd's market benchmark. If the book being priced has a difference performance the benchmark can be overwritten.

The exposure rating of business is an assumption regarding the propensity for the book to experience large losses. It can be better than average, average or worse than average. This determines the parameter of the exposure curve to be used. For Cargo, an average rating means that an exposure curve with a parameter c=6.5 would be used. For a better than average business the parameter would be 7 and for a worse than average 6.

The loss ratio rating of the business specifies the level of the loss ratio. There are 3 levels. LMX referring to the Lloyd's market and which is the highest loss ratio. Global is 5% point lower than the benchmark. It corresponds to clients that have a global presence hence benefitting from a geographical diversification. Foreign domestic correspond to local carriers. As these clients are local, they know very well the market they operate in, hence their risk selection is of better quality as they sit closer to the risks. The loss ratio for foreign domestic clients is 10% point lower than the benchmark.

The maximum line size has then to be specified and the income for each year is recorded.

ANV Syndicate Management Limited

Excess of Loss Rating Engine

Policy Information

Risk ID		Underwriter	Actuarial
Assured Name	All Cargo		
Inception Date	17/05/2015		
New/Renewing			
Currency	USD		
Quote Ind	Quote		

Class Profile

	Class 1	Class 2	Class 3	Class 4	Class 5
Name	Cargo				
Benchmark LR :	75,0%				
Manual LR					
Selected LR:	75,0%				

Exposure Rating of Business	Average
Loss Ratio Rating of Business	LMX
Maximum line size	400 000 000

Income

YOA	Cargo
2005	169 308 425
2006	648 424 533
2007	728 112 612
2008	1 001 591 021
2009	1 311 357 468
2010	1 587 196 120
2011	1 677 618 335
2012	1 773 742 338
2013	1 783 621 923
2014	1 808 200 347
2015	1 911 728 923

2. Layers

Up to 15 layers can be priced. A limit, and excess point is specified for each layer. An AAD can also be added on each layer. The AAL is calculated based on the number of reinstatements. On the first layer the AAL is $40m, this means that there are 3 reinstatements. Each layer can cover up to 5 different classes. In this example it only covers the Cargo book.

ANV Syndicate Management Limited

Excess of Loss Rating Engine

Please Press Ctrl + Alt +F9 to Recalculate

Always recalculate the work book

Save | Update | ResetFields

Layer Profile

	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Lay
Attached Classes	Cargo	Cargo	Cargo	Cargo	Cargo	
Limit	10 000 000	20 000 000	60 000 000	300 000 000	600 000 000	
Excess	10 000 000	20 000 000	40 000 000	100 000 000	400 000 000	
AAD	10 000 000	0	0	0	0	
AAL	40 000 000	60 000 000	120 000 000	300 000 000	600 000 000	
Limit xs Excess xs AAD	10m xs 10m xs 10m	20m xs 20m xs 0m	60m xs 40m xs 0m	300m xs 100m xs 0m	600m xs 400m xs 0m	
ANV Share	5,0%	5,0%	5,0%	5,0%	5,0%	

3. Claims On Levelling

Claims information is recorded in the first 3 columns. If a claim is closed then no development factor is applied to it. An inflation factor of 2% is applied. The ultimate loss columns shows the on levelled claims. The last column is used to flag the claims that are CAT.

211 Loss Count 211

Suggested Inflation: 3,0%

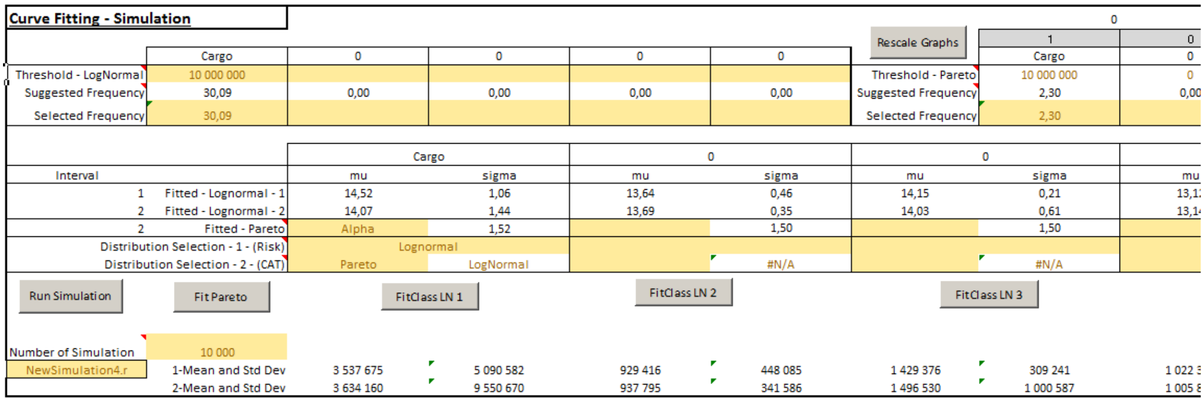
Cargo

Claims threshold 200 000 | Selected Inflation 2,0%

YOA	Incurred	Status	LDF	LDF Selected	Comments	Inflation Factor	UltimateLoss	CAT Indicator
2009	839 089		1,00			1,13	944 950	
2009	2 320 747		1,00			1,13	2 613 538	
2010	981 262		1,00			1,10	1 083 609	
2011	4 644 952	Closed	1,00			1,08	5 027 845	
2012	5 145 879	Closed	1,00			1,06	5 460 848	
2013	11 726 381	Closed	1,00			1,04	12 200 127	
2010	3 891 928		1,00			1,10	4 297 862	
2011	2 752 928		1,00			1,08	2 989 095	
2012	8 638 839		1,01			1,06	9 303 286	
2013	3 418 082		1,06			1,04	3 754 251	
2013	10 000 000		1,06			1,04	10 983 503	
2013	2 607 144		1,06			1,04	2 863 557	
2014	8 566 129	Open	1,26			1,02	11 011 810	
2011	2 129 263	Closed	1,00			1,08	2 304 783	
2011	1 800 269	Closed	1,00			1,08	1 948 669	
2007	1 637 440	Closed	1,00			1,17	1 918 522	

4. Curve fitting

The screenshot below shows that for Cargo the threshold for Risk vs. CAT distribution has been set at $10m. On average, adjusted for the exposure, there are 30.09 claims below $10m and 2.3 claims above $10m. By clicking the relevant button, a Lognormal distribution is fitted to claims below $10m and above $10m. A Pareto curve is also fitted on claims above $10m. For the Risk type losses the Lognormal is chosen and for the CAT type losses the Pareto is chosen. 10000 simulations are run in R. The process starts by clicking the button "Run Simulation".



Curve Fitting - Simulation

	Cargo	0	0	0	0	Rescale Graphs	1 / Cargo	0 / 0
Threshold - LogNormal	10 000 000					Threshold - Pareto	10 000 000	0
Suggested Frequency	30,09	0,00	0,00	0,00	0,00	Suggested Frequency	2,30	0,00
Selected Frequency	30,09					Selected Frequency	2,30	

Interval		Cargo mu	Cargo sigma	0 mu	0 sigma	0 mu	0 sigma	mu
1	Fitted - Lognormal - 1	14,52	1,06	13,64	0,46	14,15	0,21	13,1
2	Fitted - Lognormal - 2	14,07	1,44	13,69	0,35	14,03	0,61	13,1
2	Fitted - Pareto	Alpha	1,52		1,50		1,50	
	Distribution Selection - 1 - (Risk)	Lognormal						
	Distribution Selection - 2 - (CAT)	Pareto	LogNormal		#N/A		#N/A	

Run Simulation | Fit Pareto | FitClass LN 1 | FitClass LN 2 | FitClass LN 3

Number of Simulation	10 000							
NewSimulation4.r	1-Mean and Std Dev	3 537 675	5 090 582	929 416	448 085	1 429 376	309 241	1 022 3
	2-Mean and Std Dev	3 634 160	9 550 670	937 795	341 586	1 496 530	1 000 587	1 005 8

5. Credibility weighting between methods

In the example below the risk and cat results are presented in the rater for each layer and for each method. For the risk component the result is shown in terms of recoveries. For the CAT component the result is shown in terms of LOL. Weightings are then chosen for the risk & the CAT component. The credibility theory is applied in the background in order to give the modeller a suggestion in terms of weights to apply to experience rating vs. exposure rating. In the example below it shows 70% to the experience vs. 30% to the exposure.

Summary

NON-CAT	Burning Cost	826,678	6,731	0	0	0
	Exposure Rating	4,551,601	6,957,928	4,690,380	1,594,082	0
	Simulation	1,796,233	2,114,559	470,294	18,480	0
CAT	Burning Cost	7.87%	46.06%	26.50%	4.38%	0.00%
	Suggested CAT ROL (Expo)	15.04%	3.69%	0.69%	0.22%	0.00%
	Suggested CAT ROL (Sim)	34.43%	35.18%	14.43%	3.42%	0.76%
	UW Cat ROL					
	Comment					
	Selected Cat ROL	34.43%	35.18%	14.43%	3.42%	0.76%
AAD proxy	Benefit of AAD (Amts)	0	0	0	0	0
	Benefit of AAD (ROL)	0.00%	0.00%	0.00%	0.00%	0.00%
Extra	Additional pricing (ROL)					
	Comment					

Weightings

NON-CAT	Suggested Burning Cost	70%	50%	0%	0%	0%
	Suggested Exposure Rating	30%	50%	100%	100%	100%
	UW Burning Cost	0%	0%	0%	0%	0%
	UW Exposure Rating	0%	0%	0%	0%	0%
	UW Simulation	100%	100%	100%	100%	100%
CAT	CAT ROL (BC)	0%	0%	0%	0%	0%
	CAT ROL (Expo)	0%	0%	0%	0%	0%
	CAT ROL (Sim)	100%	100%	100%	100%	100%
	CAT ROL (UW)	0%	0%	0%	0%	0%

6. Results

Once the credibility weighting for risk & CAT element between the different methods is done, the reinstatement count on each layer is chosen together with the percentage at which the price is reinstated. On the example below the first layer has 3 reinstatement at 100% each. The initial LOL was 27.6%. The reinstatements brought it downs to 21.6%. If the market price for that layer was 35% ROL and that 10% brokerage fee was applied then the final loss ratio to the reinsurer would be 68.7%. If the target loss ratio for the business was 74.7% then it represents a favourable deviation from the business plan of 8.8%. Overall the programme loss ratio is at 31.5%, even if money is lost on the 2nd layer the profitability on layer 4 & 5 provides an overall return to the reinsurer for the whole programme that is 136.8% better than the business plan.

	Reinstatement Count	@	@	@	@	@
RIPs in terms	1	100%	100%	100%	100%	100%
	2	100%	100%	100%	100%	100%
	3	100%	100%	100%	100%	100%
	4	100%	100%	100%	100%	100%
	5	100%	100%	100%	100%	100%
	Selected Loss Cost	**2,759,798**	**8,794,232**	**11,631,871**	**7,662,355**	**2,372,745**
ROL with RIPs	LOL	27.6%	44.0%	19.4%	2.6%	0.4%
	LOL with 1 Reinst	22.2%	32.4%	16.5%	2.5%	0.4%
	LOL with 2 Reinst	21.7%	30.8%	16.3%	2.5%	0.4%
	LOL with 3 Reinst	21.6%	30.6%	16.2%	2.5%	0.4%
	LOL with 4 Reinst	21.6%	30.5%	16.2%	2.5%	0.4%
	LOL with 5 Reinst	21.6%	30.5%	16.2%	2.5%	0.4%
	Selected	LOL with 3 Reinst	LOL with 2 Reinst	LOL with 1 Reinst	LOL	LOL
	Loss Cost Selected	**2,163,233**	**6,157,203**	**9,889,109**	**7,662,355**	**2,372,745**
	ROL	**21.6%**	**30.8%**	**16.5%**	**2.6%**	**0.4%**
	GG Slip Premium	3,500,000	6,000,000	15,000,000	45,000,000	30,000,000
	Slip ROL	35.0%	30.0%	25.0%	15.0%	5.0%
	Brokerage	10%	10%	10%	10%	10%
	GN Slip Premium	**3,150,000**	**5,400,000**	**13,500,000**	**40,500,000**	**27,000,000**
	ANV GN Slip Premium	157,500	270,000	675,000	2,025,000	1,350,000
	Loss Ratio	68.7%	114.0%	73.3%	18.9%	8.8%
	Income Warning	**4,477,500**	**This needs actuarial review**			
	SBF LR	**74.7%**	**74.7%**	**74.7%**	**74.7%**	**74.7%**
	SBF Premium	2,895,894	8,242,574	13,238,432	10,257,503	3,176,365
	Deviation from SBF LR	**8.8%**	**-34.5%**	**2.0%**	**294.8%**	**750.0%**
	Programme Premium	4,477,500	4,477,500	4,477,500	4,477,500	4,477,500
	Programme LR	31.5%	31.5%	31.5%	31.5%	31.5%
	Programme Dev. from SBF	**136.8%**	**136.8%**	**136.8%**	**136.8%**	**136.8%**
	Deviation warning from SBF					
	UW Comments -deviation from SBF -any other					

VIII. Bibliography

Actuarial papers:

- Stefan Bernegger (1997):
  "The swiss re exposure curves and the MBBEFD distribution class"
  Casualty Actuarial Society
  www.casact.net/library/astin/vol27no1/99.pdf

- Howard C. Mahler (1986):
  "An actuarial note on credibility parameters"
  Casualty Actuarial Society
  www.casualtyactuaries.com/pubs/proceed/proceed86/86001.pdf

- *David R. Clark*
  "Basics of reinsurance pricing"
  Casualty Actuarial Society
  https://www.casact.org/library/studynotes/clark6.pdf

- *David R. Clark*
  "Credibility for a Tower of Excess Layers"
  Casualty Actuarial Society
  https://www.casact.org/pubs/forum/11wforumpt2/Clark.pdf

- *Gary Patrick and Isaac mashitz*
  "Credibility for treaty reinsurance excess pricing"
  Casualty Actuarial Society
  www.casualtyactuarialsociety.net/pubs/dpp/dpp90/90dpp317.pdf

- B. Aban, Mark M. Meerschaert, Anna K. Panorska
  "Parameter estimation for the truncated pareto distribution"
  Michigan State University
  www.stt.msu.edu/~mcubed/tpareto.pdf

- *Mark Flower (2006)*
  "Reinsurance pricing: practical issues and considerations"
  Institute and Faculty of Actuaries
  http://www.actuaries.org.uk/research-and-resources/documents/reinsurance-pricing-practical-issues-and-considerations

- *D. E. A. Sanders (1995)*
  "Pricing in the London market"
  Institute and Faculty of Actuaries
  http://www.actuaries.org.uk/research-and-resources/documents/pricing-london-market

- *D. E. A. Sanders, A. N. Hitchcox, R. Manjrekar (1998)*
  "Aspects of pricing in the London market"
  Institute and Faculty of Actuaries
  http://www.actuaries.org.uk/research-and-resources/documents/aspects-pricing-london-market

Actuarial study materials:

- Christian Y. Robert (2014):
  "Theorie des valeuirs extremes"
  ISFA – Universite Lyon 1

- Pierre-E. Therond (2013):
  "Theorie de la credibilite"
  ISFA – Universite Lyon 1

- Pierre Jal
  "Reassurance Non Vie"
  ISFA – Universite Lyon 1

Websites:

Lloyd's – The world's specialist insurance market
www.lloyds.com